Eur. Phys. J. C manuscript No.
(will be inserted by the editor)

Problems of studying $\gamma\gamma \to \gamma\gamma$ scattering at e^+e^-, e^-e^- and $\gamma\gamma$ colliders

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Received: date / Accepted: date

Abstract The process of elastic scattering of photons (scattering of light by light) has attracted much attention in recent years. It goes through a loop where all the charged particles contribute to the cross section. To date, this $\gamma\gamma \to \gamma\gamma$ process has been experimentally studied in the Delbrück scattering and splitting of photons in the Coulomb field of the nucleus, as well as at the LHC in the scattering of virtual photons in ion-ion collisions. Hopes for further study of this process are associated with high-luminosity e^+e^- colliders (SuperKEKb, FCC, CEPC, ILC, CLIC) and gamma-gamma colliders based on the Compton scattering of laser photons on electrons. In this article, we show that there are some very serious background processes where the annihilation of electrons and positrons (real and virtual) produces a pair of photons flying into the detector, and the the remaining products fly away from the detector at small angles.

1 Introduction

The scattering of light by light in a vacuum is undoubtedly one of the most beautiful processes. The cross section of this process was theoretically predicted more than half a century ago [1–5]. The cross section of the process is very small for optical photons, but it is quite accessible for measurements at photon energies greater than $m_e c^2$.

The interaction of photons goes through the box diagram shown in Fig. 1, where all charged particles contribute to the amplitude proportional to the fourth power of its electric charge (because the diagram has four vertices). The cross section of the process is shown in Fig. 2 (upper) and in Fig. 2 (bottom) multiplied by the energy squared $s = W_{\gamma\gamma}^2$, where one can see steps from various particles. At small energies quarks are still not free, but for simplicity we included u and d quarks at the π-meson mass and s-quark at K-meson mass. Amplitudes from fermions interfere, therefore the cross section is proportional to $(\Sigma e_i^4)^2$. At the $W_{\gamma\gamma} > 2m_W c^2$ the dominant contribution gives W-bosons.

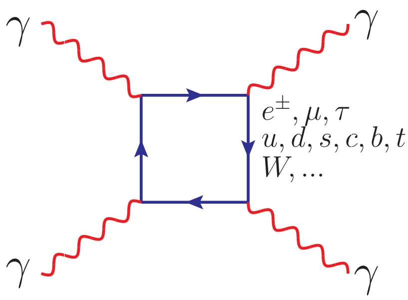

Fig. 1 Diagram for light-by-light scattering.

As one can see, the cross section of $\gamma\gamma$ scattering is sufficiently large at energies of 1–100 MeV. It was proposed half a century ago to study this process at e^+e^- storage rings through collisions of virtual photons, which are almost real [6]. Although two-photons interactions at e^+e^- colliders were actively studied since 1970, but there were no serious attempts to study elastic photon-photon scattering via the box diagram, such processes $\gamma^*\gamma^* \to \gamma\gamma$ were observed only via C-even resonance production.

Related to elastic photon-photon scattering is elastic photon scattering by a nuclei (Delbrück scattering) and photon splitting in a nuclear field. They also proceed through a box diagram. These processes have been thoroughly studied at VEPP-4M in Novosibirsk using a beam of tagged photons with an energy of 120-450 MeV [7, 8].

A new wave of interest to the $\gamma\gamma \to \gamma\gamma$ process is associated with the development of high energy linear e^+e^- colliders and photon colliders [9–21]. Also ion-ion collisions at the LHC present good possibilities for studying light-by-light scattering [22–25].

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After a suggestion by [22], the ATLAS and CMS collaborations performed a first measurement of light-by-light scattering in high-energy heavy-ion collisions at the LHC [26–28] at $W_{\gamma\gamma} \approx 5$ GeV.

There are proposals to create special photon colliders [29–31] to study the process $\gamma\gamma \to \gamma\gamma$ at $W_{\gamma\gamma} \approx 1$ MeV, where the cross section is maximum, about 2 μb.

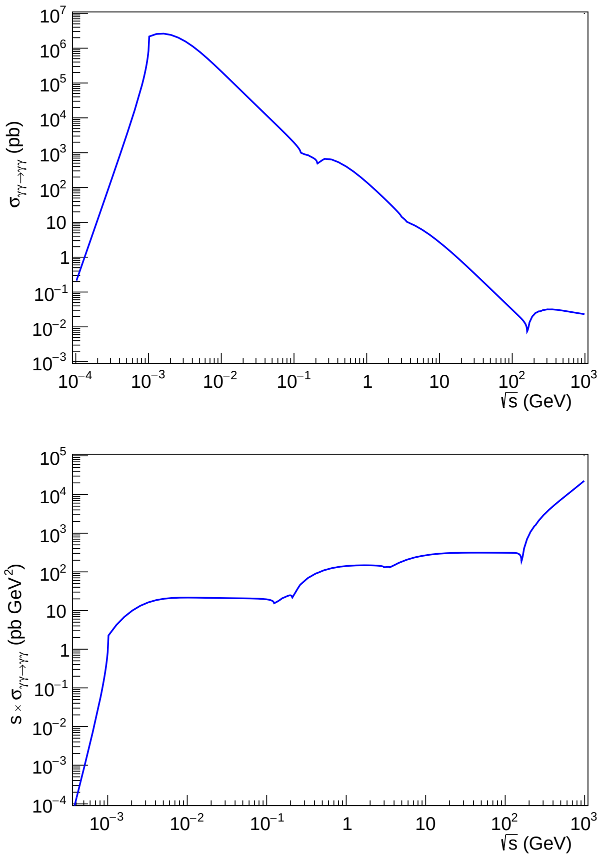


Fig. 2 Cross section for light-by-light scattering.

There is a desire to study this process at e^+e^- colliders with high luminosity SuperKEKB, FCC-ee, CEPC, at future high energy e^+e^- linear colliders ILC and CLIC, at high energy $\gamma\gamma$ colliders, believing that there will be the best conditions for studying $\gamma\gamma \to \gamma\gamma$. Recently, D. d′Enterria and H.S. Shao have considered the possibility to observe the true tauonium in $\gamma\gamma$ collisions at Super KEKB and FCC [32]. The $\gamma\gamma \to \gamma\gamma$ process was considered as one of the main backgrounds along with the decays of resonances into two photons. We doubted this statement. Is $\gamma\gamma \to \gamma\gamma$ scattering really the dominant background process at e^+e^- colliders in the energy range $W_{\gamma\gamma} \sim 3.5$ GeV for a reaction with two finite coplanar photons?

Indeed, the process of $\gamma\gamma \to \gamma\gamma$ at e^+e^- colliders proceed via the diagram shown in Fig. 3. Only photons are detected at large angles, electrons leave the detector at small angles. This process has 6 vertices, therefore its cross-section is proportional to α^6, where $\alpha = e^2/\hbar c \approx 1/137$. At the same time, the process $e^+e^- \to \gamma\gamma$ with two final photons is proportional α^2. Of course, in this case photons have energies equal E_0, such events can be easily cut off. Similarly, if only one electron or an positron emits ISR (initial state radiation) photon at small angle one can kinematically distinguish that the final photons were produced in a collision where one of the initial colliding particles had the energy E_0. This event can be rejected. However, if both e^+ and e^- emit ISR photons, as shown in Fig. 4, then there is no way to distinguish this process with $\sigma \propto \alpha^4$ from the $\gamma\gamma \to \gamma\gamma$ process with $\sigma \propto \alpha^6$. Big trouble! This problem was not noticed by QED experts [6].

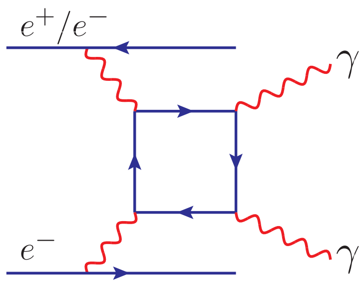


Fig. 3 Diagram for light-by-light scattering in e^+e^- collisions.

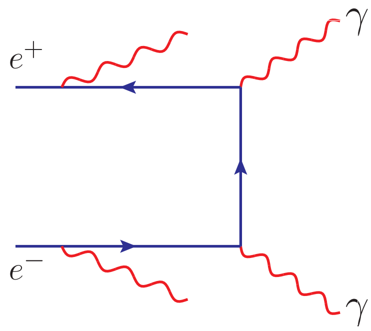


Fig. 4 e^+e^- annihilation diagram in which photons are completely indistinguishable from the case of light by light scattering.

In this paper, we consider these and other QED background for light-by-light scattering $\gamma\gamma \to \gamma\gamma$ at e^+e^-, e^-e^- and $\gamma\gamma$ colliders. For this, we use both analytic considerations and simulations using the CompHEP version 4.5.2 [33]. We assume that two photons after $\gamma\gamma$ scattering are detected at the angles $150^\circ > \theta > 30^\circ$, with all other particles flying in 10° cones around each beam.

2 Cross section for light-by-light scattering at e^+e^-, e^-e^- colliders

The process is described by the diagram shown in Fig. 3. The spectrum of colliding equivalent photons is given by well known formula [34]

$$dn_{\gamma} \approx \frac{\alpha}{\pi}\left[\left(1-x+\frac{x^2}{2}\right)\ln\frac{q^2_{\max}}{q^2_{\min}}-1+x\right]\frac{dx}{x} \equiv f(x)dx, \quad (1)$$

where $x=\omega/E_0$, ω is the energy of an equivalent photon and E_0 is the initial energy of electrons/positrons, $q^2_{\min}=m^2x^2/(1-x)$. The maximum value of $|q|$ is determined by the cut on the acoplanarity angle $\Delta\phi$ between the scattered photons. In our calculations we assumed $\Delta\phi=0.01$. Since the dependence on q^2 is logarithmic, with sufficient accuracy we can put $q_{\max}=(W_{\gamma\gamma}/2)\Delta\phi$, where the invariant mass of the colliding photons $W^2_{\gamma\gamma}\approx 4\omega_1\omega_2$.

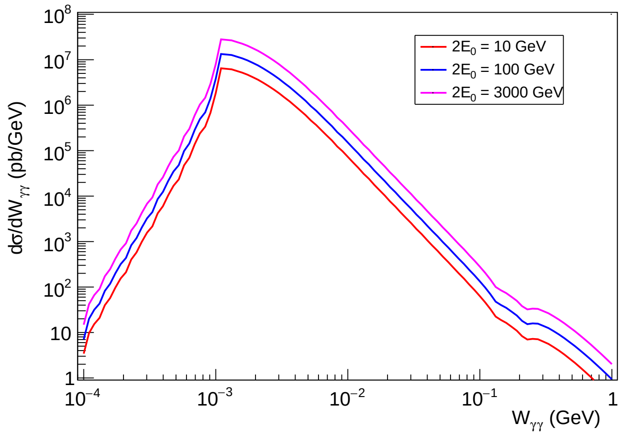
The $\gamma\gamma$ luminosity, normalized to $L_{e^+e^-}$, is given by

$$dL_{\gamma\gamma}=\int dn_1\,dn_2 = 2zdz\int f(x)f\left(\frac{z^2}{x}\right)\frac{dx}{x}, \quad (2)$$

where $z=W_{\gamma\gamma}/2E_0$ and $z^2=xy$. The distribution of luminosity on z is obtained by integrating on x from $x=z^2$ to $x=1$. However, if we restrict the maximum value of the rapidity $|\eta|=(1/2)\ln(x_{\max}/x_{\min})$, then $x_{\min}=ze^{-\eta}$, $x_{\max}=ze^{\eta}$. This rapidity η is connected with the pseudorapidity angle as $\eta=\ln[\tan(\theta/2)]$. The angle 30° corresponds to $\eta=1.31$. So, by restricting the value of η we approximately restrict angles of final photons rather than directly simulating scattered angles. The effective cross section of the process $e^+e^-\to e^+e^-\gamma^*\gamma^*\to e^+e^-\gamma\gamma$.

$$d\sigma/dW=\frac{\sigma_{\gamma\gamma\to\gamma\gamma}(W)}{2E_0}\frac{dL_{\gamma\gamma}}{dz}, \quad (3)$$

where $\sigma_{\gamma\gamma\to\gamma\gamma}(W)$ for $150^\circ>\theta>30^\circ$. This cross section of this process, called the "effect", is shown in Fig. 5. In Fig.6 (curve 1), it is compared with similar distributions of background processes, which we will discuss later.



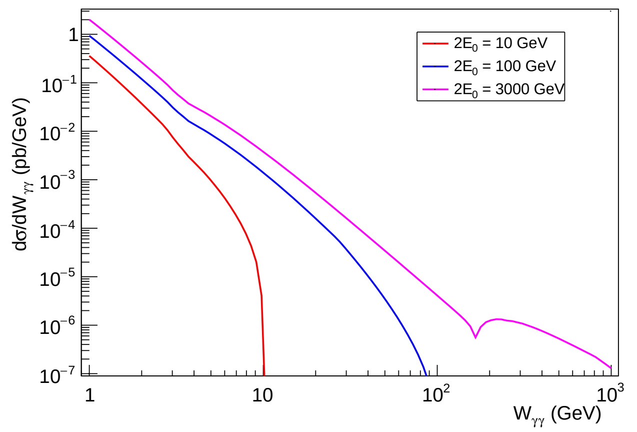


Fig. 5 The effective cross section for light-by-light scattering in e^+e^- collisions.

3 Background process $e^+e^-\to\gamma\gamma\gamma\gamma$

In the process shown in Fig. 4, ana electron and positron emit the ISR photons (at small angles) and then annihilate, producing a pair of photons with large angles. These photons have small total transverse momentum and a small acoplanarity angle, similar to the photons in $\gamma\gamma\to\gamma\gamma$.

The approximate cross section for this process can be found as follows.The spectrum of electrons/positrons after emission ISR photons is given by the same formula as for equivalent photons (1) with the substitution $\omega\to E_0-E$:

$$dn_e\approx\frac{\alpha}{2\pi}\frac{1+x^2}{1-x}\ln\frac{q^2_{\max}}{m^2}dx \quad (4)$$

where $x=E/E_0$. The luminosity distribution $dL_{e^+e^-}/dW_{e^+e^-}$ can be obtained in the same way as it was done in Eq. 2.

The cross section for $e^+e^-\to\gamma\gamma$ above an angle θ_0 is well known [5]

$$\sigma_{e^+e^-\to\gamma\gamma}=\frac{2\pi\alpha^2}{W^2_{e^+e^-}}\left(\frac{1+\cos\theta_0}{1-\cos\theta_0}-\cos\theta_0\right). \quad (5)$$

The cross section for angles $150^\circ>\theta>30^\circ$ is

$$\sigma_{e^+e^-\to\gamma\gamma}=\frac{2.3\times10^{-31}}{W^2_{e^+e^-}[\text{GeV}]^2}\ \text{cm}^2. \quad (6)$$

Similarly to (3)

$$d\sigma/dW=\frac{\sigma_{e^+e^-\to\gamma\gamma}(W)}{2E_0}\frac{dL_{e^+e^-}}{dz}, \quad (7)$$

where $z=W_{e^+e^-}/2E_0$, $L_{e^+e^-}$ for events when both electron and positron emitted ISR photons, normalized to the "usual" luminosity of the collider. This cross section is shown in Fig. 6, curves "2". The conditions used are: rapidity $\eta_{\max}=1.31$, maximum acoplanarity angle $\Delta\phi=0.1$. The strange shape of the curves is due to the fact that at $z=W_{\gamma\gamma}/2E_0<0.27$ the luminosity is limited by the rapidity threshold. In addition, the cross section has a logarithmic divergence as the electron energy approaches E_0. It becomes smoother when

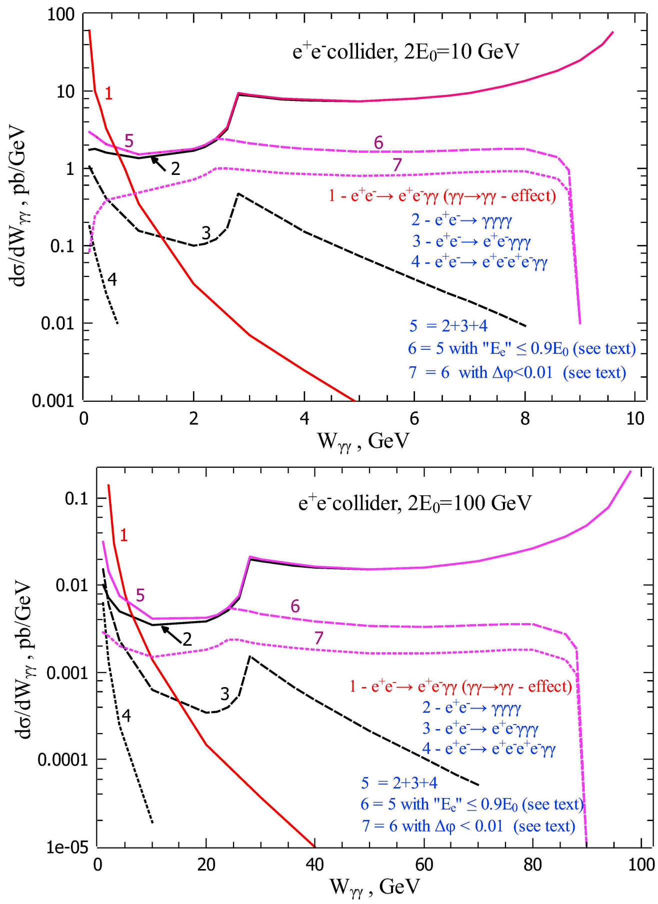


Fig. 6 Cross sections for light-by-light scattering and backgrounds in e^+e^- collisions.

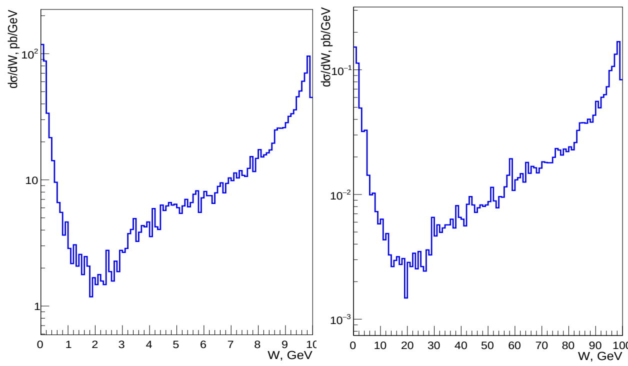


Fig. 7 Cross section for $e^+e^- \to \gamma\gamma\gamma\gamma$ with an angular cut only, see the text.

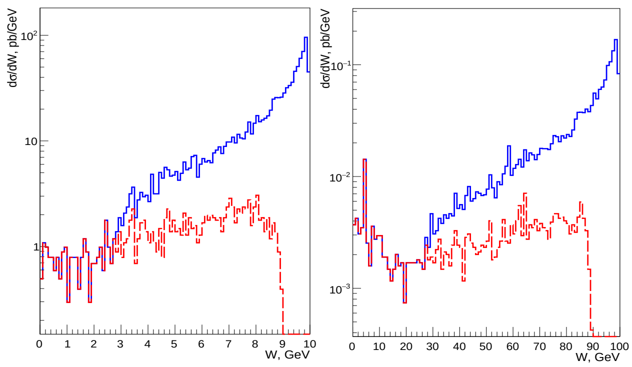


Fig. 8 Cross section for $e^+e^- \to \gamma\gamma\gamma\gamma$ with cuts $|p_{t,1} - p_{t,2}|/(p_{t,1} + p_{t,2} < 0.05$ and $|\Delta\phi| < 0.1$, red curves with $E_e < 0.9E_0$, see the text.

imposing a limit on the maximum energy of colliding electrons/positrons. This can be seen in curve "6" (the sum of all background processes) where the cutoff $E_e < 0.9E_0$ is applied. In the experiment, the energies of colliding particles can be found from the energies and angles of the registered photons. We will return to this figure after considering other background processes.

This important process was also simulated using CompHEP. It was required that two photons have angles $150^\circ > \theta > 30^\circ$ and other two photons have angles $\theta < 10^\circ$ or $\theta > 170^\circ$. Fig.7 shows cross sections with only the angular cuts.

Fig.8 shows the same cross sections as in Fig. 7 with additional cuts on the relative difference of transverse momenta $|p_{t,1} - p_{t,2}|/(p_{t,1} + p_{t,2} < 0.05$ and $|\Delta\phi| < 0.1$, the red curves also have a cut on the maximum energy of colliding electrons (as discussed above) $E_e < 0.9E_0$. Fig. 9 shows same cross sections as Fig. 8, but with a narrow cut on the acoplanarity angle: $|\Delta\phi| < 0.01$. It is seen that a narrower cut on $|\Delta\phi|$ reduces the cross section at small W. The cross sections obtained using CompHEP are close to analytical predictions. Some of the difference is due to different cuts on the angles: in CompHEP the angles are simulated directly, while in analytical calculations the angular cut was done using pseudorapidity angle.

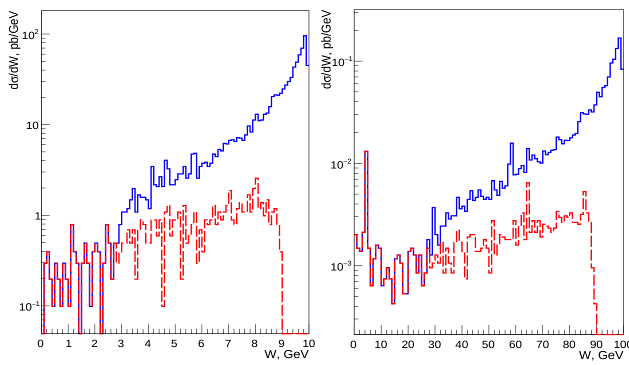


Fig. 9 Cross section for $e^+e^- \to \gamma\gamma\gamma\gamma$ with cuts as in Fig. 8, but smaller acoplanatity angle: $|\Delta\phi| < 0.01$.

Thus, it can be seen that this background process ($\propto \alpha^4$), which exists only at e^+e^- collides (absent in e^-e^- collisions) after all cuts exceeds the effect ($\propto \alpha^6$) at $W > 1$ GeV for $2E_0 = 10$ GeV and at $W > 10$ GeV for $2E_0 = 100$ GeV.

4 Background process $e^+e^- \to e^+e^-\gamma\gamma\gamma$

This process exists (is the same) both at e^+e^- and e^-e^- colliders. The most important diagram of this process is shown

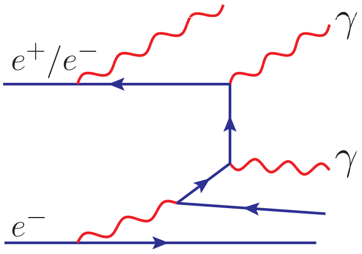


Fig. 10 Annihilation of an electron/positron that has emitted an ISR photon with a positron/electron from a virtual e^+e^- pair.

in Fig. 10. Here e^+ (or e^-), which emitted ISR photon, annihilates with e^- (e^+) from the virtual e^+e^- pair. This background process is proportional to α^5 (while the effect $\propto \alpha^6$). The number and spectrum of electrons emitted ISR photon were discussed earlier and are given by (4). The number of the sea electrons/postrons in the electron is [35, 36]

$$dn_e^e = \left(\frac{\alpha^2}{8\pi^2}\right)\ln^2\left(\frac{q_{\max}^2}{m^2}\right)\times \\ \times\left[2(1+x)\ln x + \frac{4+3x-3x^2-4x^3}{3x}\right]. \quad (8)$$

As before, having the spectra of colliding e^+ and e^-, we can calculate the e^+e^- luminosity distribution, and multiplying it by the $e^+e^- \to \gamma\gamma$ cross section (6), obtain $d\sigma/dW$. The results are shown in Fig. 6, curves "3". The cutoff $E_e < 0.9E_0$ (discussed in the previous section) will further decrease this background (in Fig. 6 this cut is applied only to the sum of all background processes). In any case, it is clear that this background process exceeds the effect approximately at $W > 2$ GeV for $2E_0 = 10$ GeV and at $W > 20$ GeV for $2E_0 = 100$ GeV. Note, that this is true for both e^+e^- and e^-e^- colliders.

5 Background process $e^+e^- \to e^+e^-e^+e^-\gamma\gamma$

The most important diagram of this process is shown in Fig. 11. It is proportional to α^6, as is the effect. The spectrum of the virtual (sea) $e^\pm$ is given by the previous formula (8). Proceeding in the same as for other processes, $d\sigma/dW$ can be calculated; the results are shown in Fig. 6, curves "4". This background is significantly smaller than the effect.

6 Background process $e^+e^- \to e^+e^-\gamma\gamma$

This process is the double bremsstrahlung process shown in Fig.12, and there are about 20 other diagrams of the $e^+e^- \to$

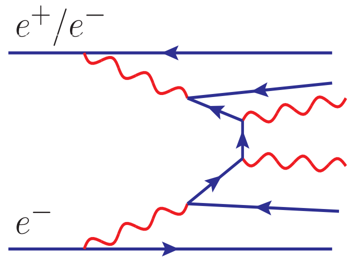


Fig. 11 e^+e^- Annihilation of electrons and positrons from virtual e^+e^- pairs.

$e^+e^-\gamma\gamma$ process. This process is proportional to α^4 (while the effect is proportional to α^6), and has the largest cross section. However, most photons are emitted at small angles, and pairs of photons flying at large angles do not peak at a small total transverse momentum and do not have a small acoplanarity angle. Therefore, this background can be subtracted.It does not add systematic error to the light-by-light scattering, but can increase the statistical error. It is difficult to make any analytical estimates of this process, but it has been simulated using CompHEP. As before, it was re-

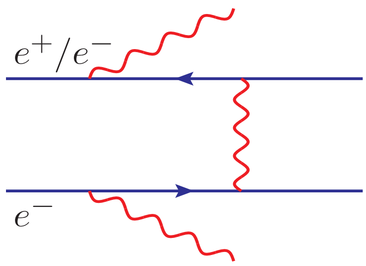


Fig. 12 e^+e^- Main diagram of the processes $e^+e^- \to e^+e^-\gamma\gamma$.

quired that two photons have angles $150^\circ > \theta > 30^\circ$ and the final electron and positron have small angles $\theta < 10^\circ$ or $\theta > 170^\circ$. Cross sections with only these angle cuts are shown in Fig.13.

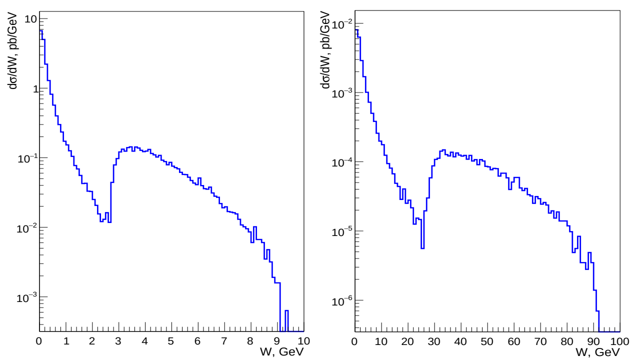


Fig. 13 Cross section for $e^+e^- \to e^+e^-\gamma\gamma$ with an angular cut only, see the text.

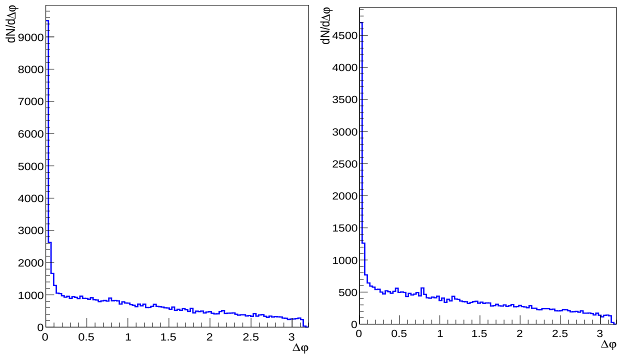


Fig. 14 The distribution on the acoplanarity angle between photons in the process $e^+e^- \to e^+e^-\gamma\gamma$ with an angular cut only, see the text.

The distribution of these event by acoplanarity angle is shown in Fig.14. Unexpectedly, we see photon pairs with small acoplanarity angles. The nature of these events is as follows. This is a process of e^+e^- annihilation into two photons, where one of the initial electrons "emits" a virtual e^+e^- pair in the forward direction, and the other colliding electron/positron has a beam energy E_0. Using energies and angles of detected photons, we can calculate energies of colliding particles and exclude such events. We applied a cut-off $E_e < 0.9E_0$, and events with small $\Delta\phi$ disappeared along with the bumps at large W in the invariant mass distributions in Fig. 13.

The cross section with additional cuts on the relative difference of transverse momenta $|p_{t,1} - p_{t,2}|/(p_{t,1} + p_{t,2}) < 0.05$ and $|\Delta\phi| < 0.1$ is shown in Fig. 15. The distribution of these events is flat at small acoplanarity angles (we do not show this figure), so for $|\Delta\phi| < 0.01$ the cross section will be additionally ten times smaller. Comparing the cross section of this process with the effect and background curves in Fig. 6, one can see that it is small and can be neglected.

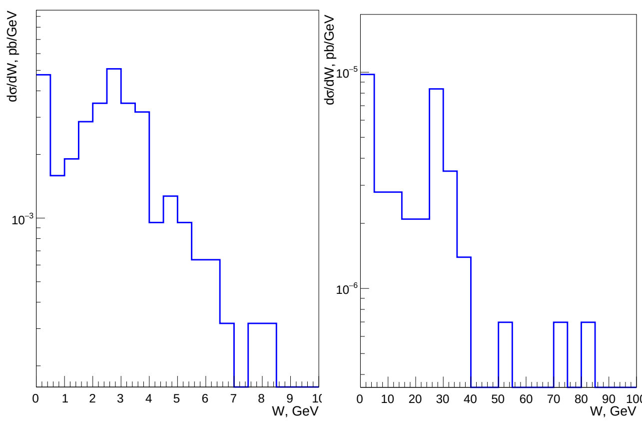


Fig. 15 Cross section for $e^+e^- \to e^+e^-\gamma\gamma$ with the cuts $E_e < 0.9E_0$, $|p_{t,1} - p_{t,2}|/(p_{t,1} + p_{t,2} < 0.05$ and $|\Delta\phi| < 0.1$, see the text.

7 Conclusion

We have found the background processes that significantly limit the possibility to study light-by-light scattering at e^+e^- colliders. Electron and positrons with $E < E_0$ (after emitting ISR photons) annihilate, producing a pair of photons indistinguishable from light-by-light scattering photons. Curve 7 in Fig. 6 shows that studying light-by-light scattering at e^-e^- colliders is possible at energies of approximately $W <$ 1 GeV for $2E_0 = 10$ GeV (SuperKEKB) and at eneries less than $W < 10$ GeV for $2E_0 = 100$ GeV (FCC-ee, CEPC).

The situation is better at e^-e^- colliders, where the main background is the annihilation of an electron that emitted the ISR photon with the virtual positron accompanying the counter electron. This background poses a problem for studying light-by-light scattering at energies of approximately $W >$ 2 GeV for $2E_0 = 10$ GeV and $W > 20$ GeV for $2E_0 = 100$ GeV.

Thus, studying $\gamma\gamma$ scattering at high energies presents a serious challenge for e^+e^- and e^-e^- colliders. Note, both these processes are absent at heavy-ion collisions and at muon colliders.

Photon colliders based on linear colliders offer a good opportunity to study light-by-light scattering at large invariant masses. Here, the background situation is also complicated by possible production of e^+e^- in the conversion region and coherent production of e^+e^- by a photon in the field of the counter beam. However, photon colliders have a high energy peak in the luminosity spectrum, which allows for suppression/subtraction of all backgrounds.

Acknowledgments

This work was supported by the Russian Science Foundation (grant number 24-22-00288).

References

1. H. Euler and B. Kockel, The scattering of light by light in Dirac's theory, Naturwiss. **23** (1935) no.15, 246-247, English translation.
2. W. Heisenberg and H. Euler, Consequences of Dirac's theory of positrons,' Z. Phys. **98** (1936) no.11-12, 714-732, arXiv:physics/0605038.
3. A. Akhiezer, L. Landau and I. Pomeranchuk, Scattering of light by light, Nature **138** (1936) no.3483, 206.1.
4. B. De Tollis, The scattering of photons by photons, Nuovo Cim. **35** (1965) no.4, 1182-1193.
5. V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, Quantum electrodynamics, Pergamon Press, 1982.
6. V. N. Baier, E. A. Kuraev and V. S. Fadin, Possibility of studying the scattering of light by light in colliding

e^+e^- beams, Sov. J. Nucl. Phys. **21** (1975) no.2, 178-182, preprint IYF 74-46
7. S. Z. Akhmadaliev, G. Y. Kezerashvili, S. G. Klimenko, V. M. Malyshev, A. L. Maslennikov, *et al.* Delbrück scattering at energies of 140-450 MeV, Phys. Rev. C **58** (1998), 2844-2850.
8. S. Z. Akhmadaliev, G. Y. Kezerashvili, S. G. Klimenko, R. N. Lee, V. M. Malyshev, A. L. Maslennikov, A. M. Milov, A. I. Milstein, N. Y. Muchnoi and A. I. Naumenkov, *et al.*, Experimental investigation of high-energy photon splitting in atomic fields, Phys. Rev. Lett. **89** (2002), 061802 [arXiv:hep-ex/0111084 [hep-ex]]
9. I. F. Ginzburg, G. L. Kotkin, V. G. Serbo and V. I. Telnov, Pizma ZhETF, 34 (1981) 514; JETP Lett. 34 (1982) 491.
10. I. F. Ginzburg, G. L. Kotkin, V. G. Serbo and V. I. Telnov, *Nucl. Instrum. Meth.* **205** (1983) 47.
11. B. Badelek *et al. Int. J. Mod. Phys. A*, **19** (2004) 5097, [hep-ex/0108012].
12. I. F. Ginzburg and S. L. Panfil, The Possibility of Observation of Heavy Dirac-schwinger Magnetic Pole, Sov. J. Nucl. Phys. **36** (1982), 850.
13. G. Jikia and A. Tkabladze, Photon-photon scattering at the photon linear collider, Phys. Lett. B **323** (1994), 453-458, arXiv:hep-ph/9312228.
14. G. J. Gounaris, P. I. Porfyriadis and F. M. Renard, Light by light scattering at high-energy: A Tool to reveal new particles, Phys. Lett. B **452** (1999), 76-82, [erratum:Phys. Lett. B **513** (2001) 431-431 erratum: Phys. Lett. B **464** (1999), 350-350]. arXiv:hep-ph/9812378.
15. I. F. Ginzburg and A. Schiller, The Visible effect of a very heavy magnetic monopole at colliders, Phys. Rev. D **60** (1999), 075016 Phys. Rev. D **60** (1999), 075016.
16. Z. Bern, A. De Freitas, L. J. Dixon, A. Ghinculov and H. L. Wong, QCD and QED corrections to light by light scattering, JHEP **11** (2001), 031, arXiv:hep-ph/0109079.
17. D. Bardin, L. Kalinovskaya and E. Uglov, Standard Model light-by-light scattering in SANC: analytic and numeric evaluation, Phys. Atom. Nucl. **73** (2010), 1878-1888, arXiv:0911.5634.
18. V. I. Telnov, Gamma-gamma collider with $W_{\gamma\gamma} \leq$ 12 GeV based on the 17.5 GeV SC linac of the European XFEL, JINST **15** (2020) no.10, P10028, arXiv:2007.14003.
19. K. I. Beloborodov, T. A. Kharlamova, G. Moortgat-pick and V. I. Telnov, Opportunities for studying *C*-even resonances at a 3–12 GeV photon collider, Eur. Phys. J. C **83** (2023) no.5, 359, arXiv:2202.04468
20. S. C. İnan and A. V. Kisselev, Polarized light-by-light scattering at the CLIC induced by axion-like particles, Chin. Phys. C **45** (2021) no.4, 043109.
21. J. Ellis, N. E. Mavromatos, P. Roloff and T. You, Light-by-light scattering at future e^+e^- colliders, Eur. Phys. J. C **82** (2022) no.7, 634 arXiv:2203.17111.
22. D. d'Enterria and G. G. da Silveira, Observing light-by-light scattering at the Large Hadron Collider, Phys. Rev. Lett. **111** (2013), 080405 [erratum: Phys. Rev. Lett. **116** (2016) no.12, 129901]
23. M. Kłusek-Gawenda, P. Lebiedowicz and A. Szczurek, Light-by-light scattering in ultraperipheral Pb-Pb collisions at energies available at the CERN Large Hadron Collider, Phys. Rev. C **93** (2016) no.4, 044907, arXiv:1601.07001.
24. M. Kłusek-Gawenda, R. McNulty, R. Schicker and A. Szczurek, Light-by-light scattering in ultraperipheral heavy-ion collisions at low diphoton masses, Phys. Rev. D **99** (2019) no.9, 093013, arXiv:1904.01243 [hep-ph].
25. P. Lebiedowicz and A. Szczurek, The role of meson exchanges in light-by-light scattering, Phys. Lett. B **772** (2017), 330-335, doi:10.1016/j.physletb.2017.06.060 arXiv:1705.06535 [hep-ph].
26. M. Aaboud *et al.* [ATLAS], Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC, Nature Phys. **13** (2017) no.9, 852-858
27. A. M. Sirunyan *et al.* [CMS], Evidence for light-by-light scattering and searches for axion-like particles in ultraperipheral PbPb collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV, Phys. Lett. B **797** (2019), 134826.
28. G. Aad *et al.* [ATLAS], Measurement of light-by-light scattering and search for axion-like particles with 2.2 nb^{-1} of Pb+Pb data with the ATLAS detector, JHEP **03** (2021), 243, [erratum: JHEP **11** (2021), 050]
29. D. Micieli, I. Drebot, A. Bacci, E. Milotti, V. Petrillo, M. R. Conti, A. R. Rossi, E. Tassi and L. Serafini, Compton sources for the observation of elastic photon-photon scattering events, Phys. Rev. Accel. Beams **19** (2016) no.9, 093401.
30. T. Takahashi, G. An, Y. Chen, W. Chou, *et al.* Light-by-light scattering in a photon–photon collider, Eur. Phys. J. C **78** (2018) no.11, 893. [arXiv:1807.00101 [hep-ex]].
31. M. Sangal, C. H. Keitel and M. Tamburini, Observing light-by-light scattering in vacuum with an asymmetric photon collider, Phys. Rev. D **104** (2021) no.11, L111101, arXiv:2101.02671 [hep-ph].
32. D. d'Enterria and H. S. Shao, Observing true tauonium via two-photon fusion at e+e- and hadron colliders, Phys. Rev. D **105** (2022) no.9, 093008 arXiv:2202.02316.
33. CompHEP, version 4.5.2 E. Boos et.al., CompHEP 4.5 Status Report, PoS **ACAT08** (2008), 008,
34. V. M. Budnev, I. F. Ginzburg, G. V. Meledin and V. G. Serbo, The Two photon particle production mechanism. Physical problems. Applications. Equiva-

lent photon approximation, Phys. Rept. **15** (1975), 181-281
35. V. N. Baier, V. S. Fadin and V. A. Khoze, Quasireal electron method in high-energy quantum electrodynamics, Nucl. Phys. B **65** (1973), 381-396.
36. V. S. Fadin and R. E. Gerasimov, Parton distributions in radiative corrections to the cross section of electron-proton scattering, Eur. Phys. J. A **57** (2021) no.3, 86, arXiv:2009.12220.