

Peculiarities of Regge cuts in QCD ¹

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Abstract

As is known from the classical theory of complex angular momenta, along with the Regge poles in the complex j - plane should be cuts that are generated by the poles. In QCD, the cuts are generated by exchanges of the Reggeized gluons, whose properties are strikingly different from the properties of Reggeons in classical theory, which leads to peculiarities of the formation of the cuts. The talk is devoted to the discussion of these peculiarities.

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1 Introduction

First of all, I would like to say a few words about the organizer of this series of conferences, Lev Nikolaevich Lipatov. In quantum chromodynamics (QCD) there are two remarkable equations of evolution, and both of them are associated with his name.

The first of the equations appeared in [1] as the reformulation of the results obtained by V.N. Gribov and L.N. Lipatov in [2], [3] in their investigation of deep inelastic e^-p^+ scattering and e^-e^+ annihilation in model field theories on the parton language. This is the equation for evolution with the transverse momentum Q of the distribution $f_h^a(x, Q^2)$ of partons a carrying the fraction x of the longitudinal momentum of the hadron h :

$$\frac{df_h^a(x, Q^2)}{d \ln Q^2} = \sum_b \int_x^1 \frac{dz}{z} \rho_b^a\left(\frac{x}{z}, Q^2\right) f_h^b f(z, Q^2) .$$

Essentially, the equation is the renormalization group equation and its form is universal. It is only the kernels (splitting functions) $\rho_b^a(\frac{x}{z}, Q^2)$ of the equation that depends on the form of the theory. These kernels were computed in QCD by Y. L. Dokshitzer [4] and independently G. Altarelli and G. Parisi [5], so that the equations are now referred to as DGLAP.

The second - the equation of evolution of the gluon distribution with the fraction of the longitudinal momentum x - was firstly derived by E. A. Kuraev, L. N. Lipatov and me [6] in non-Abelian theories with the Higgs mechanism of mass occurrence to eliminate infrared singularities. Subsequently, its applicability was shown in QCD by I. I. Balitsky and L. N. Lipatov [7], so that it acquired the name BFKL.

QCD is a unique theory in which all its elementary particles (both quarks and gluons) are Reggeized in perturbation theory. The Reggeization of an elementary particle means that the analytical continuation $f(j, t)$ of the t -channel partial wave $f_j(t)$ from physical values of j to the complex angular momentum plane (j -plane), given by the Gribov-Froissart formula [8], coincides at physical j with $f_j(t)$, despite being present in it the term with Kronecker delta-symbol due to exchange of this elementary particle. The gluon Reggeization is extremely important for theoretical description of QCD processes in the Regge kinematics (high energy \sqrt{s} and limited momentum transfer $\sqrt{-t}$). It provides also the factorized form of production amplitudes in the multi-Regge kinematics, where all particles in the final state have limited(not growing with s) transverse momenta and are combined into jets with a limited invariant mass of each jet and large (growing with s) invariant masses of any pair of jets. In particular, it is the basis of the BFKL equation.

The gluon Reggeization provides a simple derivation of the BFKL equation not only in the leading logarithmic approximation (LLA), when in the radiation corrections in each order of perturbation theory only the leading powers of $\ln s$ are retained, but in the next-to-leading logarithmic approximation (NLLA) as well, because due to the Reggeization amplitudes of QCD processes in Regge and multi-Regge kinematics with the adjoint representation of the colour group in cross-channels, which are used in its derivation with the help of the unitarity relations, are determined by the gluon Regge pole and have a simple factorized form (see [9] and links in it).

It is known from the classical theory of complex angular momenta [8], [10], along with moving with t Regge poles (Reggeons), there must be cuts in the j -plane, which are commonly referred to as Regge cuts. It is known also that the cuts generated by the poles are also moving.

In amplitudes with positive signature (symmetry with respect to the replacement $s \leftrightarrow u \simeq -s$), in which the real parts of the main logarithmic terms cancel, Regge cuts appear already in the LLA (in particular, the BFKL Pomeron is a two-Reggeon cut). These cuts appear as the results of solution of the BFKL equation, that has the strongest foundation in this approximation. Therefore, there are no particular problems with study of these cuts.

Problems appear in the next-to-next-to-leading logarithmic approximation (NNLLA), when the cuts appear in amplitudes with negative signature. These cuts can not be analysed using the BFKL equation, because the amplitudes with negative signature are used themselves for the derivation of this equation. Due to signature conservation law Regge cuts in amplitudes with negative signature must be at least three-Reggeon and can appear only in the NNLLA.

Regge cuts violate the pole factorization of amplitudes

$$\mathcal{A}_{gg}^{g'g'} \mathcal{A}_{qq}^{q'q'} = \left(\mathcal{A}_{gq}^{g'q'} \right)^2, \quad (1)$$

so such a violation may indicate the appearance of the cut contributions. The first observation of the violation of the Regge pole factorization was made [11] when considering the two-loop amplitudes for parton (gg, gq and qq) elastic scattering. It was observed in the non-logarithmic (not containing $\ln s$) terms corresponding the two-loop amplitudes to the NNLLA.

It turned out that the terms violating the pole Regge form in the two-loop approximation are infrared singular. In the dimensional regularization with the space-time dimension $D = 4 - 2\epsilon$, they have a pole of second order in ϵ . This means that the the terms violating the Regge pole form can be studied using the infrared factorization techniques.

It was done [12], [13] for two- and three-loop amplitudes of parton scattering. The non-logarithmic double-pole contribution received at two-loops in [11] was confirmed and all non-factorizing single-logarithmic infrared singular contributions at three loops were found.

It was natural to explain the observed violation by Regge cut contributions. The first explanation was done in [14]. It is based on the consideration of Feynman diagrams, and we call it diagrammatic approach. This approach was described in more details in [15].

Note that in contrast to the Reggeized gluon, which contributes only to amplitudes with the adjoint representation of the colour group (colour octet in QCD) in the t -channel, the Regge cuts can contribute to various representations. The Regge cut contributions for all possible colour states in two and three loops were calculated in the diagrammatic approach in [16], and in four loops [17] - [19].

Another approach to calculation of Regge cut contributions was used in [20]. It is based on representation of scattering amplitudes by Wilson lines and using the shock wave approximation, and we call it Wilson line approach. The cut contributions in two and three loops were calculated and the observed violation of the pole Regge form was explained. However, the explanation differs from given in [14]. The main difference between two approaches is the consideration of the colour structure of the cuts. It was subsequently noticed that the colour structure of the cuts assumed in [20] disagrees with the limit of large colour number N_c , what was paid attention to in the papers [17] - [19].

This drawback was eliminated in the papers [21] - [25], where a general recipe for separating pole and cut contributions was proposed and explicit calculations up to four loops were performed. However, the proposed recipe does not have any serious justification and, in our opinion, is not true.

Thus, the properties of the three-Reggeon cuts are poorly understood.

It must be said that also in the classical theory of complex angular momentum, the properties of Regge cuts were studied much less well than the properties of the poles. Unlike

the well-known criteria for the Reggeization of elementary particles in the perturbation theory [26] - [28] there is no criteria for determining the cut. But in QCD it is impossible to use even those few results that were obtained in the classical theory. There are at least two reasons for this. First, in QCD all studies are based on perturbation theory, whereas in the classical theory the main subject of investigation was position and nature of the j -plane singularities which is not related to any fixed order of perturbation theory. Second, unlike the classical theory, where the most important Reggeon was Pomeron with vacuum quantum numbers, having no relation to any elementary particle, in perturbative QCD the basic Reggeon is the Reggeized gluon possessing colour.

2 Regge cuts in the classical theory

Before appearance of QCD, the most effective tool for studying strong interactions at high energies \sqrt{s} was the theory of complex angular momenta j [8], [10]. Significant successes in the systematization of hadrons and in the description of the processes of their interactions at large s were achieved assuming that the only singularities of the partial scattering amplitudes in the complex j -plane are poles (Regge poles, or Reggeons) moving with the square of the transferred momentum t . However, it was soon realized that this assumption contradicts both experiment, where deviations from the pole factorization (1) were observed, and theory, which inconsistency with only pole singularities was proved. It was shown that along with the Regge poles in a complex j -plane there should also be cuts.

The tools of complex angular momenta turned out to be useful in the field theory of elementary particles. In papers [26] - [28], the concept of Reggeization of elementary particles was introduced and it was proved that electron in Quantum Electrodynamics (QED) with massive photon do Reggeize. It was shown [29], [30] that it is not so, however, for photon, so in amplitudes with photon exchanges there is no Regge pole. It was also shown [31], [32] that the high energy asymptotic of the light-light scattering amplitude is determined by a fixed branch point that is not related to moving with t cuts generated by Reggeon exchanges.

A significant difference between Regge cuts in QCD and the classical theory of complex angular momenta is related to different ideas about Regge poles. In the classical theory, they are associated with infinite series of ladder diagrams Fig.1.

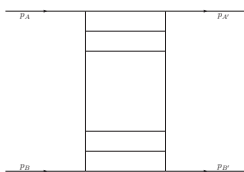


Fig. 1. Feynman diagrams for Regge pole in the classical theory.

Such series give scattering amplitudes with asymptotic $A(s, t) \sim s^{\alpha(t)}$ corresponding to the contribution of the Regge pole with the trajectory $j = \alpha(t)$. It's easy to see that the two-particle intermediate state in the s -channel for the amplitudes with two Reggeons with trajectories

$$\alpha(t) = \alpha(0) + \alpha'(0)t, \quad (2)$$

in the t -channel gives the discontinuity

$$\Delta_s A(s, t) \sim \frac{s^{2\alpha(0)-1-\alpha'(0)t/2}}{2\alpha'(0) \ln s}, \quad (3)$$

which corresponds to moving with t square root branch point at

$$j_c(t) = 2\alpha(0) - 1 - \alpha'(0)t/2. \quad (4)$$

Using this fact and considering two-particle intermediate state in the s -channel unitarity relation for the amplitude corresponding to the diagram Fig.2, D. Amati, S. Fubini and A. Stanghellini [33], [34] came to the conclusion about the existence of the Regge cut with the branch point moving with t and passing through $2\alpha(0) - 1$ for $t = 0$. Such cut and corresponding diagrams Fig.2 are called AFS cuts and AFS diagrams.

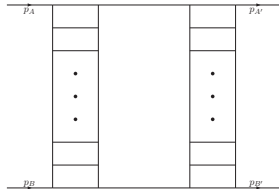


Fig.2. Diagrammatic representation of the AFS cut.

It is worth noting that D. Amati, S. Fubini and A. Stanghellini considered their consideration not as proof of the existence of the cut, but only as an argument in its favour. This argument was soon called into question and the mechanism they proposed for formation of cuts was soon criticized. J. C. Polkinghorne [35] drew attention to the possibility of cancellation of the two-particle cuts by another unitarity cuts, such as three-particle cuts given in Fig.3.



Fig.3. Diagrammatic representation of the three-particle contributions to the discontinuity of the AFS diagrams.

Subsequently mutual cancellation of contributions of all possible unitary cuts and therefore absence of a Regge cut in the total contribution of the AFS diagram was proved in [36]. A similar conclusion was made by S. Mandelstam [37] based on t -channel unitarity.

The cancellation of contributions of various unitary cuts is explained by the planar structure of the AFS diagram. Denoting the amplitude as $A_1(s_1, t_1, t_2)$ where s_1 is the squared invariant mass and t_1 and t_2 are the squared Reggeon momenta, it is easy to obtain, that the total contribution of the AFS diagram is proportional to the integral

$$\int_{-\infty}^{+\infty} ds_1 A_1(s_1, t_1, t_2) , \quad (5)$$

which goes to zero due to the planar structure and decrease of the particle-Reggeon scattering amplitude in the classical theory with increase of the particle-Reggeon invariant mass. Indeed, the integration contour should lie below all singularities for negative s_1 and above for positive. But due to decrease of $A_1(s_1, t_1, t_2)$ at large s_1 one can rotate the right side of the contour in the upper half-plane to the negative half-axis, and to obtain zero because there are no singularities at negative s_1 , due to planarity of the diagram.

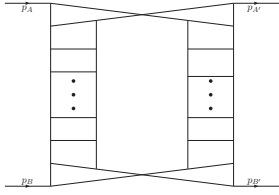


Fig.4. Diagrammatic representation of the Mandelstam cut.

To avoid such nullification S. Mandelstam [38] suggested more complicated diagrams, like on Fig.4, as a source of Regge cuts. The essential point here is that these diagrams are not planar, so that Reggeon-particle scattering amplitude have both right and left cuts.

Since then, AFS type diagrams have been rejected and it was believed that only non-planar diagrams can lead to Regge cuts.

3 j -plane singularities in QCD

In QCD, the situation with the cuts in the complex angular momenta plane is completely different. It differs in many ways, although it seems that in QCD the problem is the same as in the classical theory of complex angular momenta: to study asymptotic behaviour of amplitudes at high energies and fixed momentum transfers. Moreover, Feynman diagrams are used to solve this problem in both cases. But in QCD we calculate the first few terms of the perturbation theory using Feynman rules and want to present the results of the calculations in the form of contributions of singularities in the j -plane. In contrast, in the classical theory of complex angular momentum, one was mainly interested in a position and a degree of singularities, which have nothing to do with the finite orders of perturbation theory.

Next, Regge poles in the classical theory are represented by infinite series of ladder diagrams in model field theories that have no relation to reality. Such representation is used just to illustrate some properties of Regge poles. In particular, it can not be applied to the main Reggeon in the classical theory, which is the Pomeron with the trajectory $j_P(t)$ and positive signature. Instead, the Reggeized gluon with the trajectory $j(t) = 1 + \omega(t)$ and negative signature, which is the main Reggeon in QCD, is well described by Feynman diagrams. But these diagrams have hot ladder structure and start with one-gluon exchange.

Besides this, the important difference is that the Reggeized gluon carries colour, while this quantum number was absent in the classical theory.

3.1 Two-Reggeon cuts in QCD

Regge cuts appear already in the LLA in amplitudes with two Reggeized gluons in the t -channel and positive signature, where real parts of the leading logarithmic terms cancel out, so that remaining piece is pure imagine in the LLA. In particular, the BFLK Pomeron is a two-Reggeon cut.

Using the pole Regge form of elastic and MRK amplitudes, one obtains from the unitarity relation the elastic scattering amplitudes as the convolution

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B} . \tag{6}$$

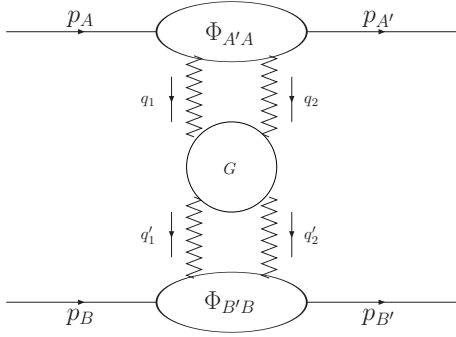


Fig.5. Diagrammatic representation of two-Reggeon exchange.

Here impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \rightarrow A'$ and $B \rightarrow B'$, G is so called Green's function for two interacting Reggeized gluon. In the angular momentum space, it is given by the operator

$$\hat{G} = \frac{1}{j-1-\hat{\mathcal{K}}}, \quad (7)$$

where $\hat{\mathcal{K}}$ presents the BFKL kernel,

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r, \quad (8)$$

$\hat{\omega}_{1,2}$ present the Reggeized gluon trajectories, and $\hat{\mathcal{K}}_r$ stands for so called real part of the BFKL kernel describing interaction of two Reggeized gluons. In the transverse momentum space, in the leading order

$$\omega_i = \omega(-\mathbf{q}_i^2) = -g^2 N_c \mathbf{q}_i^2 \int \frac{d^{2+2\epsilon}l}{2(2\pi)^{(3+2\epsilon)} l^2 (\mathbf{q}_i - \mathbf{l})^2}. \quad (9)$$

For Reggeons with transverse momenta \mathbf{q}_1 and \mathbf{q}_2 and colour indices c_1 and c_2 .

$$[\mathcal{K}_r(\mathbf{q}_1, \mathbf{q}_2; \mathbf{k})]_{c_1 c_2}^{c'_1 c'_2} = -T_{c_1 c'_1}^a T_{c_2 c'_2}^a \frac{g^2}{(2\pi)^{D-1}} \left[\frac{\mathbf{q}_1^2 \mathbf{q}_2'^2 + \mathbf{q}_2^2 \mathbf{q}_1'^2}{\mathbf{k}^2} - \mathbf{q}^2 \right], \quad (10)$$

$$\mathbf{q}'_1 = \mathbf{q}_1 - \mathbf{k}, \quad \mathbf{q}'_2 = \mathbf{q}_2 + \mathbf{k}.$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.

The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory. The kernel is universal (process independent).

In the Pomeron (colour singlet and positive signature) channel the leading singularity is a square root branch branch point [6], [9] at

$$\omega_P = \frac{4N_c \alpha_s}{\pi} \ln 2. \quad (11)$$

Thus, in the BFKL approach, two-Reggeon cuts appear already in the LLA. But their properties differ drastically from two-Reggeon cuts in the classical theory. In particular, the BFKL Pomeron is a fixed branch point (11), instead of moving branch points (4) in the classical theory. Then, it arises due to planar diagrams, which is evident from the fact that its intercept ω_P (11) is not suppressed at large number of the colours N_c . It means that the Mandelstam arguments [37] don't work in QCD. Evidently, this is due to the different nature of Reggeons. It is clear from the lowest order contribution to amplitudes with positive signature, which is given by diagrams with two-gluon exchange. This contribution has both s - and u -channel discontinuities, although it is given by planar diagrams.

It should be noted that (11) gives the position of the leading (rightmost) singularity in the colourless channel. The existence of this singularity does not at all exclude the presence of other singularities. Generally speaking, structure of j -plane singularities is rather complicated

and is different for different representations of the colour group in the t -channel. It was shown [39] in the non-Abelian theories with the Higgs mechanism of mass formation that there are there also moving poles and cuts in amplitudes with positive signature. Unfortunately, structure of j -plane singularities of the amplitudes (6) in QCD is not well investigated.

It may seem that the amplitude (6) contains different from the Reggeized gluon pole singularities also in the channel with gluon quantum numbers and negative signature. It turns out, however, that this is not the case thanks to the bootstrap relations (see [9] and links in it).

These relations are quite simple in the elastic case:

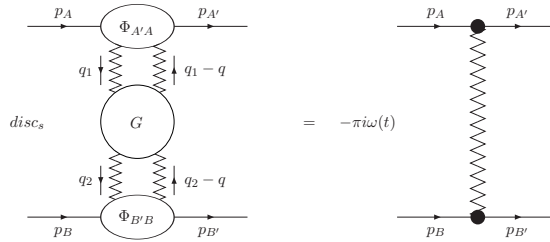


Fig.6. Diagrammatic representation of the bootstrap relation.

There is an infinite number of the bootstrap relations for production amplitudes in the MRK, and they provide the pole Regge form not only in the LLA, but in the NLLA as well (see [9] and links in it).

It should be noted that appearance of a Reggeon with negative signature in the channel with two Reggeons with the same signature is forbidden by the signature conservation law [8]. The point here is that Fig.5 and Eqs. (6-10) do not actually represent the two-Reggeon exchange in the sense of the classical theory. Actually they represent amplitudes, reconstructed by analyticity from the s -channel discontinuities of elastic amplitudes, coming in the unitarity relations from intermediate states with Regge and multi-Regge kinematics. It is the amplitudes that are called in the BFKL approach the amplitudes with the exchange of two Reggeized gluons.

Thus, there is a huge difference between cuts considered in the classical theory and in QCD: in nature of Reggeons, in their interaction, and in their colour. This difference makes impossible to apply experience of classical theory to perturbative QCD, especially taking into account that here we we want to identify with the cut contributions with terms of finite orders of perturbation theory, have no relation either to the position or to the nature of the j -plane singularities of the partial amplitudes.

3.2 Three-Reggeon cuts in the diagrammatic approach

By analogy with the two-Reggeon cut, the contribution of the three-Reggeon cut is depicted by Fig.7 and is represented as

$$\Phi_{A'A}^{3R} \otimes G^{3R} \otimes \Phi_{B'B}^{3R} . \tag{12}$$

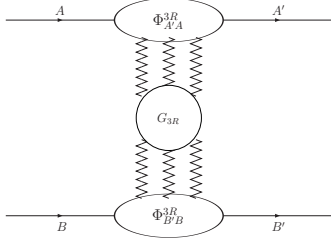


Fig.7. Diagrammatic representation of the three-Reggeon exchange.

Reggeized gluons with positive signature, which differs from Pomeron by C-parity).

This assumption looks rather natural, although, as I know, its strict proof does not exist, as well as its check in perturbation theory.

In the leading (two loop) approximation Reggeon interaction must be neglected. The exchange of non-interacting Reggeons is depicted in Fig.8 *a*. In the momentum space, the interaction of Reggeons with particles depends only on the total energy of the process and on the transverse momenta of the Reggeons. Therefore in the momentum space the lowest order contribution is depicted by Fig.8*b*, where solid lines correspond to the propagators $\frac{1}{\mathbf{k}_{i\perp 2}}$.

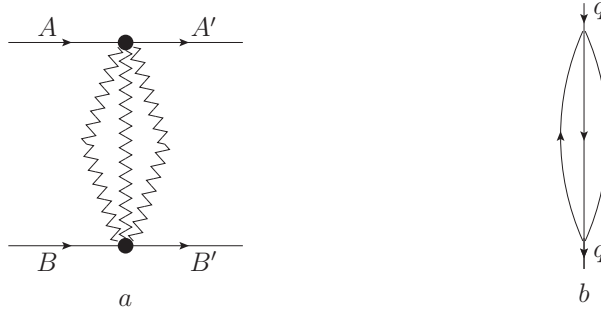


Fig.8. Diagrammatic representations of the two-loop contribution.

The colour structure of the cut is determined using the results [12], [13], based on the infrared factorization. It will be discussed in the next subsection. For separation of pole and cut contributions, difference in their energy dependence was used in the three-loop approximation, in which the cut contribution is depicted by the Reggeon diagram Fig.9*a*,

$$\hat{\mathcal{G}}^{3R} = \frac{1}{\omega - \hat{\mathcal{K}}_{3R \rightarrow 3R}}, \quad \omega = j - 1, \quad (13)$$

$$\hat{\mathcal{K}}_{3R \rightarrow 3R} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\omega}_3 + \hat{\mathcal{K}}_r^{12} + \hat{\mathcal{K}}_r^{13} + \hat{\mathcal{K}}_r^{23}, \quad (14)$$

\mathcal{K}_r^{ij} describes interaction of Reggeized gluons i and j . It is supposed that pair interaction between Reggeons is described by the BFKL kernel. In fact, such interaction was assumed many years ago in the BKP equation [40], [41] for the Odderon (colourless state of three

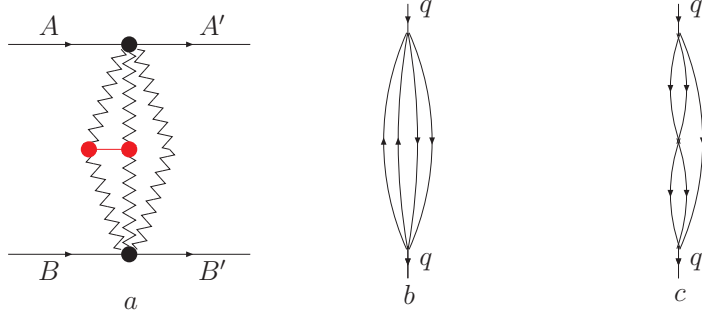


Fig.9. Diagrammatic representation of three-loop contribution.

where the horizontal line represents the BFKL kernel, and its momentum parts by two diagrams, Fig.9b and Fig.9c. Accordingly, in the four-loop approximation the Reggeon diagrams of the cut contribution are depicted in Fig.10 and their momentum parts in in Fig.11.

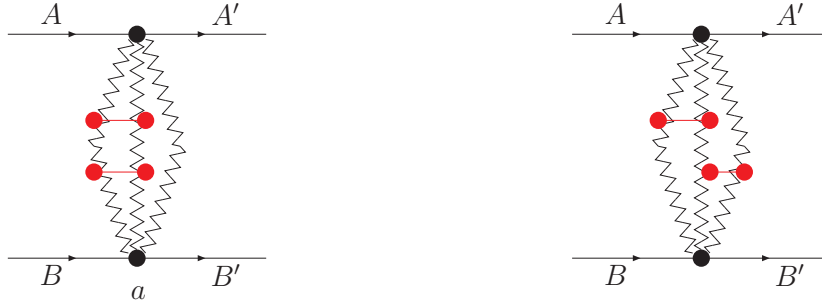


Fig.10. Reggeon diagrams for the four-loop contribution.

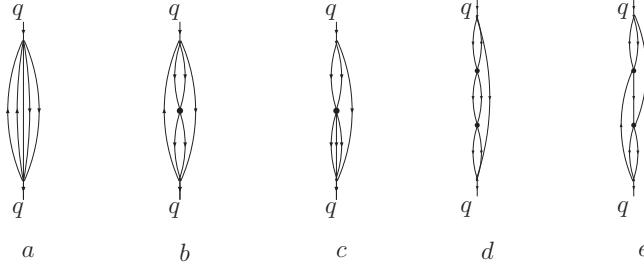


Fig.11. Transverse momentum diagrams for the four-loop contribution.

3.3 Colour structure of the cuts

A crucial question is the colour structure of Reggeon cuts. This question is simple in the two-Reggeon case, because in the product of two adjoint representations there is only one representation of given dimension and parity. But it becomes quite non-trivial in the case of three-Reggeon cuts. The Reggeon cut contributions are obtained as the sums of their momentum parts with the colour coefficients. In the diagrammatic approach the colour coefficients are determined from two- and three-loop approximations.

Due to the signature conservation the cut with negative signature must be the three-Reggeon one. Since our Reggeon is the Reggeized gluon, the three-Reggeon cut first contribute to amplitudes corresponding to the diagrams depicted in Fig.12.

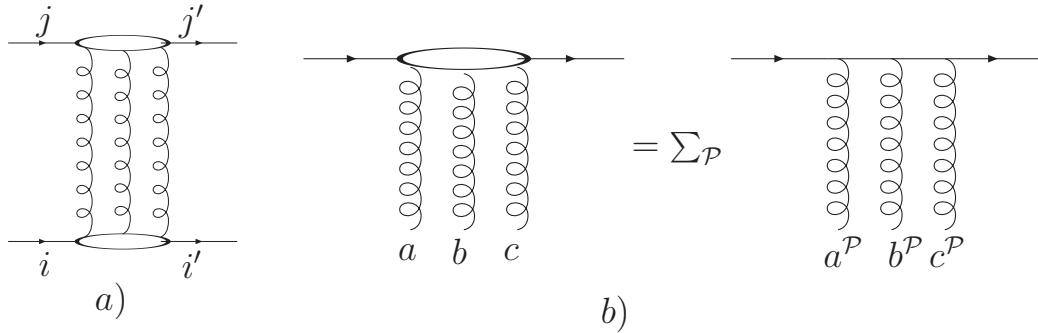


Fig.12. Three-gluon exchange diagrams.

Remind that unlike the Reggeon, which contributes only to amplitudes with the adjoint representation of the colour group (colour octet in QCD) in the t -channel, the cut can contribute to various representations. Besides the octet (**8**), possible representations are singlet (**1**) for quark-quark and quark-gluon scattering and **10**, **10*** and **27** for gluon-gluon scattering (remind that we consider only negative signature). The colour coefficients of the diagrams Fig.12 depend on the representation. It occurs that they are equal for all diagrams for the representations different from the adjoint. For these representations, we attribute all diagrams to the contribution of the three-Reggeon cut. Note that the sum of the contributions of all diagrams with equal coefficients does not depend on the gluon gauge, which makes our definition gauge invariant.

As for the adjoint representation, the colour coefficients are equal separately for two planar and four non-planar diagrams, but differ for planar and non-planar diagrams. In this representation, the contribution to the amplitude can come from both the Regge pole and the three-Reggeon cut, and the problem of separating these contributions arises. In the diagrammatic approach, this problem is solved using the results [12], [13] for the infrared singular contributions to two- and three-loop amplitudes. The colour coefficients of the three-loop contributions differ from the one-loop coefficients only by common factors, different for for the pole and cut contributions. Comparison of the obtained results with the results of [12], [13] based on the infrared factorisation allows to determine the colour coefficients of the cut contribution. It occurs that they also are equal for all diagrams, as well as for the representations different from the adjoint. The colour coefficients of the four-loop contribution are determined by considering that the separation into pole and cut contributions is specified by two and three loops.

In the Wilson line approaches certain assumptions are made about the colour structure of the cuts from the beginning. Using these assumption demanded introduction of the three-Reggeon cut-one Reggeon mixing for the explanation of the violation of the pole factorization found in [11]-[13]. As it was already discussed, the assumption made in the first version of this approach [20] contradicts to the known results of N=4 super-Yang-Mills theory in the planar limit. In the modern version, used in [21] - [25], for the separation of pole and cut contributions is based on the assertion that the diagrams for Regge cuts are non-planar. As it was already discussed, this assertion, which comes from the classical theory of complex angular momenta, is inapplicable in QCD. In addition, separate contributions of planar and non-planar diagrams are not gauge invariant.

4 Conclusion

A remarkable property of QCD, extremely important for describing processes in the Regge and the multi-Regge kinematics, is the gluon Reggeization. Thanks to it, the amplitudes of processes with adjoint colour group representation in cross-channels and negative signature have the pole Regge form in the LLA and the NLLA, which ensures a simple derivation of the BFKL equation.

As is known from the classical theory of complex angular momenta, Regge poles generate cuts. The j -plane cuts appear in amplitudes with positive signature already in the LLA as the results of the BFKL equation, so that their investigation has a solid foundation. It shows that properties of these cuts differ drastically from properties of two-Reggeon cuts in the classical theory. In particular, the BFKL Pomeron is a fixed branch point, instead of moving branch points in the classical theory. Then, it arises due to planar diagrams, which is evident from the fact that its intercept ω_P (11) is not suppressed at large number of the colours N_c . It means that the Mandelstam arguments [37] don't work in QCD.

The situation is completely different with cuts in amplitudes with negative signature, which themselves are used to derive the BFKL equation. At these amplitudes, Regge cuts appear only in the NNLLA. Evidence of their appearance is the violation of the pole factorization of elastic amplitudes factorization [11]-[13]. Unfortunately, the experience of the classical theory of complex angular momenta cannot be used for their investigation, as it is seen from the case of amplitudes with positive signature, both due to the different nature of Reggeons and due to differences in the purposes and approaches to study of j -plane singularities.

There are currently two approaches to explaining the violation of factorization by contributions from three-Reggeon cuts: the diagrammatic approach, [14]-[19], which is based on the analysis of Feynman diagrams, and the Wilson line approach, [20] - [25], based on using of representation of scattering amplitudes by Wilson lines and shock wave formalism. The main difference between these two approaches is the colour structure of the three-Reggeon cut contributions. In both approaches the violation of the pole factorization found in [11]-[13] was explained (but in different ways) and four-loop cut contributions are calculated for all colour group representations in the t -channel. In the Wilson line approach, a general recipe for separating pole and cut contributions was proposed. The recipe proposed in the first version of the Wilson line approach [20] contradicts the maximally extended supersymmetric Yang-Mills theory in the planar limit. In the modern version of this approach [21] - [25] separation of the cut contributions as contributions of non-planar diagrams is based on the Mandelstam arguments [37], [38]. But these arguments are not valid in QCD, as it clearly seen in the case of the two-Reggeon cuts, precisely for which these arguments were formulated. Besides this, separate contributions of planar and non-planar diagrams are not gauge invariant. In our opinion, all this makes one doubt the proposed recipe.

Further development of theory of QCD processes in the Regge region call for better understanding of the three-Reggeon cuts.

Standard derivation of the BFKL equation requires calculations of contribution of these cuts not only in elastic amplitudes, but in amplitudes with the multi-Regge kinematics as well.

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