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STATISTICS OF RANDOM QUASI 1D HAMILTONIAN WITH SLOWLY VARYING PARAMETERS. PAINLEVE AGAIN

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НОВОСИБИРСК

Statistics of random quasi 1D Hamiltonian with slowly varying parameters. Painlevé again

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Abstract

The statistics of random band–matrices with the width and strength of the band slowly varying along the diagonal is considered. The Dyson equation for the averaged Green function close to the edge of spectrum is reduced to the Painlevé I equation. The analytical properties of the Green function allow to fix the solution of this equation. The former appears to be the same as that arose within the random–matrix regularization of 2d-gravity.

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1. The random band–matrices are now usually considered as the theory of quasi 1D systems. For example it is thought that band matrices may adequately depict the properties of electrons in thin wires [1] (see also [2, 3]).

In this note we would like to consider the natural extension of the usual band–matrix model. We will investigate the statistical features of quasi–banded matrices with the parameters of the band (width, strength of interaction etc.) slowly varying along the diagonal.

For application to condensed matter physics or energy level statistics in complex quantum systems it may be also useful to add some regular diagonal part to the random Hamiltonian (see [4]). Thus we deal with the Hamiltonian

$$H = H^0 + V \ , \ \ H^0_{ij} = h(i)\delta_{ij} \ , \qquad (1)$$

where $h(i)$ is some smooth monotonic function and the elements of random matrix V_{ij} are Gaussian distributed.

As usual, due to the Whick theorem, the Gaussian ensemble may be defined by the second moment:

$$\overline{V_{ij}V_{mn}} = \frac{F(i,j)}{b}(\delta_{jm}\delta_{in} + \delta_{jn}\delta_{im}) \ . \qquad (2)$$

Here $F(i,j) = F(j,i)$ and $b \gg 1$ is the typical width of the band. We are interested in the quasi band case. Therefore we suppose that $F(i,j)$ decreases rather rapidly when $|i-j| > b$, but is a very slow smooth function of $i+j$. In particular the sum over j

$$\frac{1}{b}\sum_j F(i,j) = W(i) \sim 1 \qquad (3)$$

is a very smooth function of i, as well as the regular function $h(i)$,

$$b\frac{dW}{di} \sim b\frac{dh}{di} \sim \frac{1}{d} \ll 1 \ . \qquad (4)$$

The correlator (2) describes the Gaussian ensemble of real symmetric matrices. One can also consider the ensemble of general Hermitean quasi band–matrices. To this end it is enough to withdraw the second term in brackets in (2). All the results obtained in this paper are relevant for both ensembles.

2. The "physical quantity" we would like to consider is the Green function $G = (E - H)^{-1}$. As follows from (2)

$$\overline{G_{ij}} = G(i)\delta_{ij} \ . \qquad (5)$$

The formal expansion in a series in V/E allows to write down the Dyson–type equation:

$$\begin{aligned}\overline{G_{ij}} &= \frac{1}{E-h(i)}\left\{\delta_{ij} + \overline{\sum_{n=1}^{\infty}\left(V\frac{1}{E-H^0}\right)^n_{ij}}\right\} = \qquad (6)\\ &= \frac{1}{E-h(i)}\left\{\delta_{ij} + \sum_k \frac{F(i,k)}{b}(\overline{G_{kk}G_{ij}} + \overline{G_{ki}G_{kj}})\right\} \ .\end{aligned}$$

Here we have performed Whick contraction of the first element in each $(V)^n$ with any other and then resumed the series. In order to simplify this *exact* equation it is easy to note that the second term in brackets is small like $1/b$ and in the leading approximation over $1/b$ one can also decouple the average of two G-s in the first term. Thus

$$G(i) = \frac{1}{E-h(i)}\left(1 + \sum_k \frac{F(i,k)}{b}G(k)G(i)\right) \ . \qquad (7)$$

Our derivation of this formula in fact repeats the calculation of Green function for full $N \times N$ random matrices, which was done many years ago [5, 6] (see also in [7] the same proof for pure band matrices).

Due to (5) only closed chains contribute to each (V^n) in (6). In terms of dual Feynman graphs each $(V^n)_{ii}$ may be thought as n-vertex polygon, while the averaging via Whick contractions may be considered as a self gluing of the edges of this polygon. Within this language in (7) we have summed up *exactly* all the planar (spherical) graphs. The nonplanar contributions lead

to corrections to G of the order of $\sim 1/b$ for real symmetric matrices and $\sim 1/b^2$ for Hermitean matrices [8]. Nevertheless the equation (7) is *exact* in $1/d$ (4).

It is easy to expand the equation (6) in the series in $1/d$. Up to $1/d^2$

$$G(i) = \frac{1}{E-h(i)}\left(1 + W(i)G(i)^2 + G(i)(AG' + BG'')\right) \ . \qquad (8)$$

Here prime means the derivative with respect to $X = i/b$ and

$$A(i) = \sum_k \frac{F(i,k)}{b}\left(\frac{k-i}{b}\right) \sim \frac{1}{d} \ , \quad B(i) = \frac{1}{2}\sum_k \frac{F(i,k)}{b}\left(\frac{k-i}{b}\right)^2 \sim 1 \ . \qquad (9)$$

In the leading approximation the solution of (8) reads:

$$G^0(i) = \frac{1}{2W(i)}\left(E - h(i) - \sqrt{(E-h(i))^2 - 4W(i)}\right) \ . \qquad (10)$$

The imaginary part of G^0 at $E = E - i0$ reproduces the usual in matrix models semicircle density. In general the $1/d^2$ corrections to (10) may be found from (8) by iteration. Due to singularity of G^0 this corrections, which are proportional to G' and G'', spreads to infinity when $E = h(i) \pm 2\sqrt{W(i)}$. Thus the nonperturbative treatment of equations (6) or (8) is necessary near the edge point.

The use of expansion (8) assumes that both functions $h(i)$ and $W(i)$ are equally slow (4). The smoothness of W is necessary because we want to consider the almost banded matrices, but one can sufficiently weaken the restriction for $h(i)$. In this case the solution of equation (7) in the Breit–Wigner form is easy to found

$$G^0(j) = \frac{1}{E - h(j) - i\gamma(j)}, \ \gamma(j) = \pi F(j,j)\left(b\frac{dh}{dj}\right)^{-1} \ ; \ \frac{1}{b} \ll \left|\frac{dh}{dj}\right| \ll \frac{1}{\sqrt{b}}. \qquad (11)$$

Here the upper bound for dh/dj ensures the smallness of the regular energy intervals $h(j+1) - h(j)$ compared to the hopping matrix elements V_{ij} (2).

3. Turning back to the slow $h(i)$ case let us introduce the new "scaling" variables

$$\begin{aligned}E - h(0) &= 2\sqrt{W(0)} + \frac{\varepsilon}{d^{4/5}} \ ,\\ X = i/b &= d^{1/5}x \ , \ G = \frac{1}{\sqrt{W(0)}} + \frac{y(x)}{d^{2/5}} \ , \qquad (12)\end{aligned}$$

$$W(X) = W(0)+X\frac{dW}{dX} \equiv W(0)+\frac{X}{d} ,$$
$$h(X) = h(0)+\alpha\frac{X}{d} , \alpha \sim 1 .$$

Here we have also made the explicit definition of the large parameter d. Substitution of (12) into (8) leads to

$$By'' + W^{3/2}y^2 = \varepsilon - \left(\frac{1}{\sqrt{W}} + \alpha\right) x . \tag{13}$$

Here $W = W(0)$. Due to (9) the term $\sim AG'$ in (8) leads to negligible correction of the order of $\sim 1/d^{4/5}$. As well the higher derivatives of G, which we have neglected in (8), contribute like some powers of $1/d$.

The equation (13) is the famous Painlevé I equation. Parameters B, W and α which still alive are of the order of 1 and may be dropped out by trivial rescaling. Being a part of Green function the function $y = y_\varepsilon(x)$, have to reproduce its analytical features. Thus the only singularity of $y_\varepsilon(x)$ is the cut along the axis $\mathrm{Im}\,\varepsilon = 0$. We have defined x (12) as a real variable. On the other hand it is natural to consider $y_\varepsilon(x)$ as an analytical function of one complex variable $x' = x - \frac{\sqrt{W}}{1+\alpha\sqrt{W}}\varepsilon$. Now the function $y(x')$ should not have singularities in the whole upper half plane.

It is amazing that our model is not the first model of random matrices whose scaling limit is described by the Painlevé equation. A few years ago the random matrix models helped to solve exactly the $2d$ quantum gravity [9]. It was shown that the second derivative of partition function of $2d$ gravity with respect to the cosmological constant is the solution of Painlevé equation. The unique solution of this equation, which is realized in Euclidean $2d$–gravity was found in [10]. Each solution of eq. (13) has an infinite set of second order poles on the complex plane $y \sim (x - x_i)^{-2}$ and no other singularities. As it was pointed out by F.David [10] (and as was known for mathematicians for many years) only one solution may have no poles in the whole half plane. This is the so called "triply truncated solution", which may have only a finite number of poles in the sector $\frac{-2\pi}{5} < Arg(x) < \frac{6\pi}{5}$ (and still has an infinite set of poles within the rest $\frac{2\pi}{5}$). Just this solution is realized in $2d$–gravity if the matrix model is regularized via analytical continuation. Moreover only this unique solution can reproduce the analytical features of the Green function in our model (2).

The most informative quantity, which may be found from the averaged Green function is the single particle density

$$\rho(i,E) = \frac{1}{\pi}\mathrm{Im}\,G(i,E-i0) \sim \mathrm{Im}\,y(x - \frac{\sqrt{W}}{2}\varepsilon) . \tag{14}$$

It is easily seen that for real ε all real solutions of (13) are unstable (have the poles at real x axis). The complex solutions, in accordance with (10), have all the same asymptotics $\mathrm{Im}\,y(x \to \pm\infty) = -\sqrt{\varepsilon - (\frac{1}{\sqrt{W}} + \alpha x)}$. Nevertheless the nonperturbative imaginary part of y at negative x allows to distinguish the triply truncated solution among the others [10, 11]

$$\mathrm{Im}\,y(x \to -\infty) = \frac{\sqrt{3\sqrt{2}}}{4\sqrt{\pi}}\frac{B^{1/4}(1+\alpha\sqrt{W})^{3/8}}{W^{9/8}}\left(\frac{\varepsilon\sqrt{W}}{1+\alpha\sqrt{W}} - x\right)^{-1/8} \times$$
$$\times \exp\left\{-\frac{4\sqrt{2}}{5\sqrt{B}}W^{1/4}(1+\alpha\sqrt{W})^{1/4}\left(\frac{\sqrt{W}}{2}\varepsilon - x\right)^{5/4}\right\} . \tag{15}$$

Thus we see that close to the edge the single particle density behaves like

$$\rho(i,E) = \frac{1}{d^{2/5}} f\left(d^{4/5}(E - E_0)\right) , \tag{16}$$

where f is a smooth imaginary part of the triply truncated solution. This result accounts exactly for the series of most singular corrections over $1/d^n$ (4,12).

Still we have not considered the corrections of the order of $1/b$. These corrections accounts for the finite width of the band (2) and also can smooth out the singularity of the zero order result (10). The accurate description of $1/b$ effects is beyond of the main subject of this paper. Here we give only the result and postpone discussion for a separate publication [12]. For the pure band–matrix (i.e. if $F(i,j) \equiv F(|i-j|)$ and $h(i)$ is negligible small) the density of states close to end–point reads

$$\rho(E) = \frac{1}{b^{2/5}}\phi(b^{4/5}(E - E_0)) \tag{17}$$

with some smooth ϕ. So we can conclude, that the scaling (12,13) should not be disturbed by the $1/b$ corrections until $d \ll b$.

4. As we have seen (12,13) the variation of band width $W(i)$ and shifting of the center of zone $h(i)$ play the same role at the edge. Therefore in the following two sections we suppose that $h = 0$.

Up to now we have supposed (12) that the edge of spectrum $E_e = h(i) + 2\sqrt{W(i)}$ is a smooth monotonic function of i. On the other hand it seems to be interesting to investigate also the statistical properties of the model (2) near the bulge of E_e. The set of scaling variables suitable for this case reads

$$\begin{aligned} W(X) &= W(0) - \frac{X^2}{2d^2} \ , \ X = i/b = \frac{B^{1/3}}{W^{5/6}} d^{1/3} x \ , \\ E &= 2\sqrt{W(0)} + \frac{B^{2/3}}{W^{5/6}} \frac{\varepsilon}{d^{4/3}} \ , \ G = \frac{1}{\sqrt{W(0)}} + \frac{B^{1/3}}{W^{7/6}} \frac{y(x)}{d^{2/3}} \ . \end{aligned} \tag{18}$$

Here again $W = W(0)$ and B are of the order of 1. Also here we give a new definition of large d. Simply substituting (18) into the same equation (8) one gets

$$y'' + y^2 = \varepsilon + x^2 \ . \tag{19}$$

Like in monotonic case (13) the first derivative y' and all higher derivatives are suppressed by some powers of $1/d$. The asymptotic solution of this equation reads

$$y = -\sqrt{\varepsilon + x^2} + \ldots \tag{20}$$

in accordance with (10). For large negative ε in vicinity of $\varepsilon + x^2 = 0$ the equation (19) reduces to the Painlevé I and the singularity of (20) is smoothed out via its triply truncated solution. Thus the only new problem will be to find y at $\varepsilon \sim 1$.

Generally speaking the equation (19) is not of the Painlevé type. Its solutions also has poles on the complex plane, but each pole turns out to be also the edge of logarithmic cut. We are interested in the set of real x solutions $y_\varepsilon(x)$ analytical in the whole lower half plane of complex parameter ε. Let us write down explicitly the real and imaginary parts of (19)

$$\begin{gathered} \varepsilon - \varepsilon - i\lambda, y = u + iv \ , \\ \left\{ \begin{array}{l} u'' + u^2 - v^2 = \varepsilon + x^2 \\ v'' + 2uv = -\lambda \ . \end{array} \right. \end{gathered} \tag{21}$$

For real energy ($\lambda = 0$) the latter equation may be thought as the Schrödinger equation with v being the wave function, $(-u)$ – the potential and eigenvalue equal to zero. Moreover it is immediately seen from (20) that this Schrödinger equation could not have any localized solution at sufficiently large positive ε. Thus the imaginary part of Green function (or the single particle density) vanishes in our model at large ε.

Our scaling equation (19) accounts only for the most singular corrections in each order over $1/d$. Nevertheless, one can see, that for the *exact* equation

(6) the imaginary part of $G(i)$ also vanishes at sufficiently large E. In fact at large E the imaginary part is small and thus it should satisfy some linear homogeneous matrix equation. But in general (except for the degenerate case) the linear equation has no nontrivial solutions.

The exact solution of equation (19) (or(21)) may be found only numerically. Nevertheless one can further clarify, how the imaginary part appears for real ε. Suppose that $y = u_0(x)$ is the real solution of (19) with smallest real ε_0. Then the corresponding Schrödinger equation at $u = u_0$ also should have the localized zero mode ψ_0 :

$$\left\{ \begin{array}{l} u_0'' + u_0^2 = \varepsilon_0 + x^2 \\ \left(-\frac{d^2}{dx^2} - 2u_0\right) \psi_0 = 0 \ . \end{array} \right. \tag{22}$$

Now it is rather simple exercise to find the solution in vicinity of ε_0:

$$\varepsilon = \varepsilon_0 + \Delta \ , \ y = u_0 - \sqrt{\Delta \frac{\int \psi_0 dx}{\int \psi_0^3 dx}} \psi_0 + O(\Delta) \ . \tag{23}$$

So we see that the square root singularity of the zero order result (10) which seemed to be smoothed out by the solution of differential equation (19) still alive at the very edge of the spectrum. This result may have even experimentally measurable consequences. One can introduce the density of eigenstates (see (14))

$$\rho(E) = \sum_i \rho(i, E) \ . \tag{24}$$

This quantity is well defined because we are working at the point of maximum thickness of the wire (band). At large negative ε the asymptotics of $\rho(E)$ may be found from (20)

$$-\varepsilon \gg 1 \ , \ \rho(\varepsilon) \sim (\varepsilon_0 - \varepsilon)^{3/2} \ . \tag{25}$$

On the other hand close to the edge

$$0 < \varepsilon_0 - \varepsilon \ll 1 \ , \ \rho(\varepsilon) \sim \sqrt{\varepsilon_0 - \varepsilon} \ . \tag{26}$$

Thus the "smooth" exact result appears to be even more singular than the zero order approximation (25).

Of course the vanishing of $\mathrm{Im}\, y$ at $\varepsilon > \varepsilon_0$ is the artifact of our approximation. Again in this section we have neglected all the $1/b$ corrections. Comparing (18) and (17) one can find that our results are valid if $d \ll b^{3/5}$.

5. Up to now we have considered the averaging of only the single Green function. The much more puzzling quantities are the correlation functions of two Green functions. In particular the correlators are sensitive to the effect of localization. For usual band–matrices the localization length turns out to be of the order of $\sim b^2$ [1] and can not be found without nonperturbative treatment of $\sim 1/b$ effects. Nevertheless near the thickening of the wire (18) some sort of compulsory localization takes place and the eigen-functions are localized due to the "geometry" of band (wire). In this section we would like to find the averaged value of the squared absolute value of G_{ij} at the thickening point. In fact what we can easily calculate is the so called "smoothed" correlator [7], valid only for sufficiently large $\operatorname{Im} E$ (at least as compared to the interval between the individual levels).

Like in (6) one can expand correlator in the series

$$\overline{|G_{ij}|^2} = \frac{1}{|E|^2}\sum_{n,m}\overline{\left(\frac{V}{E}\right)^n_{ij}\left(\frac{V}{E^*}\right)^m_{ji}} \ . \tag{27}$$

We are mostly interested in the case of small imaginary part of E. The crucial observation is that in the planar limit all the Whick contractions between $(V)^n$ and $(V)^m$ may be done explicitly, while the sum over $(V/E)^s$ which are contracted within one of the multipliers in (27) may be resumed back to $G(i)$ or $G(i)^*$ (5):

$$\overline{|G_{ij}|^2} = \sum_{n=0}\sum_{i_1,\dots,i_n}|G|^2_i\frac{F(i,i_1)}{b}|G|^2_{i_1}\cdots\frac{F(i_n,j)}{b}|G|^2_j \equiv \sum_{n=0}\Gamma_n(i,j) \ , \tag{28}$$

where $|G|^2_i \equiv |G(i)|^2$. Like the equation (7) this formula accounts *exactly* for all $\sim 1/d$ corrections. All the $\sim 1/b$ corrections to $|G_{ij}|^2$ have been neglected in (28). Unfortunately the price for this approximation is rather high. The $\sim 1/b$ corrections will blow up the correlation function at very small $\operatorname{Im} E$, thus preventing the observation of the correlations in neighbor levels.

The recursion formula for Γ-s (compare with(7,8)) reads

$$\begin{aligned}\Gamma_{n+1}(i,j) &= \sum_k |G|^2_i\frac{F(i,k)}{b}\Gamma_n(k,j) = \\ &= W|G|^2\Gamma_n + A|G|^2\Gamma'_n + B|G|^2\Gamma''_n + \dots \ ,\end{aligned} \tag{29}$$

where $A, B, |G|^2$ are taken at the point i and the prime means the derivative with respect to $X = i/b$. Now if one uses the scaling variables (18) and also

introduces the additional variable (time) instead of n the differential equation for $\Gamma(t,x,x_j)$ is easy to write down

$$t = \frac{n}{d^{2/3}} \ , \ \dot\Gamma = \Gamma'' + 2Re\, y\,\Gamma \ . \tag{30}$$

Here and below we choose $W(0) = B = 1$ for simplicity. It is seen immediately from (21) that Γ spreads to infinity if $\operatorname{Im}\varepsilon \to 0$ due to zero mode of the Schrödinger equation in the r.h.s. of (30).

For small $\lambda = -\operatorname{Im}\varepsilon$ it is convenient to consider the linearized version of the system (21)

$$\begin{aligned}&u = u_0 + f \ , \ v = v_0 + g \ , \ f, g \sim \lambda \ , \\ &\begin{cases} f'' + 2u_0 f = 2v_0 g \\ g'' + 2u_0 g = -\lambda - 2v_0 f \ . \end{cases}\end{aligned} \tag{31}$$

Here u_0, v_0 are the solutions of (21) for $\lambda = 0$ and hence v_0 is also the zero mode of operator $\frac{d^2}{dx^2} + 2u_0$. Thus one can write down the consistency conditions for two equations (31):

$$2\int v_0^2 g = 0 \ , \ \lambda\int v_0 = -2\int v_0^2 f \ . \tag{32}$$

The solution of (30) may be found as a sum over eigenmodes of the operator:

$$\begin{aligned}-\frac{d^2}{dx^2} - 2u &= -\frac{d^2}{dx^2} - 2u_0 - 2f \ , \\ \Gamma(t,x,x_j) &= \frac{1}{bd^{1/3}}\sum |n\rangle\langle n|\exp(-\epsilon_n t) \ ,\end{aligned} \tag{33}$$

where ϵ_n and $|n\rangle$ are the eigenvalues and normalized eigenmodes and the overall factor in Γ was found from the initial condition (see (28)). For small λ and large t only the almost zero mode ($n = 0$) survives in (33), which is:

$$|0\rangle = \frac{v_0(x)}{\sqrt{\int v_0^2}} \ , \epsilon_0 = -2\frac{\int v_0^2 f}{\int v_0^2} = \lambda\frac{\int v_0}{\int v_0^2} \ . \tag{34}$$

Finally the "smoothed" [7] correlator reads

$$\overline{|G_{ij}|^2} = \frac{1}{bd\operatorname{Im} E}\,\frac{v_0(x_i)v_0(x_j)}{\int v_0 dx} \ , \ \sum_j\overline{|G_{ij}|^2} = \frac{\rho(i,E)}{\pi\operatorname{Im} E} \ . \tag{35}$$

Here we use that $\lambda = d^{4/3}\mathrm{Im}\, E$ (18). We have also shown the sum rule, which this correlator evidently satisfy. Function $v_0(x)$ may be found numerically as a solution of the simple system of differential equations (21).

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References

[1] Y. V. Fyodorov and A. D. Mirlin, Phys. Rev. Lett. **67** (1991) 2405, **69** (1992) 1093, **71** (1993) 412.

[2] K. B. Efetov, A. I. Larkin, Zh. Eksp. Teor. Fiz. **85** (1983) 764 [Sov. Phys. JETP **58** (1983) 444].

[3] J.-L. Pichard, in *Quantum Coherence in Mesoscopic Physics*, ed. by B. Kramer (Plenum Press, N. Y., 1991).

[4] E. Wigner, Ann. Math. **62** (1955) 548, **65** (1957) 203;
G. Casati, B. V. Chirikov, I. Guarneri, F. M. Izrailev, Phys. Rev. **E 48** (1993) R1613.

[5] L. A. Pastur, Theor. Mat. Phys. (USSR) **10** (1972) 67

[6] J. Verbaarschot, H. A. Weidenmüller, and M. Zirnbauer, Ann. Phys. (N.Y.) **153** (1984) 367.

[7] E. Brezin and A. Zee, Phys. Rev. **E 49** (1994) 2588.

[8] G. 't Hooft, Nucl. Phys. **B72** (1974) 461.

[9] E. Bresin and V. A. Kazakov, Phys. Lett. **B236**, (1990) 144); D. J. Gross and A. A. Migdal, Phys. Rev. Lett. **64** (1901) 127; M. R. Douglas and S. H. Shenker, Nucl. Phys. **B335** (1990) 635.

[10] F. David, Nucl. Phys. **B348** (1991) 507; P. G. Silvestrov and A. S. Yelkhovsky, Phys. Lett. **B251** (1990) 525; A. R. Its and A. V. Kitaev, Mod. Phys. Lett. **A5** (1990) 2079.

[11] F. David, Phys. Lett. **B302** (1993) 403.

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Statistics of Random Quasi 1D Hamiltonian with Slowly Varying Parameters. Painleve Again

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Статистика случайных квази 1-мерных гамильтонианов с медленно меняющимися параметрами. Опять Пенлеве

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