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The State Scientific Center of Russia
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SB RAS

V.N. Khudik, O.A. Shevchenko

COLLISIONLESS DAMPING OF A QUASILINEAR LANGMUIR WAVE PACKET

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V.N. Khudik, O.A. Shevchenko

The State Scientific Center of Russia
The Budker Institute of Nuclear Physics
630090, Novosibirsk, Russia

Abstract

Collisionless damping of an one-dimensional packet of Langmuir waves that is localized in the direction of its propagation is described by using quasilinear equations. A peculiar minimum principle is proposed for reducing this quasilinear problem to a simple graphic analysis of the initial spectrum of the packet.

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1 Introduction

There are many examples when the quasilinear approach appears as very useful to determine the turbulent transport of particles in chaotic electromagnetic fields (see, e.i. [1, 2]). But quite often the complexity of problems studied does not enable one to advance further than the qualitative estimations. In this paper we consider a problem of another kind. Making certain simplifying assumptions, we can find the accurate solution of one quasilinear problem.

It is well-known that in the spatially homogeneous case, the result of interaction of one-dimensional Langmuir waves with resonant electrons depends on the level of spectral energy density of the oscillations [3]. If the spectral energy density is small, then the waves are damped exponentially and the distribution function of the resonant particles remains undistorted. (It is obvious that this result is also valid for the wave packets which are localized in space). But if the energy density is large, then the amplitude of oscillations vary only slightly and the formation of a plateau in the distribution function takes place. The spatial boundedness of the packet changes essentially this result.

The damping of the localized Langmuir wave packet has been studied for the particular case of the parabolic distribution of the spectral energy density [4]. It has been shown that the interaction of resonant particles with Langmuir oscillations results in decreasing the packet length at a constant rate. The purpose of this paper is to find the time evolution of quasilinear packets with generally shaped spectrum. We shall show that the character of damping of such packets differs essentially from the case of parabolic spectrum. As in paper [4], we assume that the spectral energy density of Langmuir waves W_k varies in space only in one direction, which coincides with the sense of the electrical field of oscillations. The distribution function of resonant particles f is supposed not to depend on spatial coordinates at the initial instant.

The contents of this paper are as follows. Sec. 2 contains a set of basic quasilinear equations and formulation of the problem. The damping of the packets with the paraboliclike spectra is investigated in Sec. 3, the damping of the packets with generally shaped spectra is studied in Sec. 4 and finally all the results are briefly discussed in Sec. 5.

2 Basic equations

Within the quasilinear approximation the influence of the oscillations on the resonant electrons causes their diffusion in the velocity space [5, 6]:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \frac{\partial}{\partial v} D \frac{\partial f}{\partial v}, \tag{1}$$

here the quasilinear diffusion coefficient D is proportional to the spectral energy density

$$D = \frac{4\pi^2 e^2}{m^2 v} W_k,$$

e, m — electron charge and mass respectively. In could plasma the group velocity can be neglected and evolution of a Langmuir wave packet in time reduces to its Landau damping

$$\frac{\partial W_k}{\partial t} = W_k \frac{\pi \omega_p}{n} v^2 \frac{\partial f}{\partial v}, \tag{2}$$

where ω_p — electron plasma frequency , n — plasma concentration. For our purpose, there is no need to distinguish between a spectral energy density and a quasilinear diffusion coefficient; only the latter one will be used in following. In the one-dimensional case there is one-to-one correspondence between the wave vector of oscillations k and the velocity of the resonant particles $v = \omega_p/k$ and the latter we shall further use as the independent variable.

In order to easily visualize the main features of the damping processes of the packet we assume that the oscillations are generated homogeneously in

the half-space $x > 0$ so that the initial coefficient of diffusion is

$$D(x,v,t)|_{t=0} = \eta(x) D_0(v), \quad \eta(x) = \begin{cases} 1, & x > 0 \\ 0, & x \le 0 \end{cases} \tag{3}$$

Here the function $D_0(v)$ is considered not to vanish in a narrow interval Δv_0 of phase velocities near some value v_0. It is also supposed that the unperturbed distribution function of resonant particles $f_0(v)$ can be approximated by the linear function in this range of velocities:

$$f_0(v) \simeq f_0(v_0) + f_0'(v_0)(v - v_0), \quad f_0'(v_0) \equiv \left.\frac{df_0}{dv}\right|_{v=v_0}. \tag{4}$$

It will be recall that there are two time scales in quasilinear problem. The first scale is the characteristic time of diffusion of particles in velocity space

$$\tau_{dif} = \frac{\Delta v_0^2}{\langle D_0 \rangle}$$

($\langle D_0 \rangle$ is average value of the function $D_0(v)$). The second scale is the characteristic time of the damping of oscillations

$$t \sim \gamma^{-1}, \quad \gamma = \frac{\pi \omega_p}{2n} v^2 |f_0'(v_0)|.$$

We suppose that the level of oscillations is so high that the diffusion time is much less than the damping time:

$$\tau_{dif} \ll \gamma^{-1}.$$

Under initial condition (3) it is evident that the distribution function f remains always unperturbed in half-space $x < 0$ while this function has a "plateau" $f = f_0(v_0)$ on velocity interval $(v_0 - \Delta v_0, v_0 + \Delta v_0)$ at sufficiently large positive x. The half-space $x < 0$ is a peculiar kind of reservoir of the unperturbed resonant particles. There is a steady flow of "fresh" particles from here into the region occupied by oscillations. These particles take up energy from the oscillations that causes the erosion of the packet and, roughly speaking, the motion of the leading front as a whole with a velocity $v_{fr} \sim (\gamma \tau_{dif}) v_0$.

Since the diffusion time is small, the time derivative of the distribution function can be neglected in equation (1):

$$\frac{\partial f}{\partial t} \sim v_{fr} \frac{\partial f}{\partial x} \sim (\gamma \tau_{dif}) v_0 \frac{\partial f}{\partial x} \ll v_0 \frac{\partial f}{\partial x}. \tag{5}$$

Taking into account this fact and smallness of Δv_0, and using the dimensionless variables

$$u = \frac{v - v_0}{\Delta v_0}, \quad \tau = 2\gamma_0 t, \quad \zeta = \frac{2x}{v_0 \tau_{dif}},$$

$$\mathcal{D} = \frac{D}{\langle D_0 \rangle}, \quad g = \frac{f - f_0(v_0)}{|f_0'(v_0)| \Delta v_0},$$

we can rewrite the quasilinear equations (1), (2) in the form

$$\frac{\partial g}{\partial \zeta} = -\frac{\partial j}{\partial u}, \quad j \equiv -\mathcal{D}\frac{\partial g}{\partial u}. \tag{6}$$

$$\frac{\partial \mathcal{D}}{\partial \tau} = -j, \tag{7}$$

For convenience, we have marked here the flow of the particles in velocity space j. The function g must be subject to the boundary condition

$$g|_{\zeta=0} = -u. \tag{8}$$

while the function $\mathcal{D}$ must satisfy the initial condition

$$\mathcal{D}|_{\tau=0} = \eta(\zeta)\mathcal{D}_0(u), \tag{9}$$

The initial function $\mathcal{D}_0(u)$ is considered not to vanish only on the interval (-1,1). In the following, we intend to determine the asymptotic form of the solution of the simplified quasilinear problem (6) – (9) in large τ for any initial spectral curve $\mathcal{D}_0(u)$.

3 Damping of a packet with paraboliclike spectra

To illustrate the characteristic features of the damping processes of the packet with generally shaped spectrum we consider several simple examples. Let us clear up first the shape of initial spectral curve that results in the erosion of the packet occurring by the dissipation wave. While examining this case it is convenient to use the equation:

$$\frac{\partial g}{\partial \zeta} = \frac{\partial}{\partial u}\frac{\partial \mathcal{D}}{\partial \tau}, \tag{10}$$

which is the consequence of the equations (6), (7) In the wave travelling with velocity V_*

$$\frac{\partial}{\partial \tau} \longrightarrow -V_* \frac{\partial}{\partial \zeta},$$

and as is seen from (10) there is a direct relationship between distribution function and quasilinear diffusion coefficient

$$g = -V_* \frac{\partial(\mathcal{D} - \mathcal{D}_0)}{\partial u}. \tag{11}$$

After substitution of (11) into the expression for flow of particles we obtain from (7) the equation for the diffusion coefficient in the dissipation wave:

$$\frac{\partial \mathcal{D}}{\partial \zeta} = \mathcal{D}\frac{\partial^2(\mathcal{D} - \mathcal{D}_0)}{\partial u^2}, \tag{12}$$

Taking into account that at the left of the packet

$$g|_{\zeta=0} = -u, \ \mathcal{D}|_{\zeta=0} = 0,$$

one can find from (11) the shape of the initial spectral curve that results in the leading front of the packet being dissipation wave:

$$\mathcal{D}_0(u) = \frac{1}{2V_*}(1 - u^2). \tag{13}$$

Velocity of the wave, determining by the normalization condition $\langle \mathcal{D}_0(u) \rangle = 1$, is equal to numerical constant: $V_* = 2/3$. Analysis of the equation (12) shows that the diffusion coefficient $\mathcal{D}$ proves to be everywhere the parabolic function of variable u at the initial spectral distribution (13):

$$\mathcal{D} = \frac{1 - u^2}{\exp(-\zeta'/V_*) + 2V_*}, \quad \zeta' = \zeta - V_*\tau + const. \tag{14}$$

It is clear that going into regime (14) takes place at sufficiently large τ.

It should be noted for following that there is not only the motion of the leading front as a whole but also its uninterrupted deformation in all cases when initial spectral curve differs from parabola. The simplest example of this kind pertains to the deformation of the leading front, which indicated in Fig. 1. In this example we suppose that the waves in the packet with phase velocities $v \approx v_*$ ($u \approx 0$) damp most slowly and the function $\mathcal{D}_0(u)$ has even symmetry and vanishes at points $u = \pm 1$. At large time the width of the

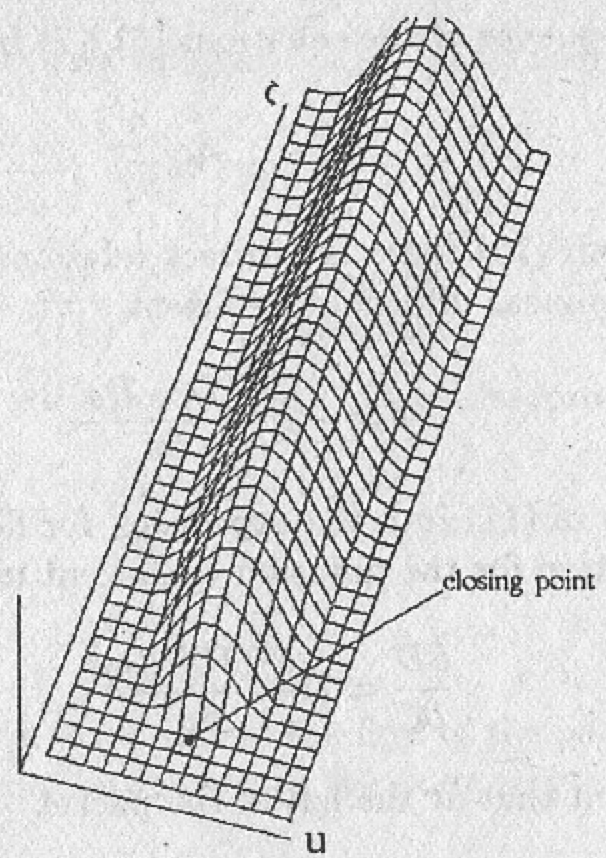

Figure 1. The damping of packet with the simplest deformation of leading front.

front becomes large and all quantities vary smoothly in space. As is seen from equation (12), diffusion processes lead to the levelling of distribution function of particles on some velocity interval, where the intensity of oscillations does not vanish [5]:

$$g|_{u\in(-\tilde{u},\tilde{u})} \approx 0, \quad \mathcal{D}|_{u\in(-\tilde{u},\tilde{u})} \neq 0,$$

here $\tilde{u} = \tilde{u}(\zeta, \tau)$ is the right boundary of interval in question. Outside of this interval the intensity of oscillations is small and attenuated diffusion processes have no influence on the distribution function:

$$g|_{u\notin(-\tilde{u},\tilde{u})} = -u, \quad \mathcal{D}|_{u\notin(-\tilde{u},\tilde{u})} \approx 0$$

When ζ increases by $\delta\zeta$ the interval boundary $\tilde{u}$ increases by $\delta\tilde{u}$ and new portion of particles is involved in the diffusion. It is an easy matter to see that there initiates the particle flow from the left end of the interval $(-\tilde{u}, \tilde{u})$ to the right that does not depend on the velocity u at given ζ:

$$j \approx g(-\tilde{u})\frac{\partial\tilde{u}}{\partial\zeta} = \tilde{u}\frac{\partial\tilde{u}}{\partial\zeta}. \tag{15}$$

Integrating the equation (7) with the particle flow (15) and taking into account boundary condition

$$\mathcal{D}|_{u=\tilde{u}} = 0,$$

we obtain the expression for the diffusion coefficient on the interval $(-\tilde{u}, \tilde{u})$:

$$\mathcal{D} = \mathcal{D}_0(u) - \mathcal{D}_0(\tilde{u}). \tag{16}$$

Thereafter the equation (7) reduces to the equation for interval boundary:

$$\frac{d\mathcal{D}_0(\tilde{u})}{d\tilde{u}}\frac{\partial\tilde{u}}{\partial\tau} = \tilde{u}\frac{\partial\tilde{u}}{\partial\zeta}. \tag{17}$$

From the last equation it follows that the boundary point $\tilde{u}$ moves in space with constant velocity, so that at $\tau \gg 1$

$$\zeta \approx V(\tilde{u})\tau, \quad V(\tilde{u}) = -\left(\frac{1}{\tilde{u}}\frac{d\mathcal{D}_0(\tilde{u})}{d\tilde{u}}\right)^{-1} \tag{18}$$

It has been supposed above that the leading front is extended with time and movement velocity $V(0)$ of closing point $\tilde{u} = 0$ is minimal. Actually this is true only when

$$\frac{dV(\tilde{u})}{d\tilde{u}} = V^2(\tilde{u})\frac{d}{d\tilde{u}}\left(\frac{1}{\tilde{u}}\frac{d\mathcal{D}_0(\tilde{u})}{d\tilde{u}}\right) > 0 \tag{19}$$

for all $0 < \tilde{u} < 1$. In particular, any initial function $\mathcal{D}_0(\tilde{u})$ that has the negative second-order derivative decreasing monotonically from the boundaries of the interval (-1,+1) to its center satisfies the condition (19).

Let us consider now the example of such deformation that is shown in Fig.2. In contrast to previous case we supposed that oscillations with phase velocity $v \approx v_0$ ($u \approx 0$) damp most rapidly. By this is meant that there is peculiar "dividing" point on the leading front after which the packet is divided into two arms. At large time τ the length of arms formed is large ($\Delta\zeta_{fr} \sim \tau$) and the intensity of oscillations does not vanish now within two velocity intervals:

$$(-1, -\tilde{u}), \quad (\tilde{u}, +1). \tag{20}$$

The distribution function has the "plateau" of its own in each arms:

$$g|_{u\in(-1,-\tilde{u})} \approx \frac{1+\tilde{u}}{2}, \quad g|_{u\in(\tilde{u},1)} \approx -\frac{\tilde{u}+1}{2}$$

and outside of intervals (20) it remains unperturbed. The intervals (20) are closed at $\zeta = \zeta_*$ (ζ_* is coordinate of the dividing point) and, as shown in

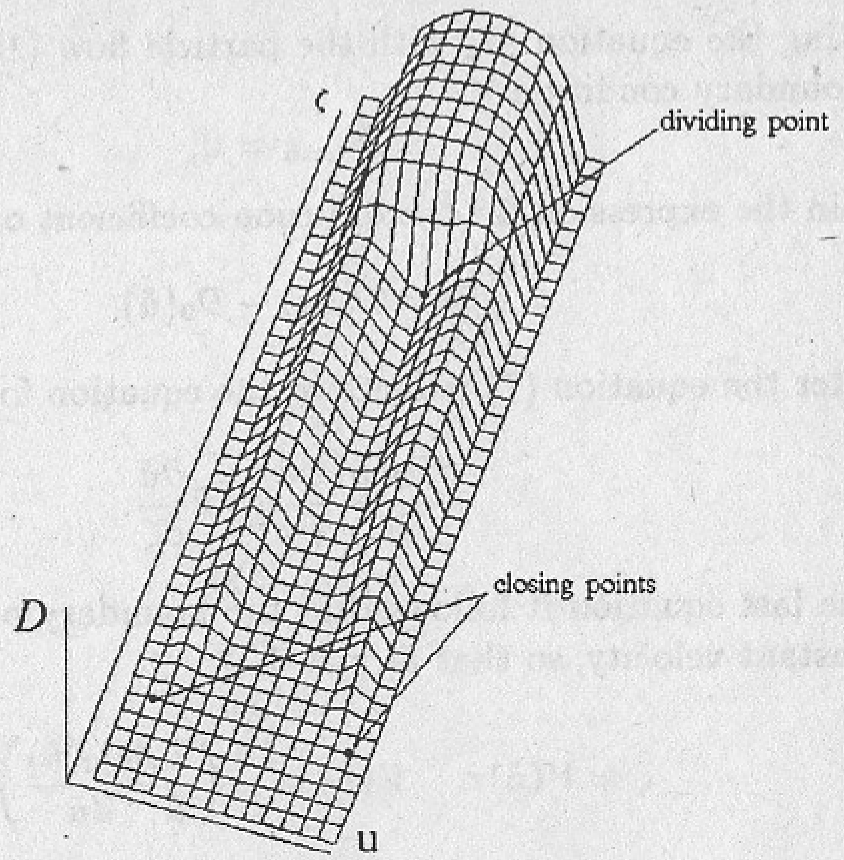

Figure 2. The damping of packet with division of leading front into two arms.

Fig.3, there arises a step in distribution function in center of velocity interval $(-1, +1)$. The diffusion coefficient vanishes never within this velocity region at $\zeta > \zeta_*$ and, as may be seen from diffusion equation (6), the blurring of the step and the formation of the plateau in the distribution function on whole interval $(-1, +1)$ occur on small space scale $\Delta\zeta_* \sim 1 \ll \Delta\zeta_{fr}$. This process is accompanied by essential dissipation of oscillation energy.

Thus, there is the dissipation wave travelling ahead dividing point with velocity V_*. In this wave the distribution function and diffusion coefficient are connected by relation (11). Denoting $\mathcal{D}_* = \mathcal{D}(\zeta_*, u, \tau)$ and integrating equation (11) over u at $\zeta = \zeta_*$ with boundary conditions

$$\mathcal{D}_*|_{u=-1} = 0, \quad \mathcal{D}_*|_{u=0} = 0, \quad \mathcal{D}_*|_{u=+1} = 0,$$

we obtain

$$V_* = 1/2\mathcal{D}_0(0), \tag{21}$$

$$\mathcal{D}_* = \mathcal{D}_0(u) - (1 - |u|)\mathcal{D}_0(0). \tag{22}$$

It is remarkable that we have found change of diffusion coefficient caused by dissipation wave not solving the equation (12) that governs $\mathcal{D}$ on small scales.

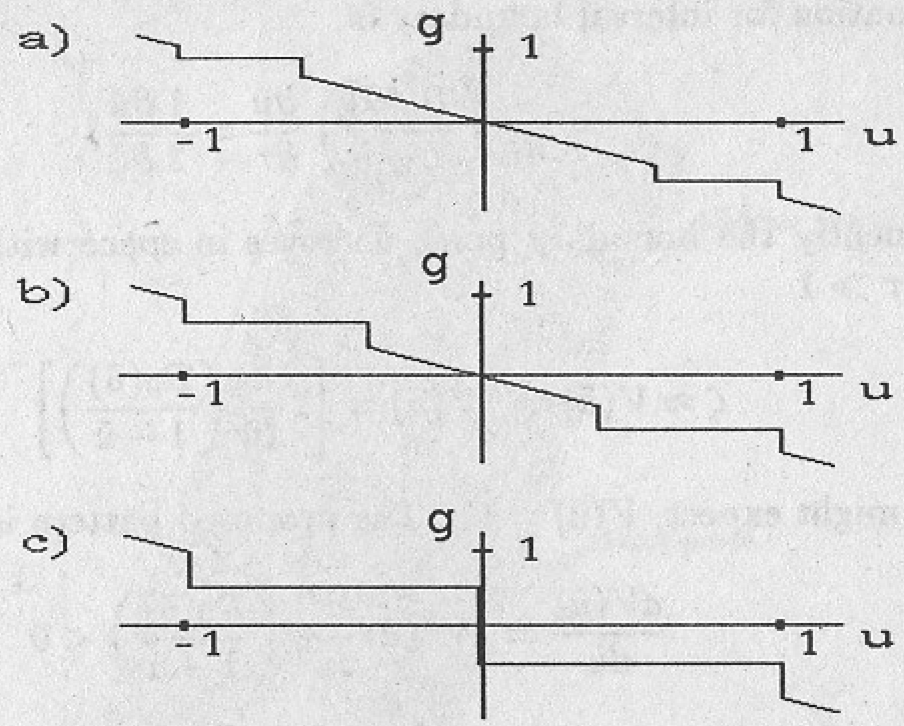

Figure 3. The distribution function at different points of the leading front: a) at $\zeta = \zeta_1$, b) at $\zeta = \zeta_2 > \zeta_1$, c) at $\zeta = \zeta_* > \zeta_2$.

We encounter with a similar situation when we deal with propagation of usual shock waves in which a change of parameters of medium is not connected with their microscopic structure.

Knowing the spectrum of oscillations at the point $\zeta = \zeta_*$, one can determine the diffusion coefficient in region $\zeta < \zeta_*$ where all functions vary smoothly in space. Clearly the damping of arms occurs independently of one another so that we can restrict our consideration to the right arm. Now the particle flow from the left end of interval $(\tilde{u}, 1)$ to the right depends linearly on the velocity u:

$$j = -\mathcal{D}\frac{\partial g}{\partial \tilde{u}} \approx -\frac{1}{2}(1-u)\frac{\partial \tilde{u}}{\partial \zeta}. \tag{23}$$

and integration the equation (7) with boundary conditions

$$\mathcal{D}|_{u=\tilde{u}} = 0, \quad \mathcal{D}|_{u=1} = 0,$$

enables to determine the expression for the diffusion coefficient on velocity interval $(\tilde{u}, 1)$:

$$\mathcal{D} = \mathcal{D}_*(u) - \frac{1-u}{1-\tilde{u}}\mathcal{D}_*(\tilde{u}) = \mathcal{D}_0(u) - \frac{1-u}{1-\tilde{u}}\mathcal{D}_0(\tilde{u}), \tag{24}$$

The equation for interval boundary is

$$-\frac{d}{d\tilde{u}}\left(\frac{\mathcal{D}_0(\tilde{u})}{1-\tilde{u}}\right)\frac{\partial\tilde{u}}{\partial\tau}=\frac{1}{2}\frac{\partial\tilde{u}}{\partial\zeta}, \tag{25}$$

Consequently the boundary point $\tilde{u}$ moves in space with constant velocity and at $\tau\gg 1$

$$\zeta\approx V(\tilde{u})\tau,\quad V(\tilde{u})=\left[2\frac{d}{d\tilde{u}}\left(\frac{\mathcal{D}_0(\tilde{u})}{1-\tilde{u}}\right)\right]^{-1}. \tag{26}$$

As one might expect, $V(0)=V_*$. The proposed pattern is true only when

$$\frac{dV(\tilde{u})}{d\tilde{u}}=2V^2(\tilde{u})\frac{d^2}{d\tilde{u}^2}\left(\frac{\mathcal{D}_0(\tilde{u})}{1-\tilde{u}}\right)<0 \tag{27}$$

for all $0<\tilde{u}<1$. In particular, any initial function $\mathcal{D}_0(\tilde{u})$ that has the negative second-order derivative increasing monotonically from the boundaries of the interval (-1,+1) to its center satisfies the condition (27). In this case the dissipation wave reduces the density of oscillation energy by 50 percent or more.

4 Damping of a packet with generally shaped spectrum

Summarizing the results obtained above, we can state that in the general case the leading edge of the packet consists of the set of dividing points, smooth arms and closing points (see Fig. 3). Dividing points and associated dissipation waves move with constant and, generally speaking, different velocities. A similar statement is true for closing points, while the smooth arms extend more and more with the passage of time.

Now at given values ζ and τ, the intensity of oscillations does not vanish within several intervals of velocity u: $(\tilde{u}_i,\tilde{u}'_i)$, $i=1,...I$. At $\tau\gg 1$ the boundaries of the intervals vary smoothly in space, so that distribution function has plateau of its own in each arm:

$$g|_{u\in(\tilde{u}_i,\tilde{u}'_i)}\approx-\frac{\tilde{u}_i+\tilde{u}'_i}{2},\quad i=1,...I.$$

Broadening of the boundaries of i-th interval with increasing spatial coordinate leads to initiation of particle flow from the left end of this interval to

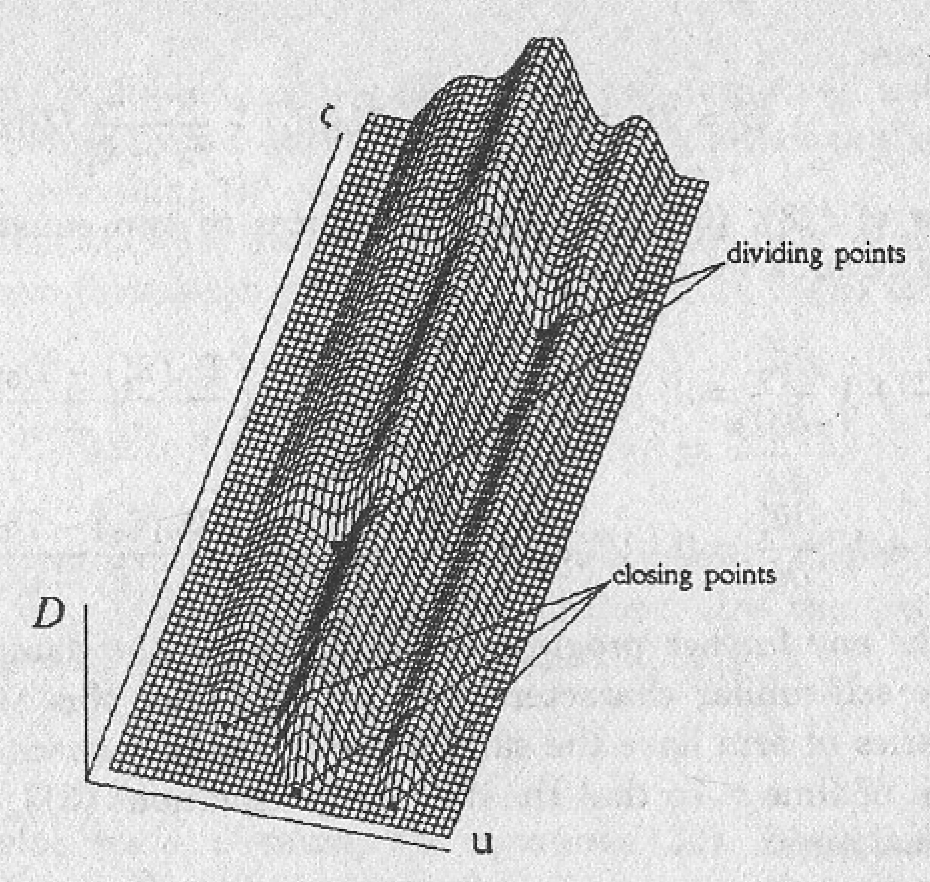

Figure 4. The damping of packet with generally shaped spectrum.

the right

$$j=\frac{1}{2}(u-\tilde{u}_i)\frac{\partial\tilde{u}'_i}{\partial\zeta}+\frac{1}{2}(u-\tilde{u}'_i)\frac{\partial\tilde{u}_i}{\partial\zeta}. \tag{28}$$

From (7), (28) it follows that the time variation of diffusion coefficient is linear function of velocity u while ζ lies within i-th arm. It is not difficult to find also variation of this coefficient caused by the passage of the dissipation wave at some preceding instant of time. For this purpose let us note that distribution function and diffusion coefficient before and after the passage of the dissipation wave are connected by relationship analogous with (11):

$$g_*^{(+)}-g_*^{(-)}=-V_*\frac{\partial(\mathcal{D}_*^{(+)}-\mathcal{D}_*^{(-)})}{\partial u}. \tag{29}$$

Within the interval $\tilde{u}_i(\zeta,\tau)<u<\tilde{u}'_i(\zeta,\tau)$, the left side of the relationship (29) does not depend on velocity u, so that diffusion coefficient changes abruptly by some linear function u after passage of dissipation wave. This enables to state that spectrum in given arm differs from the initial by some linear function of velocity. Taking into account the boundary conditions

$$\mathcal{D}|_{u=(\tilde{u})}=0,\quad\mathcal{D}|_{u=(\tilde{u}'_i)}=0,$$

we obtain:

$$\mathcal{D} = \mathcal{D}_0(u) - \frac{u-\tilde u_i}{\tilde u_i' - \tilde u_i}\mathcal{D}_0(\tilde u_i') - \frac{u-\tilde u_i'}{\tilde u_i - \tilde u_i'}\mathcal{D}_0(\tilde u_i). \quad (30)$$

In view of (28), (30) equation (7) reduces to two equations for smoothly varying boundaries $\tilde u_i$, $\tilde u_i'$:

$$\frac{\partial \tilde u_i}{\partial \tau} + V\frac{\partial \tilde u_i}{\partial \zeta} = 0,\ V(\tilde u_i, \tilde u_i') = -\left[2\frac{\partial}{\partial \tilde u_i}\left(\frac{\mathcal{D}_0(\tilde u_i') - \mathcal{D}_0(\tilde u_i)}{\tilde u_i' - \tilde u_i}\right)\right]^{-1}. \quad (31)$$

$$\frac{\partial \tilde u_i'}{\partial \tau} + V'\frac{\partial \tilde u_i'}{\partial \zeta} = 0,\ V'(\tilde u_i, \tilde u_i') = -\left[2\frac{\partial}{\partial \tilde u_i'}\left(\frac{\mathcal{D}_0(\tilde u_i) - \mathcal{D}_0(\tilde u_i')}{\tilde u_i - \tilde u_i'}\right)\right]^{-1}. \quad (32)$$

To make any further progress we shall assume the damping of the packet to have self-similar character at $\tau \gg 1$. Under this assumption, mobile boundaries of arm have the same velocity at given space point ζ and given moment of time τ, so that the differential equations (31), (32) reduce to the functional ones:

$$V(\tilde u_i, \tilde u_i') = V'(\tilde u_i, \tilde u_i') = \frac{\zeta}{\tau}. \quad (33)$$

If one of boundaries (i.g., the right) is immobile then functional connection between $\tilde u_i, \tilde u_i'$ is modified:

$$V(\tilde u_i, \tilde u_i') = \frac{\zeta}{\tau},\quad \tilde u_i' = const. \quad (34)$$

The solution of the equations (33), (34) is multiple-valued function $\tilde u = \tilde u(V)$ ($V = \zeta/\tau$). Our main interest is the inverse function $V = V(\tilde u)$ determining displacement velocity of each boundary point in space. It is single-valued and its extremum points coincides with branch points of function $\tilde u(V)$. Furthermore the function $V(\tilde u)$ has no discontinuities for smooth initial spectra.[1] This means, in particular, that immobile boundary of some arm coincides always with the boundary of initial interval and hence in the equation (34) we must put

$$\tilde u_i' = +1. \quad (35)$$

The boundaries of the same arm are closed at the minimum points of the function $V(\tilde u)$. These points correspond to the closing points of the leading front of the packet. Further, the boundaries of the different arms arise and then come apart at the maximum points of this function. These

[1]The function $\mathcal{D} = \mathcal{D}_0(u)$ is assumed to have bounded continuous second-order derivative on the interval (-1,+1).

points correspond to the dividing points. Thus, one can extract all essential information about leading front from equations (33), (34), although they are appropriate only for describing the smooth arms.

A set of equations (33), (34) has simple graphic solution. Let us draw a parabola on u-$\mathcal{D}$ plane that is tangent to spectral curve $\mathcal{D} = \mathcal{D}_0(u)$ at point $(\tilde u, \mathcal{D}_0(\tilde u))$:

$$P_\alpha(u, \tilde u) \equiv -\alpha(u-\tilde u)^2 + \beta(u-\tilde u) + \mathcal{D}_0(\tilde u),\quad \beta \equiv \frac{d\mathcal{D}_0(\tilde u)}{d\tilde u}, \quad (36)$$

and let coefficient of square term be inversely proportional to displacement velocity of point $\tilde u$: $\alpha = 1/2V$, $V \equiv \zeta/\tau$. One can check that this parabola is tangent to the spectral curve at the second point $(\tilde u', \mathcal{D}_0(\tilde u'))$:

$$P_\alpha(u, \tilde u) = P_\alpha(u, \tilde u'),$$

if and only if the velocities $\tilde u$, $\tilde u'$ satisfy the equations (33). Equations (34) with refinement (35) mean that the parabola $P_\alpha(u, \tilde u)$ can pass through points $(-1, 0)$, $(+1, 0)$ of u-$\mathcal{D}$ plane making an angle with spectral curve.

To determine displacement velocity V of any boundary point $\tilde u$ of the leading front one can use peculiar minimum principle. Let us consider a set of parabolas $\{P_\alpha(u, \tilde u)\}$, that lie under spectral curve

$$P_\alpha(u, \tilde u) \le \mathcal{D}_0(u) \quad (37)$$

on whole interval $-1 \le u \le +1$. Parabolas with great values of coefficient α are very similar to vertical line segment lying under spectral curve. It is apparent that they belong to the set $\{P_\alpha(u, \tilde u)\}$; while parabolas with small values of coefficient α are very similar to straight line that is tangent to spectral curve at point $(\tilde u, \mathcal{D}_0(\tilde u))$ and do not belong to the set. Hence there is parabola belonging to this set - let us denote it by $\mathcal{P}(u, \tilde u)$ - that has minimum value of coefficient $\alpha = \alpha_{min}(\tilde u)$. Displacement velocity V as function of boundary point $\tilde u$ is determined by formula:

$$V(\tilde u) = 1/2\alpha_{\min}(\tilde u). \quad (38)$$

The parabola $\mathcal{P}(u, \tilde u)$ is tangent to the spectral curve $\mathcal{D}_0(\tilde u)$ at all common points except at the boundary ones where these curves can make an angle with each other. The classification of arising situations by amount of common points is given below:

1. When the curves $\mathcal{P}(u, \tilde u)$ and $\mathcal{D}_0(u)$ have only one common point $(\tilde u, \mathcal{D}_0(\tilde u))$,

they are tangent to each other and have the same curvature at the common point.[2] The point $\tilde{u}$ is a closing point and its displacement velocity is

$$V(\tilde{u}) = -[d^2\mathcal{D}_0(\tilde{u})/d\tilde{u}^2]^{-1}.$$

The parabola 1 in Fig. 5 is tangent to spectral curve at the point that has a minimal displacement velocity.

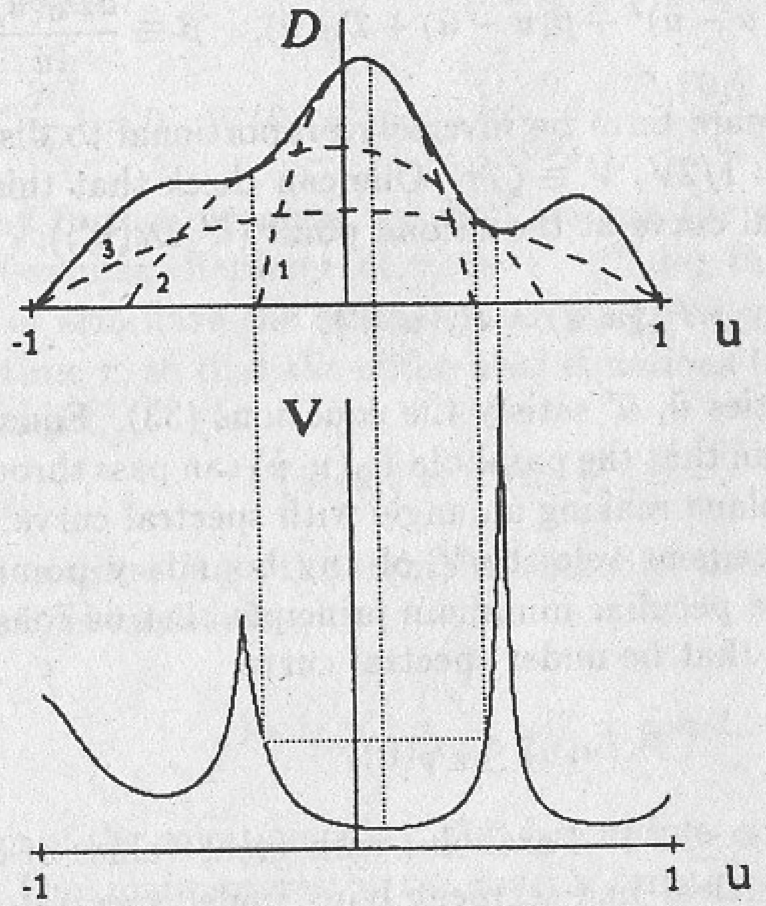

Figure 5. Application of the minimum principle to analysis of damping of the packet shown in Fig.4. The parabolas 1, 2, 3 correspond respectively to situations 1, 2, 3 of presented classification.

2. When the parabola $\mathcal{P}(u,\tilde{u})$ is tangent to spectral curve at the second point $(\tilde{u}', \mathcal{D}(\tilde{u}'))$, both the points $\tilde{u}$, $\tilde{u}'$ are boundary points of the same arm and have the equal displacement velocities (this case is illustrated in Fig.5 by the parabola 2). The displacement velocities of all points lying within interval $(\tilde{u}, \tilde{u}')$ do not exceed $V(\tilde{u})$, that guarantees a self-consistency of the solution presented. If $\tilde{u}' = -1$ or $\tilde{u}' = +1$ then the parabola and the spectral curve can make an angle with each other.

[2]This point coincides with a local minimum point of the second-order derivative of the function $\mathcal{D} = \mathcal{D}_0(\tilde{u})$.

3. When the parabola $\mathcal{P}(u,\tilde{u})$ is tangent to spectral curve at three points $(\tilde{u}, \mathcal{D}(\tilde{u}))$, $(u', \mathcal{D}(u'))$ and $(u'', \mathcal{D}(u''))$, all these points have the same displacement velocity. The middle point is a dividing point. The parabola 3 in Fig.5 corresponds to the case that $u' = -1$, $u'' = +1$.
4. In the special case that parabola $\mathcal{P}(u,\tilde{u})$ coincides with spectral curve on some interval, all points of this interval have the same displacement velocity. In particular, coincidence of the parabola with spectral curve on all interval (-1,+1) means that all points of the leading front have the same displacement velocity and erosion of the packet is carried out by dissipation wave discussed in section 3.

As the last example, let us consider the packet with initial diffusion coefficient having N segments, on which the second-order derivative of function $\mathcal{D}_0(\tilde{u})$ is strictly monotonic. In this case a number of arms, which the leading front divides into, is not more than $(N+1)/2$. And the velocity of the slowest point is equal

$$\min V(\tilde{u}) = \min[-d^2\mathcal{D}_0(\tilde{u})/d\tilde{u}^2]^{-1}.$$

The graphic approach enables also to determine the damping of packet in case, when function $\mathcal{D}_0(\tilde{u})$ is unsmooth. For this, it will suffice to note that one can consider such a function as a limit of some set of smooth functions, each being studied by using the minimum principle.

5 Conclusion

As a result of interaction of particles and waves, there arises a motion of the leading front of the packet as a whole that is accompanied by the linear front extension. By applying the minimum principle, we can describe all structure features of the leading packet front in detail: motion and position of dividing and closing points, deformation and form of arms. Our analysis has been based on the assumption that the damping of the packet has a self-similar character. Computer simulation of quasilinear equations (6), (7) with conditions (8), (9) confirms a self-similarity of diffusion processes after time $t \gg \gamma^{-1}$ and numerical results are in good agreement with the analytical consideration.

Let us discuss conditions of applicability of results obtained and possibility of generalization of theirs.

When temperature of plasma not being equal to zero, we should take into account the group velocity of Langmuir waves in the equation (2) [7]. In this case, it is obvious that the velocity of motion of the leading front as a whole increases by average group velocity of waves in packet. And deformation of the front does not undergo essential changes if dispersion of the group velocities is negligible:

$$\Delta v_g \sim v_g(\Delta v_0/v_0) \ll (\gamma\tau_{dif})v_0.$$

We have dropped the derivative $\partial f/\partial t$ in the equation (1). One can take into account this term and get the expression for the displacement velocity of boundary points little differing from (38).

We have investigated time evolution of one-dimensional Langmuir wave packets with the leading front in the shape of step (3) at the initial instant. At smooth variation of initial diffusion coefficient on space scale $\sim \Delta L_0$, entering into self-similar regime described above takes place after time

$$t \gg \max\{\gamma^{-1}, \Delta L_0/[(\gamma\tau_{dif})v_0)]\}, \quad (39)$$

except when the initial spectral curve coincides with parabola. In this case the velocities of the points of the leading front differ little from one another after time (39). In consequence of anomalously slow deformation of the leading front, entering into self-similar regime (14) occurs after time

$$\ln(\gamma t) > \max\{1, \Delta L_0/[\tau_{dif})v_0]\}.$$

As for the back front of quasilinear packet, it remains motionless (at plasma temperature $T = 0$).

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V.N. Khudik, O.A. Shevchenko

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В.Н. Худик, О.А. Шевченко

Бесстолкновительное затухание квазилинейного пакета ленгмюровских волн

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