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A.R. Zhitnitsky

# TORONS, CHIRAL SYMMETRY BREAKING AND U(1) PROBLEM IN QCD AND SQCD

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НОВОСИБИРСК

TORONS, CHIRAL SYMMETRY BREAKING AND $U(1)$ PROBLEM IN QCD AND SQCD

A.R.Zhitnitsky

The new class of self-dual solutions with fractional topological charge in SU(2) gauge theories is considered. The solution is defined on manifold with boundary, it has topological charge $Q = 1/2$ and action $S = \frac{8\pi^2}{g^2} Q$. The contribution of the corresponding fluctuations to chiral condensates $\langle \tilde{\varphi}\varphi \rangle$, $\langle \lambda^2 \rangle$ in SQCD is calculated and it is consistent with Konishi anomaly. For the fermion condensate in QCD (with $N_f = N_c = 2$, $m \to 0$) we find $\langle \bar{\psi}\psi \rangle = -\pi^2 2^4 e^{5/12} \Lambda^3$, $\Lambda^3 \equiv M_0^3 \frac{1}{g^4(M_0)} \exp\{-4\pi^2/g^2\}$. The unbound resonances of the continuum at $\lambda = 0$ play a crucial role in this calculation.

## 1. Introduction

At this time the best-known example of a nonperturbative fluctuation is the instanton [1]. The integral nature of the topological charge $Q$ is in that case related to the compactification of the space to sphere, i.e. with the identification of all infinitely distant points. The choice of other boundary conditions could result in fractional topological charge. In particular, in gluodinamics with the SU(N) gauge group the introduction of so-called twisted boundary conditions [2] permits solutions of the classical equations - torons [3] - with $Q = k/N$, $k = 0, 1, \ldots N-1$ and with action $S = 8\pi^2/g^2 N$. However, this solution is defined in a box of sizes $L_\mu$ and exists only if the ratios of the sizes $L_\mu$ of the box satisfies certain relations. Besides that the introduction into the theory of fundamental representation of fields (quarks) is rather difficult because of special (twisted) boundary conditions. Moreover, the standart quasiclassical calculations of gluino condensate $\langle \lambda^2 \rangle$ in SYM [4], based on 't Hooft's solution [3] are unreliable because $g(L \to \infty) \to \infty$. Therefore, 't Hooft's solution [3] can be considered only as an illustrative example with fractional $Q$.

The solutions with fractional $Q$, can be formulated [5,6] in some way other than 't Hooft's solution. Our solutions [5,6] is defined on two Riemann sheets (for SU(2) group); the physical space is one of them. There are an alternative points of view on toron* solution. They are connected with the consideration of manifold with boundary or description on orbifolds (in a more details see Refs. [5,6]).

What physical effects arise due to solutions with fractional $Q$? These effects appear most glaringly in supersymmetric variants of the theory. In particular, in supersymmetric Yang-Mills theory (SYM) with SU(2) gauge group, instantons give zero contribution to $\langle \lambda^2 \rangle$ and can ensure nonzero values only for the correlator $\langle \lambda^2(x), \lambda^2(0) \rangle$ (for review, see Refs. [7,8]).

* We keep the term "toron" introduced in Ref. [3] for self-dual solution in $\sigma$-model [5] and in gauge theories [6] as well. By this means we emphasize the fact that the solution minimizes the action $S$ and carries topological charge $Q = 1/2$, i.e. possesses all of the characteristics ascribed to the toron

The torons generate the condensate $\langle\lambda^2\rangle$. Corresponding calculations, based on 't Hooft's toron solution were carried out in Ref. [4]. Analogous calculations based on our solution, which may be understood as the point defect with size $\Delta\to 0$ and which admitts introduction of the fields in the fundamental representation, were carried out in Refs. [5,6].

This is precisely the purpose of the current letter to find a toron measure and to calculate the chiral condensates in the theories with fundamental representation of fields - SQCD and QCD.

The difference with the SYM case [6] is that in SYM toron effects may be reconducted to fermionic zero modes (ZM); in SQCD and QCD quasi-zero modes at $\lambda\to 0$ play a crucial role. Moreover, ZM of quarks are absent in toron's background, it may be seen from anomaly equation.

## 2. The toron measure and condensates in SQCD

We shall follow the notations of [6] and consider the toron measure in SYM [6]:

$$Z_{SYM} = C\,\frac{M_0^4 d^4x_0}{g^4}\cdot\frac{d^2\varepsilon}{M_0}\cdot \exp\left\{-\frac{4\pi^2}{g^2}\right\} \tag{1}$$

Here the factor $\exp\left\{-\frac{4\pi^2}{g^2}\right\}$ is connected with the classical action; $d^4x_0$ corresponds to integration over the four collective variables (translations of toron); $M_0^4/g^4$ is the regulator contribution, corresponding to the four ZM mentioned above; lastly, $d^2\varepsilon/M_0$ is connected with two gluino ZM; $C$ - is the numerical const, $C\neq 0$.

The expression (1) ensure nonzero value for $\langle\lambda^2\rangle$ and correct renormalization group dependence as well [6]:

$$\langle g^2\lambda^2\rangle = 2\frac{C}{g^2}\Lambda^3_{1\text{-}loop}\,,\quad \Lambda^3_{1\text{-}loop} = M_0^3\exp\left\{-\frac{4\pi^2}{g^2}\right\} \tag{2}$$

We pass now to the analysis of toron measure in SQCD. In this case we have additionally the two factors $d_B$ and $d_f$ connected to bosonic and fermionic guadra-tic functional integration around the toron.



$$Z_{SQCD} = Z_{SYM}\,(d_f)^{N_f}(d_B)^{-N_f}$$

$$d_f = Det\left[\frac{-i\hat{\mathcal{D}}-im}{-i\hat{\partial}-im}\right],\quad d_B = Det\left[\frac{-\mathcal{D}^2+m^2}{-\partial^2+m^2}\right] \tag{3}$$

Here the regulator contribution is undermined, although not explicitly indicated. Formal manipulation of $d_f$ allows us to write [9]:

$$m\frac{d}{dm}\ln d_f = Tr\left[\frac{m^2}{-\hat{\mathcal{D}}^2+m^2} - \frac{m^2}{-\hat{\partial}^2+m^2}\right] = \tag{4}$$

$$= Tr\left[\frac{m^2}{-\mathcal{D}^2+m^2} - \frac{m^2}{-\partial^2+m^2}\right] - Tr\left[\frac{m^2\gamma_5}{-\hat{\mathcal{D}}^2+m^2}\right].$$

The symbol $Tr$ denotes a trace over space-time, Dirac and color indices. As is well known, the last term in (4) is connected with the index of Dirac operator, actually independent of $m$ and equal to topological charge $Q$ of background field [10]. The first term in (4) cancels with $d_B$ and we obtain:

$$m\frac{d}{dm}\ln Z_{SQCD} = QN_f\,,\quad Z_{SQCD}\sim\left(\frac{m}{M_0}\right)^{QN_f} \tag{5a}$$

$$-Tr\left(\frac{m^2\gamma_5}{-\hat{\mathcal{D}}^2+m^2}\right) = Q \tag{5b}$$

In particular, in instanton case with $Q=1$, the expression (5) corresponds simply to ZM contribution, i.e. $Z\sim(m)^{N_f}$.

In toron case with fractional topological charge ($Q = 1/2$ for SU(2)) the ZM of quarks are absent, but formula (5b) is still correct. The l.h.s. of (5b) in this case is determined by quasi-zero modes ("unbound resonance at $\lambda = 0$ [11]). From mathematical point of view, this phenomenon is connected to definition of our solution on manifold with boundary and the index theorem should be modified (in a more detail see Ref. [5]).

The formula (5) implies that for $m \to 0$ (chiral limit) the toron measure $Z_{SQCD}$ depends on "$m$" as $\prod m_i^{1/2}$, i.e. we have fractional degree of "$m$" for each flavor.

Collecting all the factors (3)-(5), we get for the toron mesure the following expression:

$$Z_{SQCD} = C \frac{1}{g^4} M_0^4 d^4x_0 \frac{d^2\varepsilon}{M_0} \left(\frac{m}{M_0}\right)^{N_f/2} \tag{6}$$

We shall calculate now the gluino condensate in SQCD. At this point the calculation proceeds exactly as in the pure SYM case (2), [6]. The only difference is that now the invariant appropriate to SQCD and the corresponding $\beta$ function will appear. Hence (for equal $m_i$):

$$\langle g^2\lambda^2\rangle = \frac{2C}{g^2} \Lambda^{3-\frac{N_f}{2}} (m)^{N_f/2}, \quad \Lambda_{1-loop}^{3-N_f/2} \equiv M_0^{3-\frac{N_f}{2}} \exp\left\{-\frac{4\pi^2}{g^2}\right\} \tag{7}$$

One finds that the "$m$", "$g$" - dependence of eq.(7) is just what is needed [8]. At this point one can make use of the Konishi anomaly [12],

$$\frac{1}{32\pi^2} \langle g^2\lambda^2\rangle = m\langle\tilde{\varphi}\varphi\rangle \tag{8}$$

and find the value of the scalar field condensates. However, an independent determination of them is possible and will provide a valuable consistency check of our approach. The results of corresponding calculations (fig. 1) is in agreement with the eq.(8) and will be published in Ref.[13].

Let's note that each kind of configurations ( instantons [8] and torons) gives a contribution satisfying all the consistency checks (in particulary (8)). However, numerical factors do not agree with each other, and they differ on factor $\sqrt{4/5}$, $C = 2^5\pi^2$ (details in Ref. 13 ). Up to now we don't know the origin of this difference.



3. The chiral condensate in QCD

We pass now to the analysis of toron measure in QCD. In this case factors $d^2\varepsilon/M_0$ and $(d_\theta)^{-N_f}$ (6) are absent, but we still have to include in the measure (6) the nonzero modes.

We pause briefly to mention the contribution of the nonzero modes. Up to logarithmic accuracy, the total contribution of the nonzero modes can be easily calculated with the help of the usual Feynman diagrams, as was done for gauge theories in Ref. [14]. The effective addition to the action is determined by Fig. 2. (for gauge fields) and Fig. 3. (for fermion fields) and equals:

$$\begin{aligned} S &= \frac{4\pi^2}{g^2} + \Delta S_g + \Delta S_f \\ \Delta S_g &= \frac{2}{3}\frac{g^2}{16\pi^2} \ln M_0^2 \int d^4x \left[\frac{1}{4}G_{\mu\nu}^2\right]_{class.} = \frac{2}{3} Q \ln M_0 \\ \Delta S_f &= \frac{g^2 N_f}{16\pi^2} \ln M_0^2 \left(1-\frac{1}{3}\right) \int d^4x \left[\frac{1}{4}G_{\mu\nu}^2\right]_{class.} = Q N_f \left(1-\frac{1}{3}\right) \ln M_0 \end{aligned} \tag{9}$$

Let's remind that the first term in eq.(4) in SQCD cancels with $d_\theta$. In the case of QCD under consideration the first term in eq.(4) is small at $m \to 0$ and up to logarithmic accuracy equals $3/2\, m^2 \ln m \to 0$ (see Appendix of Ref.[13]). So, this term gives a negligible contribution to the toron measure.

Collecting all factors together we obtain the following expression for the toron density in QCD with $N_c = 2$ :

$$Z_{QCD} = K \frac{M_0^4}{g^4} d^4x_0 \left(\frac{m}{M_0}\right)^{N_f/2} e^{-4\pi^2/g^2} \exp\left\{-\frac{1}{6}(2-N_f)\ln M_0\Delta\right\} \tag{10}$$

Here $\Delta \to 0$ is the regulator of our toron solution, which may be understood as the point defect with size $\Delta \to 0$. The important difference with the supersymmetric case is that in supersymmetric theories the regulator $\Delta$ in the expression for the toron mesure is absent because of cancellation of NZM contribution. In QCD - case the dependence on $\Delta$ appears. In particulary for $N_f = 1$ we have $Z_{QCD} \sim d^4x\, \Delta^{-1/6}$. It is obviously that the limit $\Delta \to 0$ can not be taken in this expression because quasiclassical calculation is correct only for small toron density $Z \ll 1$. Indeed, it may be shown [13] that interaction is very essential and we can't calculate anything in this case. The case of the QCD with $N_f = 2$, $m \to 0$ (in more general $N_f = N_c$)* calls for particular attention. In this case the toron measure:

$$Z_{QCD}(N_f=2) = K \Lambda^3_{1-loop}\, d^4x_0 \cdot m^1, \quad \Lambda^3_{1-loop} \equiv M_0^3 \frac{1}{g^4} \exp\{-\frac{4\pi^2}{g^2}\} \qquad (11)$$

do not depand on $\Delta$. From technical point of view this case reminds supersymmetric theories because of cancellation of NZM.

We pass now to the calculation of chiral condensate in QCD with $N_f = N_c = 2$, $m \to 0$. In this case quasi-zero modes play a crucial role, as it was in the analysis of toron measure (5), (6). Namely these modes can cancel the small quark mass $m \to 0$ in front of the expression $Z_{QCD}$(11) and can ensure the nonzero values for condensate $\langle \bar\Psi \Psi \rangle$ at $m \to 0$.

By definition, we have:

$$\langle \bar\Psi \Psi \rangle_{mink} = -i \langle \psi^+ \psi \rangle_{Eucl.} = -i K \Lambda^3 m \int d^4x_0\, tr \frac{-im}{[-\hat D^2 + m^2]} \qquad (12)$$

Here $\psi$ - any from flavors $(u, d)$ and factor $\psi^+\psi$ gets replaced by Green-function in the toron background. The evaluation of eq.(12) is now straightforward:

$$\int d^4x_0\, tr \frac{m^2}{[-\hat D^2 + m^2]} = \int d^4x_0\, tr \frac{m^2(1+\gamma_5)}{[-D^2 + m^2]} - \qquad (13)$$

$$- \int d^4x_0 \frac{m^2 \gamma_5}{[-\hat D^2 + m^2]} \underset{m \to 0}{\simeq} Q = 1/2$$

The first term in (13) gives a negligible contribution as before (see text under the eq.(10)); the last term in (13) actually independent of $m$ and equal to topological charge $Q$

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* It is exactly theory of strong interaction at $N_f = N_c = 3, m \to 0$



of background field (4), [10]. Antitoron gives the same value, so that:

$$\langle \bar\Psi \Psi \rangle_{mink.} = -K \Lambda^3_{1-loop} \qquad \Lambda^3_{1-loop} \equiv \frac{M_0^3}{g^4} \exp\{-\frac{4\pi^2}{g^2}\} \qquad (14)$$

The constant $K$ may be calculated and it is equal to: $K = \pi^2 2^4 \exp\{5/12\}$ [13]. It is our main result. The mechanism of chiral symmetry breaking under consideration reminds the one of Ref. [15]. In both cases $\langle \bar\Psi \Psi \rangle \neq 0$ because of quasi-zero modes at $\lambda \to 0$. The difference is that now the quasi-zero modes at $\lambda \to 0$ are inherent property of configurations with fractional $Q$. In Ref. [15] this effect is due to instanton interaction. As is well known, the nonvanishing of the condensate (14) indicates spontaneous breaking of chiral symmetry and correlator

$$\Pi_5 \underset{q \to 0}{=} \int dx\, e^{iqx} \langle T\{u^+ \gamma_5 d(x), d^+ \gamma_5 u(0)\} \rangle = -\frac{\langle \bar\Psi \Psi \rangle_{mink.}}{m} \qquad (15)$$

must tend to infinity when $m \to 0$. The evaluation of l.h.s. (15) is now straightforward. We replace $d(x) d^+(0)$ by massive Green-function in the toron background and integrate over the collective coordinates $d^4x_0$. We don't know the closed form for the inverse of the operator $\hat D + m$ for non-vanishing masses. Fortunately, the evaluation of integral $d^4x_0\, d^4x$ (15) reduces to well-known expression (5b), which is actually independent of $m$ and equal 1/2 and to expressions like the first term in (13), which gives a negligible contribution at $m \to 0$ [13]. The results of corresponding calculations is in agreement with eg. (15) and will be published.

4. <u>U(1) - problem and $\theta$ - periodicity puzzle</u>.

As is well known it is very difficult to satisfy the anomaly ward identities ( $WI$ )

$$\int d^4x \langle T\{Q(x), Q(0)\} \rangle = \frac{1}{2} m \langle \psi^+ \psi \rangle = \frac{1}{2} m K \Lambda^3 \qquad (16)$$

$$\int d^4x \langle T\{Q(x), \frac{u^+\gamma_5 u + d^+\gamma_5 d}{\sqrt{2}}(0)\}\rangle = \frac{-\sqrt{2}}{m}\int d^4x \langle Q(x) Q(0)\rangle = \tag{17}$$
$$= -\frac{\sqrt{2}}{2} K \Lambda^3 \,, \qquad Q \equiv \frac{1}{32\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$$

in the standart instanton picture (see Ref. [16] and references there in). On the other hand, the U(1) problem arises with condensate (14). Without any spontaneous breaking of chiral symmetry there is no U(1) problem. So, in any consistent mechanism for chiral breaking the U(1) problem must be solved in automatic way. We shall compute now the correlators (16), (17) in agreement with W.I. An independent determination of W.I. will provide a valuable consistency check of our approach.

The evaluation of eqs.(16), (17) in now straightforward because the integrals $d^4x\, d^4x_0$ ( $d^4x_0$ from the measure (11)) are factorised, and calculation reduced to eqs.(12,13) and $\int d^4x\, Q = 1/2$ .

It is very important that r.h.s. (16) is of the order of $m^1$ and r.h.s. (17) is of order $m^0$ as it should be. Furthermore, the condensate $\langle\bar\psi_L\psi_R\rangle$ depends on $\theta$ as

$$\langle\bar\psi_L\psi_R\rangle = \exp\{-i\theta/2\}\langle\bar\psi_L\psi_R\rangle_{\theta=0} \tag{18}$$

because:

$$\frac{d}{d\theta}\langle\bar\psi_L\psi_R\rangle_{\theta=0} = -i\int d^4x \langle Q(x), \bar\psi_L\psi_R\rangle_{\theta=0} = \tag{19}$$
$$= -i\left[\int d^4x \cdot Q(x)\right]\langle\bar\psi_L\psi_R\rangle = -\frac{i}{2}\langle\bar\psi_L\psi_R\rangle$$

in agreement with W.I. [16].

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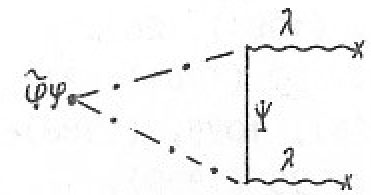


Fig. 1


Fig.2

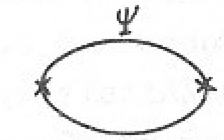


Fig.3

— · — · — · scalar $\varphi$

——— quark $\Psi$

~~~~~ gluino $\lambda$

- - - - - - gluon $a_\mu$

x external field
~~~~~

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А.Р.Житницкий

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