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INSTANTONS IN QCD III. QUARK PROPAGATORS AND MESONS CONTAINING HEAVY QUARK

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Quark Propagators and Mesons
Containing Heavy Quark

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ABSTRACT

We study the instanton-induced effects on the propagators of light quarks, using the pseudoparticle ensemble obtained previously. It is shown how massless quarks develop some effective mass as they «jump» from one instanton to another. Correlation functions for mesons made of a light quark and a very heavy antiquark are also calculated, the results are in good agreement with the available experimental data.

1. INTRODUCTION

During the last few years rapid progress was reached in understanding of the role of instantons in QCD. Instead of a semi-qualitative phenomenological model [1] of early eighties, we now have the quantitative theory of interacting pseudoparticles, studied both analytically [2] and by the direct computer experiments [3, 4]. This theory explains various phenomena associated with the chiral symmetry breaking in the QCD vacuum.

At the present stage of the development of this theory there is plenty of things to do: instead of studies of some simplified theories (with the SU(2) colour, one quark flavour etc.) one may now deal with real QCD, calculate various quantities and compare them directly to experimental data. This is the third paper of the series, devoted to such kind of studies. In the previous two [5] (called below C1 and C2) we have generated ensemble of pseudoparticles, calculated the quark condensate value and also studied the correlation functions for the pseudoscalar and scalar currents.

In this work we study the light quark propagator in the «instanton liquid». There are two reasons for doing this. The first one is mostly euristic: we want to understand in details how a «bare» (or «current») quark becomes «dressed» (or «constituent») one, at which distances in takes place, what is the corresponding spin structure of the propagator, etc. However, as the quark propagator is not a gauge invariant quantity, these studies make sence strictly speaking only for some fixed gauge field configurations and they cannot have direct physical meaning. The idea of massive «constituent quarks» (although existing for decades) is not well defined, so comparison with the phenomenology here is obviously model-dependent.

The second reason is pragmatic: the quark propagator is an important building block for construction of the correlation functions of «white» (gauge invariant) operators. One step toward this aim is made in the second part of this work, were our results for the light quark propagator are used for studies of the correlation functions of the mesonic currents. We restrict our present discussion to operators of the type $\bar{Q}\Gamma q$, where Q is very heavy antiquark, q is the massless quarks, and Γ is a gamma matrix fixing the quantum numbers of a meson. As it was first emphasized in Ref. [6], such mesons are a kind of a «hydrogen atom» of hadronic physics, and it is a pity that we know so little about them from experiment.

Before we come to technical details, let us discuss the main physical questions considered in this and subsequent works in more general terms. First af all, we remind the reader, that the «instanton-liquid» is definitely not the complete picture of the QCD vacuum, for such important penomenon as «colour confinement» is not explained in it. However, instantons do produce «quark effective mass» of the order of 300 MeV. Therefore, a quark—antiquark pair or three quarks do obtain the instanton-induced energy comparable to masses of mesons and baryons.

Moreover, this theory predict that «dressed» quarks strongly interact with each other, and the magnitude of such interaction is prescribed by this theory. One example in which such interaction is crucial was demonstrated in the previous work C2. In the pion case instead of mass $2\cdot m_{eff}\simeq 600$ MeV, one get a meson which is massless in the chiral limit (and very light in the real world).

Considering other hadronic channels, say baryons, we may formulate the following intriquing question: do instantons provide interaction of such «dressed» quarks which is strong enough to form bound states? Can it be so, that these bound states are in good correspondence to physical lowest hadronic excitations?

It is related with more general question: are the confinement forces really very important for the understanding of the lightest hadrons? In literature one may find multiple attempts to describe these particles without confinement. Either they try to understand baryons in terms of slightly bound nonrelativistic «constituent» quarks, or in terms of the pion field (skirmions), or both (chiral

bags, etc.). The theory of instantons seem to provide good description for both these ingredients (consitiuent quarks and the pion field), so it is a reasonable framework to study this general question.

In a technical language, the interaction of quarks propagating in the vacuum show up as some deviation of the mesonic (or baryonic) correlation functions from the product of two (or three) quark propagators. But in order to study such deviations one has to measure the propagators first. Thus, the present work is the necessary step toward the studies of these interaction.

2. LIGHT QUARK PROPAGATION IN THE «INSTANTON LIQUID»

Quark Green function $S(x, y)=\langle\bar{\psi}(x)\psi(y)\rangle$ (or the propagator) in a given gauge field $A^a_\mu(x)$ can be obtained by the inversion of the operator $(i\hat{D}+im)$, where $\hat{D}$ is the «Dirac operator» $\gamma_\mu\left(i\partial_\mu+\frac{g}{2}t^aA^a_\mu\right)$. The most straightforward way to do it is to find all eigenfunctions ψ_λ and eigenvalues λ of the operator $\hat{D}$, and then use the general expression

$$S(x,y)=\sum_\lambda\frac{\psi_\lambda(x)\,\psi^+_\lambda(y)}{\lambda+im}. \qquad (1)$$

In the chiral limit $m\to0$, the most delicate part of such sum is related with λ around zero. As it is well known, for one instanton (or antiinstanton) the operator $\hat{D}$ has zero eigenvalue, corresponding to the famous fermionic zero mode (below ZM) discovered by 't Hooft. As it was explained in detailes in Refs [2, 3], for gauge fields being a superposition of pseudoparticles some eigenvalues λ are close to zero, and the corresponding eigenvectors are, in turn, a superposition of such zero modes. The states belonging to such «zero mode zone» play an outstanding role in QCD, forming the quark condensate, the pions etc. In this paper, devoted to the quark propagator, we also demonstrate the very important role of these states. Due to this fact, for the quark states belonging to ZM zone we do not make any approximations, performing explicit diagonalization of the Dirac operator in the ZM subspace and then using the exact relation (1).

In principle, the same should also be made for the nonzero modes (NZM) as well, but this is very difficult to do in practice. While zero modes are similar to a localized quark bound states (in the so called 5-th time), the NZM are the «scattering states». Obviously, in a complicated «instanton liquid» there are multiple rescatterings of quarks, so these states are complicated. In addition, the sums over NZM are infinite and should be regularized. Thus, we do not attempt straightforward diagonalization of NZM in this work.

Instead the NZM contribution to the propagator is estimated in some approximations, based on the exact expression in the one-pseudoparticle background field found in [7]

$$S^{NZM}(x+z,y+z)=\Big(-\frac{\hat{\Delta}}{2\pi^2\Delta^4}\Big)\Big[1+\frac{\rho^2}{x^2y^2}(\sigma_-x)(\sigma_+y)\Big]h_xh_y+$$
$$+\Big(-\frac{1}{4\pi^2\Delta^2}\Big)\Big[\Big(\frac{\rho}{\rho^2+x^2}\Big)(\sigma_-x)(\sigma_+\gamma)(\sigma_-\Delta)(\sigma_+y)\Big(\frac{1+\gamma_5}{2}\Big)+ \qquad (2)$$
$$+\Big(\frac{\rho^2}{\rho^2+y^2}\Big)(\sigma_-x)(\sigma_+\Delta)(\sigma_-\gamma)(\sigma_+y)\Big(\frac{1-\gamma_5}{2}\Big)\Big]h_xh_y/x^2y^2,$$

where $h_x=1/(1+\rho^2/x^2)^{1/2}$, $h_y=1/(1+\rho^2/y^2)^{1/2}$, $\Delta=x-y$, z is the position of the instanton center and 4-dimensional extension of Pauli matrices defined as $\sigma_\mu^\pm=(\vec{\sigma},\pm i)$. Below we use its simplified version, valid if we take the following traces

$$S^{nonflipping}=\frac{1}{12\Delta}\operatorname{Tr}(\hat{\Delta}S)=\Big(-\frac{1}{2\pi^2\Delta^3}\Big)h_xh_y\Big[1+\frac{\rho^2(xy)}{x^2y^2}+$$
$$+\frac{\Delta^2\rho^2(xy)(2\rho^2+x^2+y^2)}{4x^2y^2(x^2+\rho^2)(y^2+\rho^2)}\Big]=S_0(1+\delta(x,y,z)),\quad S_0=-\frac{1}{2\pi^2\Delta^3}. \qquad (2a)$$

Here we have introduced correction $\delta(x, y, z)$ to the free quark propagator S_0. It is significant if the measurement points x, y are «inside» the instanton, say, at the distance of the order ρ from z. If the distance exceeds ρ, one may speak about interference of two waves, one coming from x to y directly, another propagating from x to z and then scattered from z to y. (By the way, first studies of the low-momentum scattering amplitude of a quark on the pseudoparticle can be found in the third paper [1], where it was used for evaluation of the fermionic determinant at nonzero temperatures.)

Consider now some complicated multipseudoparticle background field. If the propagation distance $\Delta=|x-y|$ is small compared to typical instanton separation R ($\sim$1 fm), significant correction are

mostly produced by one pseudopaticle, if both points are close to its center. (We remind here, that the «instanton liquid» is reasonably dilute, $(\rho/R)^4\simeq 1\%$, and that the mean interaction of pseudoparticles is strongly repulsive at small distances.) Therefore we estimate such corrections δ_I according to (2) for each pseudoparticle, and then select the dominant one δ_{max} (the largest in absolute value). This one is taken into account, while others are ignored.

Such approximation is definitely correct if the distance between x, y is small, but at $|x-y|$ comparable to the interparticle distance it is not reliable. In order to get some insight into the total magnitude of the «nondominant» corrections we have also calculated the product of the correction factors over all pseudoparticles, $\prod_{I,A}(1+\delta_{I,A}(x,y,z))$, see below. (Let us also note, that powerful numerical algorithm for evaluation of the relativistic propagators in arbitrary backround field was developed by O.V. Zhirov [8], and we are now trying to make evaluation of quark propagators without any approximations.)

Let us now qualitatively compare the ZM part of the propagator with the NZ one (2). First of all, for small $\Delta=(x-y)$ the NZ part is obviously dominant: its leading term is of the order of Δ^{-3}. It is, of course, just the free propagator S_0 plus some corrections, both singular and regular at $x\to y$. The ZM part is obviously finite at $x\to y$.

One important observation deals with the chiral structure of these two contributions. Let us consider «chirality flipping» and «nonflipping» parts of the propagator, which may be distinguished by the following traces:

$$S^{flipping}=\frac{1}{12}\operatorname{Tr}S,$$
$$S^{nonflipping}=\frac{1}{12\Delta}\operatorname{Tr}(\hat{\Delta}S). \qquad (3)$$

Now, as the NZM contribution (2) has one gamma matrix in all terms, it belongs to the «nonflipping» part. The ZM contribution has both «chirality flipping» and «nonflipping» parts. Note however, that each pseudoparticle can be considered as a «'t Hooft vertex», emitting a quark-antiquark pair of opposite chiralities. For one pseudoparticle ZM term in the propagator is just $S^{ZM}=\psi_0(x)\psi_0^+(y)/im$ where the quark mass acts as a regulator (unnecessary in the discussion of the collectivized ZM zone). Such contribution has no gamma matrices and is therefore the chirality flipping one, as the mass term. Thus, final chirality of a quark, jumping from one pseudoparticle to another, depends on whether the number of such jumps is even or odd. As we show shortly, in the distance range to be considered the «one jump» contribution always dominates over the «two jumps» one, so our main ZM contribution remains chirality flipping. At larger distances the contribution of even and odd number of jumps becomes, of course, comparable, so both flipping and nonflipping parts of the propagator have similar behaviour.

Let us now proceed to discusssion of the so called «effective mass» issue. As it was noted in [6], at small distances the chirality-flipping part of a massive quark propagator can be written as

$$S^{flipping}(\Delta) \xrightarrow[\Delta\to 0]{} m/4\pi^2\Delta^2 . \tag{4}$$

Comparing it with the definition of the quark condensate, one get then a general expression for the «x-dependent effective mass» at small distances

$$m_{eff}(\Delta) = \frac{\pi^2}{3}\Delta^2 \,|\langle \bar{\psi}\psi \rangle| \tag{5}$$

(note, that it is different from the Politzer result [9], which is a kind of a radiative correction to it).

As for the large distance behaviour of the quark propagator in the «instanton liquid», it should correspond to some finite value of the effective mass $m_{eff}(\Delta\to\infty)$. Its value was estimated analytically by Dyakonov and Petrov [2] under some approximations, strongly simplifying the problem (for example, they have assumed random positions and orientations of pseudoparticles etc.). Our present studies is an attempt to answer the same questions using much more realistic calculations.

At this point we have finished our discussion of qualitative features of the ZM and NZM contributions and proceed to the particular results obtained. Their basis is a set of pseudoparticle configurations recorded during numerical experiments reported in C1 (and used in C2). We have used those with 16 pseudoparticles in a box of the size $(2\Lambda_{PV}^{-1})^4$ with the periodic boundary conditions, thus the pseudoparticle density is $1\Lambda_{PV}^4$. As in C1, C2, in order to avoid any ajustment of any free parameters, we use one «standard» value $\Lambda_{PV}=220$ MeV, as suggested by most accurate existing deep-inelastic data.

Results for the ZM contributions to chirality-flipping (solid points) and nonflipping parts of the propagator (open points) are shown in Fig. 1. They are obtained by means of numerical diagonalization of the Dirac operator in the ZM subspace, and then calculation of the propagator by the formula (1). Averaging over positions of x, y is based on about 1000 pair of points (at each distance $|x-y|$) for each of 10 recorded configurations used. In order to make averaging over point pairs more effective, they were selected with proper weight function, see C1, C2 (of course, we have checked that results are not sensitive to such trick). In order to make our results less sensitive to still uncertain general normalization of the instanton density, we have plotted the ratio of the trace of the propagator to the quark condensate value, measured by the same method for the same configurations. Therefore, the chirality-flipping part is by definition equal to unity at $x\to y$. One more technical remark: here and below we, unlike in C2, include finite size corrections on the torus directly in the data points, so the propagators shown should be compared to usual propagators in infinite space-time.

Two set of data shown in Fig. 1 display quite different shape and magnitude. Chirality-flipping part is much larger and changes more rapidly than the nonflipping one: both features are explained by the fact that at such distances it is dominated by «one-jump» contribution.

In order to see whether we have reached the asymptotic behaviour at our largest distances, we have compared the data for chirality-flipping part with the propagator of a massive quark

$$S^{flipping}(x,0) \xrightarrow[|x|\to\infty]{} Z^2 m D(m,x) , \tag{6}$$

where we have introduced some «quark wave function renormalization factor» Z. Two such curves, with masses given in Fig. 1, are shown there. One may see, that (due to some so far unclear reason) Z happen to be very close to unity. Data points do agree with these curves at distances above say 1/2 fm, which means that quark becomes nearly completely «dressed» at such distances. Unfortunately, just in this region (0.5 — 1 fm) it is not possible to distinquish between these curves, and so far the «constituent» mass value is not predicted accurately by these data.

Now we proceed to NZ modes contribution, shown in Fig. 2. For convenience, we also put here the chirality-nonflipping part of the

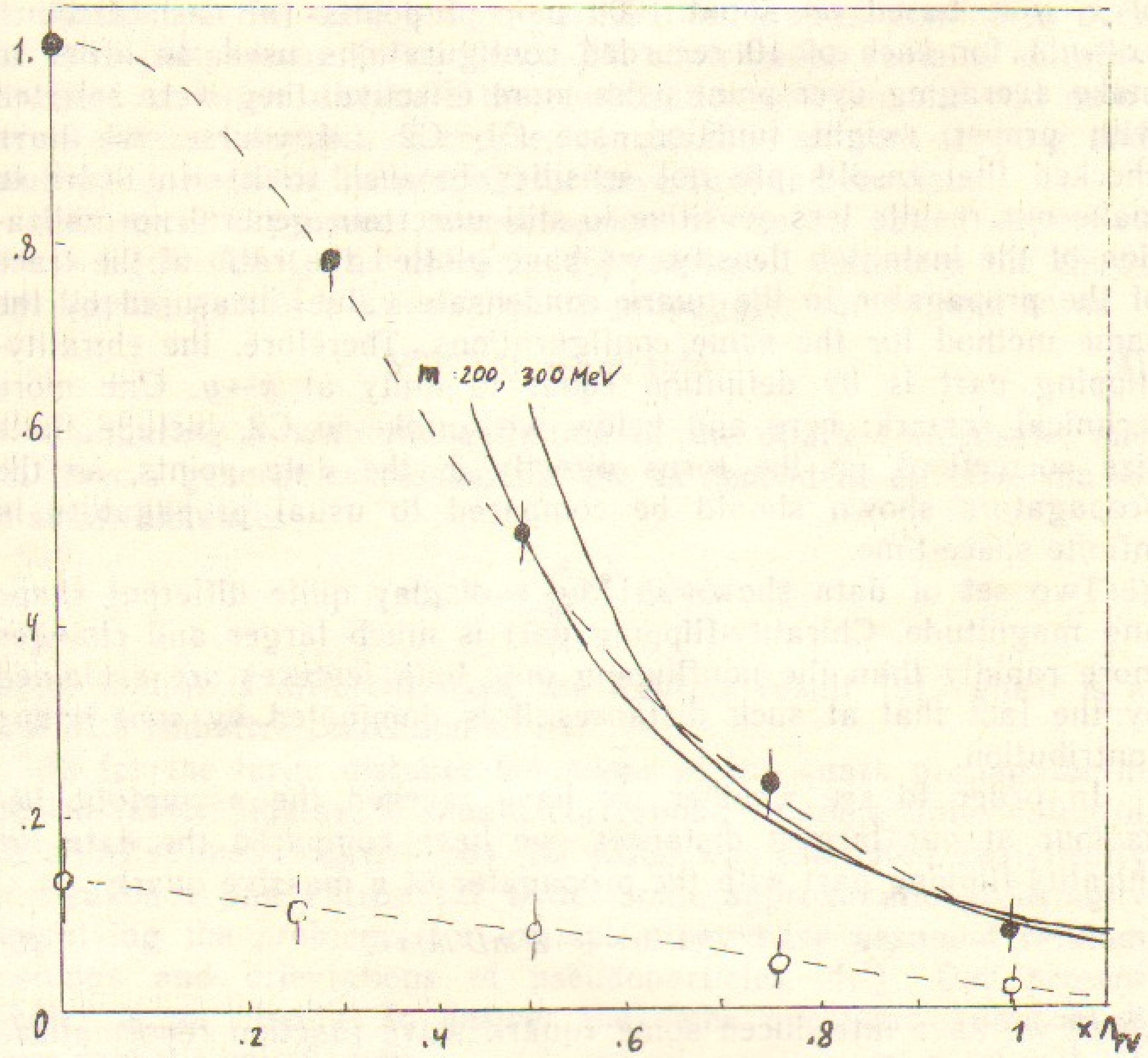

Fig. 1. The ZM contribution to the quark propagator, divided by the quark condensate $\langle\bar{\psi}(x)\,\psi(0)\rangle/\langle\bar{\psi}\psi\rangle$ versus distance x (in Λ_{PV}^{-1}). Closed and open points are the chirality-flipped and chirality-conserved parts, respectively. Solid lines correspond to propagators of massive «dressed» quarks (with mass values shown in the figure) with the factor $Z=1$.

ZM contribution (now in absolute normalization). One can see the absolute dominance of the NZM part in this region. The dashed region is our estimate for total chirality-nonflipping propagator, three dashed lines compare it with a massive propagator

$$S^{nonflipping}(x,0)\to Z^2(i\hat{\partial}_x)\,D(m,x) \qquad (7)$$

with the mass values as indicated in Fig. 2. Description of such simple fit is good enough, with the mass about $1.5\Lambda_{PV}$ (or 330 MeV). Definitely, it is quite reasonable value for a «constituent quark mass».

The crossed points in Fig. 2 correspond to the «product approximation» for the NZM part (considered above in this section). They drop somewhat more strongly than those in the «dominant pseudoparticle» approximation, demonstrating the level of uncertainties of our calculation.

3. MESONS CONTAINING A HEAVY QUARK. EXPERIMENTAL DATA AND PREVIOUS WORKS

We have mentioned in the Introduction, that the quark propagator is not quite physical quantity, so it is better to study some gauge-invariant correlation functions. Now we are going to do it, keeping at the same time as close to the results obtained as possible.

We study quite specific objects, mesons made out of the massless («dynamic») quark and of the infinitely heavy («static») antiquark. First detailed studies of the correlation functions of such type

$$K(x)=\langle(\bar{Q}\,\Gamma q)_x(\bar{q}\,\Gamma^+Q)_0\rangle\,, \qquad (8)$$

(where Γ are some gamma matrices, defining quantum numbers of the mesons considered) were made in Ref. [6], and now we briefly recapitulate some formulae and results of this work.

First of all, very heavy mass M_Q of Q allows us to use more familiar nonrelativistic language. Therefore, both points x and y to be considered will differ only in their (Euclidean) time components: the superheavy quark does not move in space. Let us call such time difference $\tau=x_0-y_0$. Also, all energies are counted from M_Q, so if we say below that the energy of some mesonic state is E it actually means the mass $E+M_Q$. One more simplification due to large M_Q

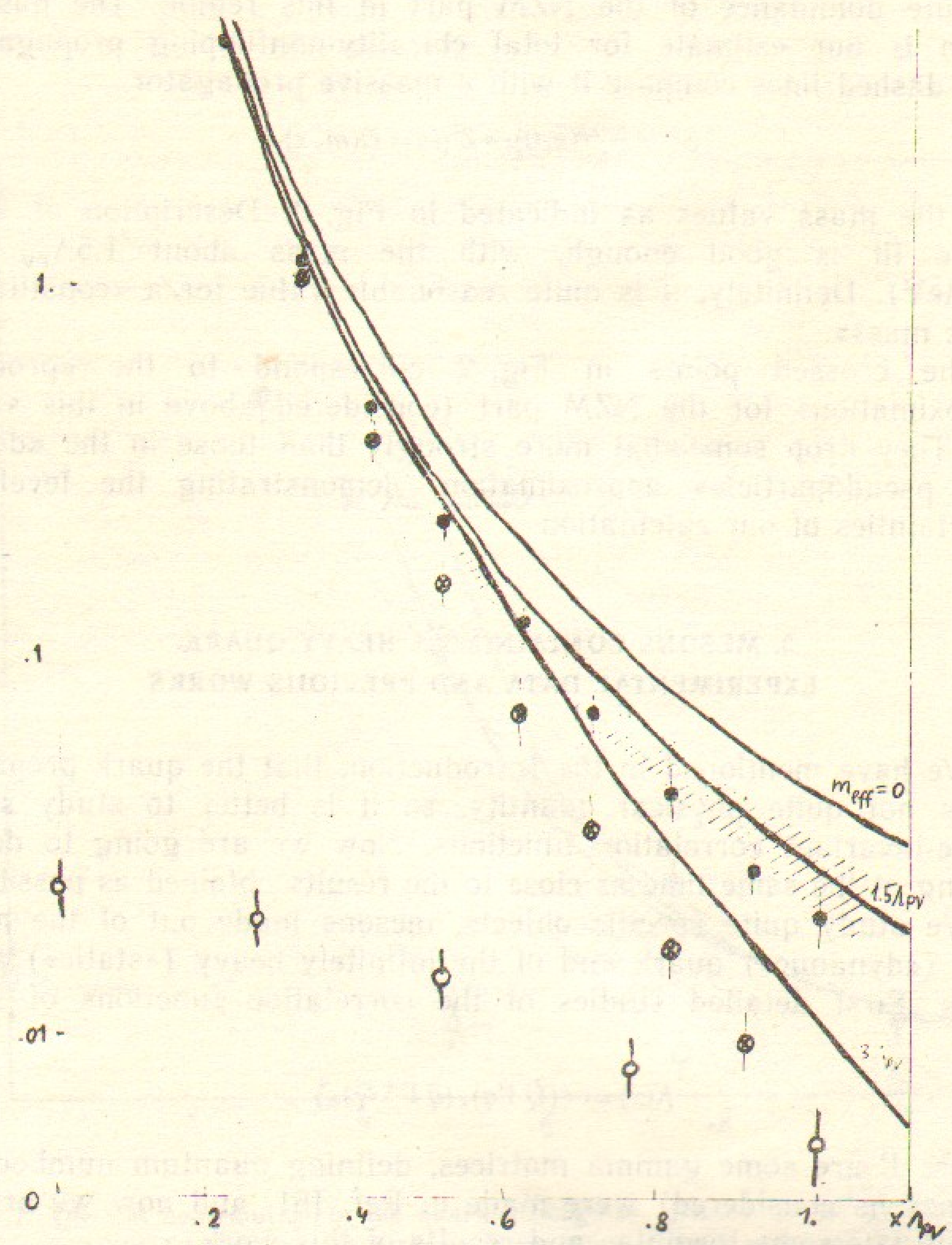

Fig. 2. Two contributions to the chirality nonflipping part of the quark propagator S (in Λ_{PV}^{-1}): due to NZM (closed and crossed points) and that due to ZM (open points). The closed points correspond to «leading pseudoparticle» approximation, while the crossed ones are the «product of all corrections», see text. The dashed region is the total propagator (if the former approximation is used), it should be compared with massive propagators at few mass values, shown by solid lines for the comparison.

limit is the absence of spin splitting: direction of the spin S is of no importance. Thus, pseudoscalar and vector mesons are degenerate, as well as scalar and axial ones, therefore we consider below only the «parity splitting» issue.

Due to asymptotic freedom, at small τ the correlation function is essentially given by the product of quark free propagators

$$K(\tau) \xrightarrow[\tau\to 0]{} K_0(\tau) \equiv \mathrm{Tr}\,|\,S_0^q(\tau)\,\Gamma S_0^{+(Q)}(\tau)\,\Gamma|\,,$$
$$S_0^q = -\gamma_0/2\pi^2\tau^3\,, \qquad (9)$$
$$S_0^Q = (1+\gamma_0)\,\delta^3(x)\,,$$

while at finite τ the propagators are modified. Several corrections were considered in [6], let us only mention the main one due to the nonzero quark condensate:

$$S_0^{(a)}(\tau) = -\frac{1}{2\pi^2\tau^3} + \frac{1}{12}\langle\bar{\psi}\psi\rangle + \dots,$$

$$K(\tau) = K_0(\tau)\left(1 - P\frac{\pi^2}{6}\tau^3\,|\langle\bar{\psi}\psi\rangle| + \dots\right). \qquad (10)$$

Here P is the parity of the state considered: $P=-1$ for pseudoscalar and vectors, $P=+1$ for scalars and axials. (Relation between the nonzero quark condensate and the parity splitting is remarkable consequence of the chiral symmetry breaking.)

Proceeding to the results of [6], we have to mention the standard parametrization of the correlation functions, used in QCD sum rules papers. Physical spectral density is written as

$$\mathrm{Im}\,K(q) = \pi\lambda_{res}^2\,\delta(q^2 - m_{res}^2) + \theta(q^2 - S_0)\,\mathrm{Im}\,K_0(q) \qquad (11)$$

with three parameters: the meson mass m_{res} and its coupling constant λ_{res}, S_0 is the so called «continuum threshold». In our nonrelativistic space-time presentation it is modified to

$$\mathrm{Im}\,K(E) = 12\pi n_{res}\,\delta(E - E_{res}) + \theta(E - E_0)\,(3E^2/\pi)\,, \qquad (12)$$

where n is the density of the light quark at the heavy one (the wave function at the origin squared). However, below we use more standard notations, in which the coupling is expressed in terms of f_Q constant. Their relation is $12n = f_Q^2 M_Q$ (the l.h.s. is finite at infinite M_Q, so the r.h.s. should be finite too) and below we give only f_Q for $Q=b$ quark. Analysis of the small-time region described in

[6] have produced predictions of these parameters, and now we mention some of them:

$$f_B \simeq 140 \text{ MeV}, \quad E_{res}(P=-1) \simeq 0.4 \pm 0.1 \text{ GeV},$$
$$E_{res}(P=1) \simeq 1.2 \pm 0.2 \text{ GeV}. \qquad (13)$$

Experimental data on heavy-flavoured mesons are still far from being complete. Let us list here only few (somewhat rounded) mass values (in MeV) for $Q=s$, c, b quarks

$$Q=s \quad m(0^-)=497; \quad m(1^-)=892; \quad m(1^+)=1270,\ 1400, \quad m(o^+)=1430;$$
$$Q=c \quad m(0^-)=1865; \quad m(1^-)=2007; \quad m(1^+)=2422\ ; \qquad (14)$$
$$Q=b \quad m(0^-)=5275\ .$$

Of course, s and even c quark are not really very heavy, they are shown for explaining the trend. (The recently found charmed meson at 2422 MeV is ascribed to the axial channel because of its strong decay to $D^*\pi$, similar to the decay of the strange axial state. Experimentally its quantum numbers are not yet fixed, but anyway all «P-wave» states should be very close in mass, much closer than the magnitude of the splitting in parity we are going to discuss.)

Using the values of the c, b quark masses from the sum rule analysis $m_s=150$ MeV, $m_c=1250$ MeV [10] and $m_b=4800$ MeV [11], and assuming that the spin splitting is caused by the $\vec{S}_1\vec{S}_2$ type interaction, we have for the «spin-averaged» negative and positive parity states

$$m(P\pm 1) \equiv \frac{3}{4} m(1^{\pm}) + \frac{1}{4} m(0^{\pm}),$$
$$m_s(P=-1) - m_s \simeq 640 \text{ MeV},$$
$$m_c(P=-1) - m_c \simeq 520 \text{ MeV},$$
$$m_b(P=-1) - m_b \simeq 475 \text{ MeV},$$
$$m_s(P=1) - m_s(P=-1) \simeq 600 \text{ MeV},$$
$$m_c(P=1) - m_c(P=-1) \simeq 450 \text{ MeV}. \qquad (15)$$

Comparing it with (13) we can see, that prediction for $E_{res}(P=-1)$ was quite correct, but that for the splitting $E_{res}(P=+1) - E_{res}(P=-1) \simeq 800$ MeV was too large. (Note however, that the difference is only twice the error value given in

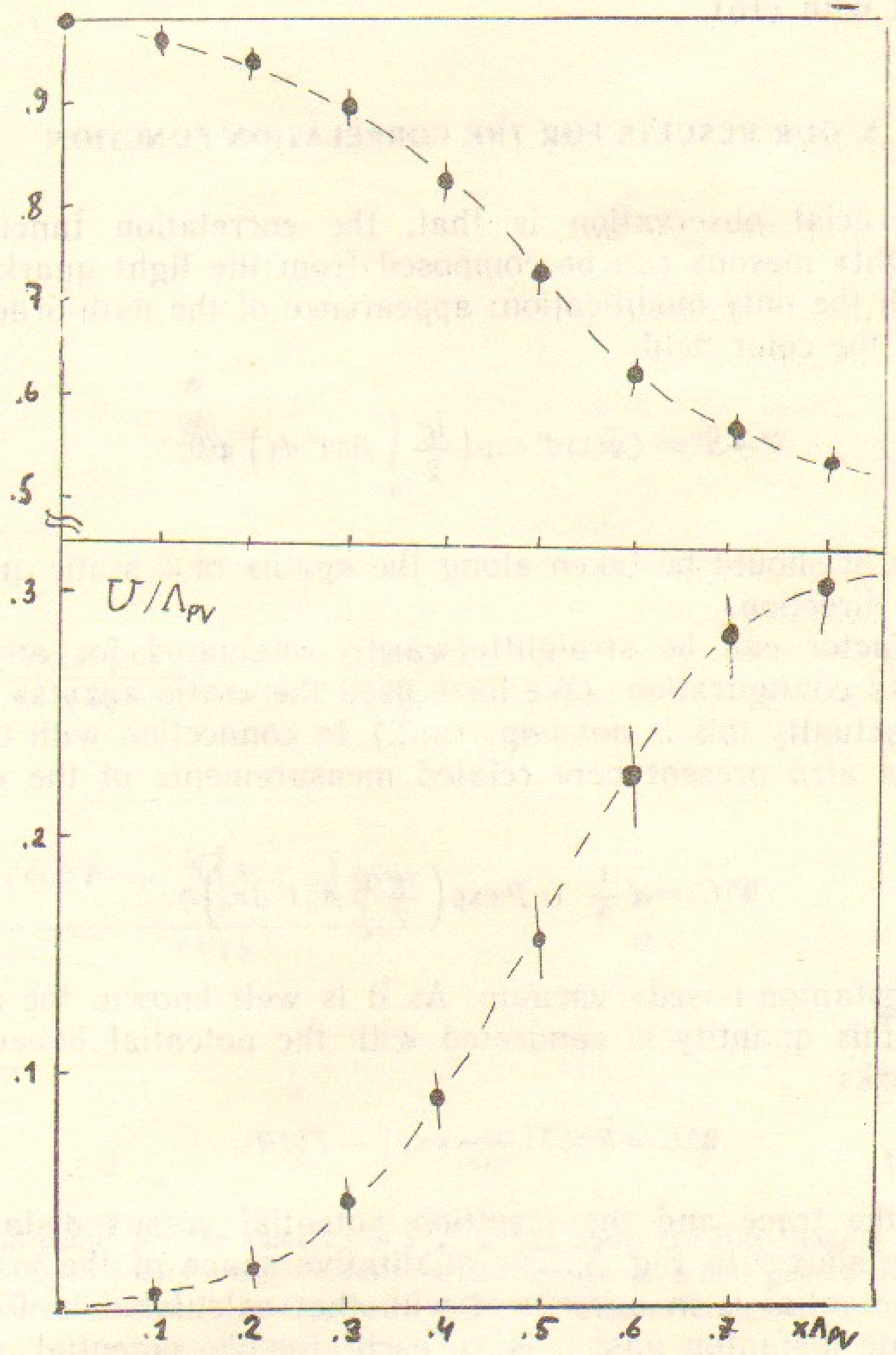

Fig. 3. Wilson loop (the upper part of the picture) and static quark-antiquark potential (lower part) versus distance x (in Λ_{PV}^{-1}).

[6].) Let us also mention here, that various potential models typically give for this splitting numbers around 500 MeV, so they are consistent with (15).

5. OUR RESULTS FOR THE CORRELATION FUNCTION

The crucial observation is that, the correlation function for «heavy-light» mesons can be composed from the light quark propagator with the only modification: appearence of the path-order exponential of the color field

$$S^q \to \tilde{S}^q = \langle \bar{\psi}(x)\, P\exp\left(\frac{ig}{2}\int_0^x A_0^a t^a\, d\tau\right) \psi(0) \rangle. \qquad (16)$$

Obviously, it should be taken along the «path» of a static quark, or over time direction.

This factor can be straightforwardly evaluated for any given gauge field configuration. (We have used the «ratio anzats», see B2 [3], but actually this is not important.) In connection with this factor, let us also present here related measurements of the «Wilson loop»

$$W(C) = \langle \frac{1}{3}\,\mathrm{Tr}\, P\exp\left(\frac{ig}{2}\oint_C A_\mu^a t^a\, dx_\mu\right) \rangle \qquad (17)$$

in our «instanton-based» vacuum. As it is well known, the average value of this quantity is connected with the potential between two static quarks

$$W(C = R\times T) \xrightarrow[T\gg R]{} \exp[-TV(R)]. \qquad (18)$$

Both the trace and the resulting potential versus distance obtained are shown in Fig. 3. The qualitative shape of the instanton-induced potential is in agreement with that calculated in Ref. [12] for a dilute instanton gas: it is an oscillator-like potential at small R, and tends to some constant at large R. This constant is twice the effective mass of a heavy quark in the vacuum, which is estimated to be

$$m_{eA}^Q = \frac{1}{2}\, V(R\to\infty) \simeq \frac{1}{6}\Lambda_{PV} \sim 40\ \text{MeV}. \qquad (19)$$

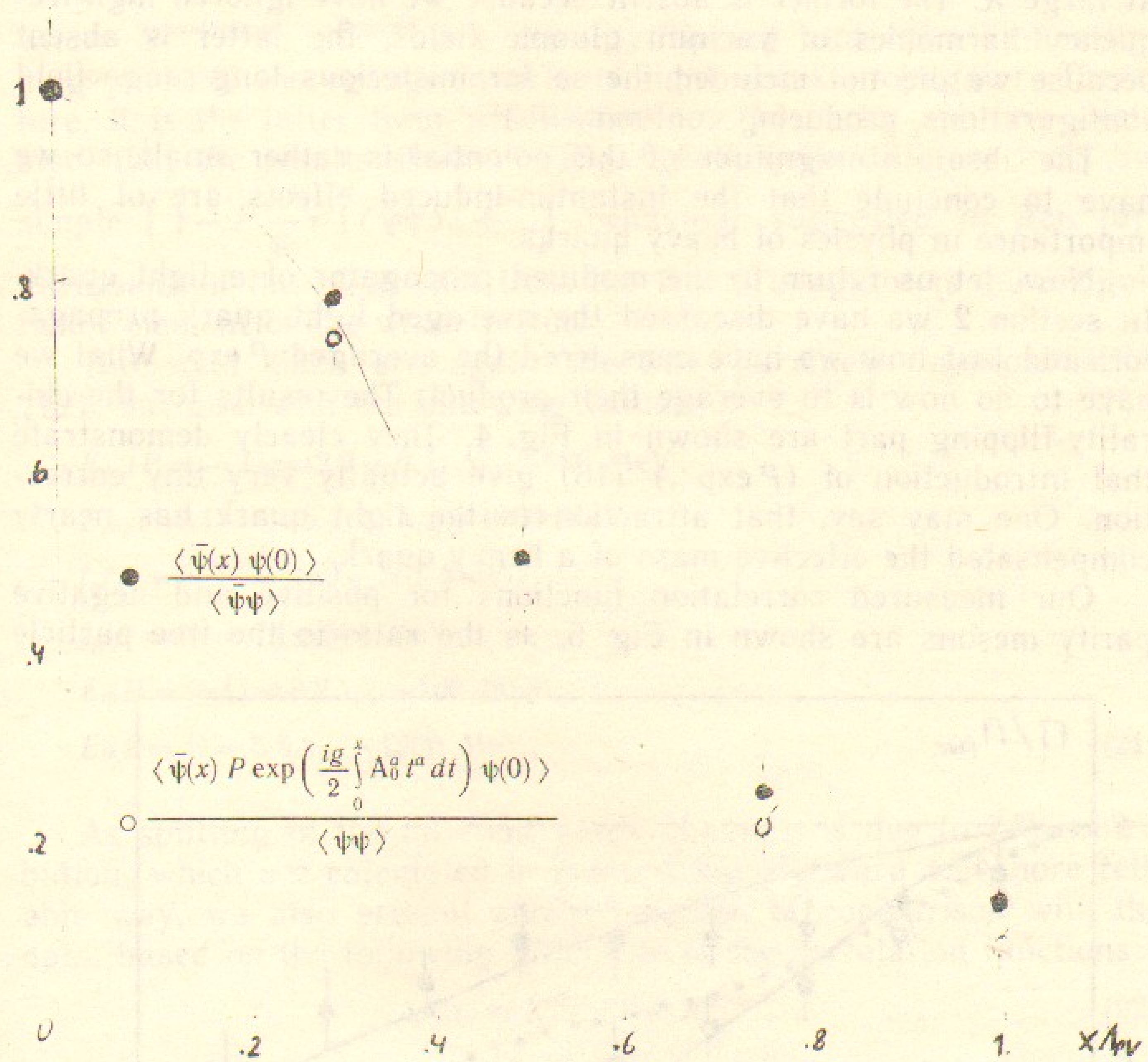

Fig. 4. Solid points are the same as in Fig. 1, they show chirality-flipping part of the propagator. Open points are the results for its «gauge invariant generalization». The solid curve is some fit by expression (23) with parameters given in the text.

(In order to avoid misunderstanding, let us note that such potential is far from being a realistic one: it contains neither the Coulomb-like potential at small R, nor the linearly rizing confining potential at large R. The former is absent because we have ignored high-frequency harmonics of vacuum gluonic fields, the latter is absent because we did not included the so far misterious long-range field configurations, producing confinement.)

The absolute magnitude of this potential is rather small, so we have to conclude that the instanton-induced effects are of little importance in physics of heavy quarks.

Now, let us return to the modified propagator of a light quark. In section 2 we have discussed the averaged light quark propagator, and just now we have considered the averaged Pexp. What we have to do now is to average their product. The results for the chirality-flipping part are shown in Fig. 4. They clearly demonstrate that introduction of (Pexp...) (16) give actually very tiny correction. One may say, that attraction to the light quark has nearly compensated the effective mass of a heavy quark.

Our measured correlation functions for positive and negative parity mesons are shown in Fig. 5, as the ratio to the free particle

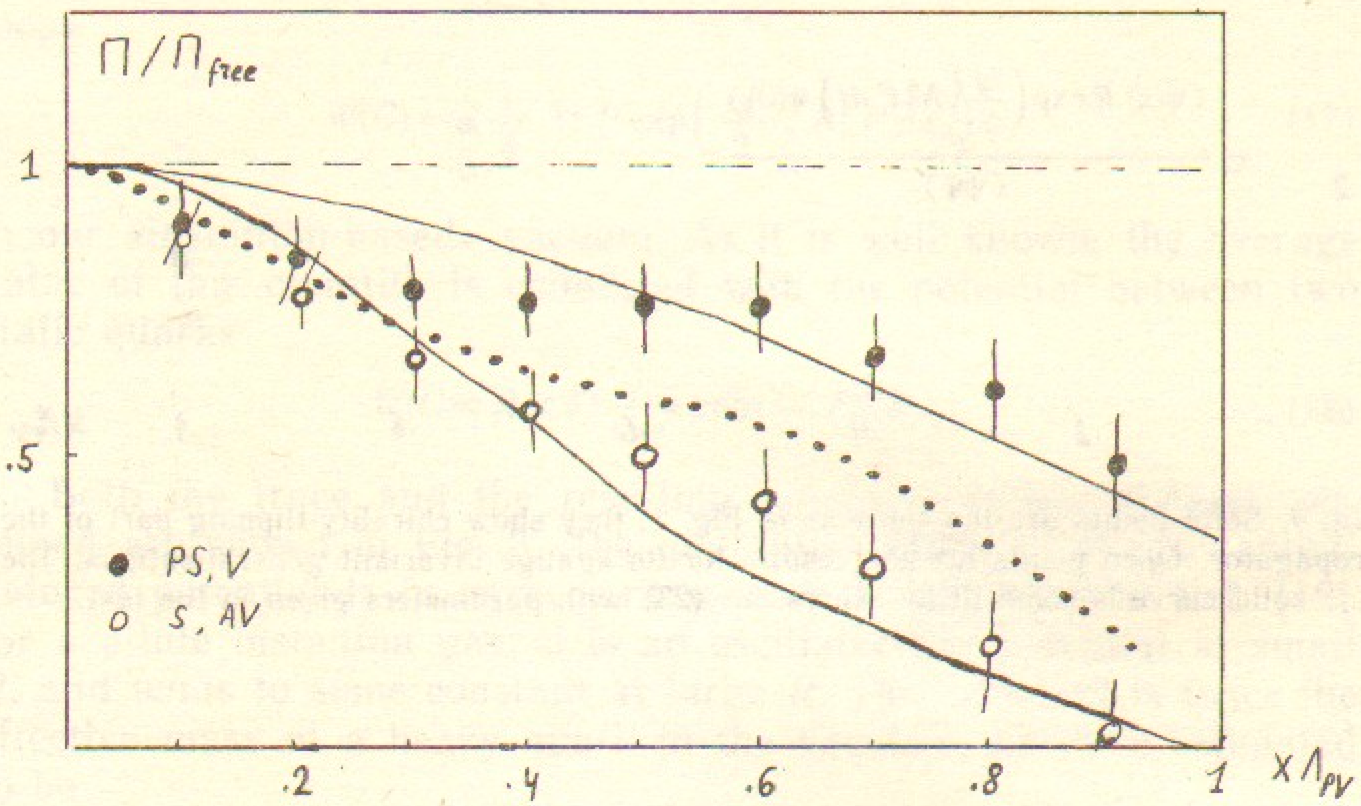

Fig. 5. Ratio of the mesonic correlator to its «asymptotically free» version, as a function of distance x (in Λ_{PV}). The dotted line is the NZM contribution, the closed and open points correspond to negative and positive parity channels, respectively. The curves are three parameter fit discussed in the text.

correlator. Note that due to $\Gamma(1+\gamma_0)\Gamma$ in this correlator, it is proportional to

$$K(\tau)\sim|\tilde{S}_{(T)}^{nonflipping}-P\tilde{S}_{(\tau)}^{flipping}|\,. \qquad (20)$$

where $S^{nonflipping}$, $S^{flipping}$ are chirality-flipping and chirality-nonflipping parts of the «gauge invariant» light quark propagator. Therefore, it is the latter term which produces the parity splitting. Note also, that the results shown in Fig. 5 look quite different from the simple $\left(1-P\frac{\pi^2}{6}\tau^3|\langle\bar\psi\psi\rangle|+...\right)$ behaviour suggested in Ref. [6]: deviations of the NZM part from the free propagator is quite significant element of the whole picture.

The solid lines are again the three-parameter approximation (12), but now with the following values:

$$E_{res}(P=-1)=(2.2\pm0.4)\,\Lambda_{PV}\sim480\text{ MeV},$$
$$E_{res}(P=1)=(4.5\pm1)\,\Lambda_{PV}\sim1000\text{ MeV},$$
$$f_B(P=-1)=(100\pm30)\text{ MeV},$$
$$f_B(P=1)=(70\pm50)\text{ MeV},$$
$$E_0(P=-1)=3.2\Lambda_{PV}\sim700\text{ MeV},$$
$$E_0(P=1)=5.5\Lambda_{PV}\sim1200\text{ MeV}, \qquad (21)$$

As splitting of the different parity channels is due to ZM contribution, which are calculated in more straightforward and more reliable way, we also present another method of comparison with the data, based on the following difference of the correlation functions:

$$\Delta K(\tau)\equiv K^{(P=-1)}-K^{(P=1)}. \qquad (22)$$

Neglecting the difference of the «continuum states» in two channels, we may prescribe this quantity to the difference of the resonance contributions

$$\Delta K(\tau)/M_Q\sim\{f^2(P=-1)\exp[fE_{res}(P=-1)\,\tau|-$$
$$-f^2(P=1)\exp[-E_{res}(P=1)\,\tau|\}\,. \qquad (23)$$

Our measurements of the chirality-flipping part of the correlator are compared with this parametrization in Fig. 4, the curve shown corresponds to the following values of the parameters

$$E_{res}(P=-1)=3\Lambda_{PV}\,, \quad E_{res}(P=1)=6\Lambda_{PV}\,,$$

$$f_B(P=-1)=0.54\Lambda_{PV}\,, \quad f_B(P=1)=0.40\Lambda_{PV}\,. \qquad (24)$$

Comparing these results to the data, discussed in the previous section, let us note few important new elements. First, the splitting between two parity states is much reduced (and is now in agreement with the data). This modification follows from the fact, that splitting is now given not by the quark condensate, but by the nonlocal quantity $\langle \psi(x)\psi(0) \rangle$ with x of the order of 0.5 fermi or so, which is several times smaller.

The second deviation from the previous estimates [6] is noticeable reduction of the coupling constant f_b. This feature, in turn, is mostly due to specific fall off of the NZM contribution. (Its measurement for B meson is in principle possible, and is very interesting from various points of view.)

The third point worth mentioning is that now we have obtained smaller coupling of the positive parity states compared to the negative parity ones. This trend is quite consistent with the nonrelativistic models, in which the former are the P-states, with larger radius compared to the S-states, and they consequently have smaller coupling to the local current.

6. CONCLUSIONS AND DISCUSIONS

1. One of the most important result of this work is the measurements of the chirality-flipping part of the light quark propagator. Being in the local ($x \to y$) limit just the quark condensate, this quantity decreases with $R=|x-y|$, and already at R about 1/2 fm it seems to approach its asymptotic behaviour, corresponding to a «dressed» massive quark. We expect such effective quark mass to be developed due to quark «hopping» from one pseudoparticle to another, but actually we see that such behaviour is observed already in the region where only one-pseudoparticle contribution is dominant.

Two more surprises came from these data. The first is related with the «quark wave function renormalization constant» Z, which happen to be close to unity.

The second (less fortunate) fact is as follows: it turned out, that even possessing reasonably good data for (the chirality-flipping part of) the propagator up to distance of about 1 fm (where it

drops by nearly one order of magnitude!) we still cannot fix the effective mass value with good accuracy. The reason is unrelated with the data quality: the curves for various masses nearly overlep in this region. And still, as our data do fall on this curve, we have rather convincing demonstration of the «early asymptotics» mentioned above.

2. Coming now to chirality nonflipping part of the propagator, let us first emphasise strong dominance of the NZM contribution up to our largest distances ($\sim$1 fm). However, we observe that the NZM contribution drops much facter than the ZM one (which is mainly due to «hopping» over one instanton and one antiinstanton). Thus, there are good reason to believe that at distances exceeding 1 fm their roles are changed, and the ZM contribution becomes dominant in this amplutude as well. Obviously, at very large distances both parts of the propagator should correspond to the same effective mass value.

It is interesting to observe, that the nonflipping part behaves roughly as some massive propagator with reasonable value of the effective mass in the whole region. Therefore, here we also see a kind of «early asymptotics». However, in this case uncertainties are larger and better data are needed to clarify this issue.

3. Considering heavy quarks in the «instanton liquid» we once more emphasize a general fact: static quarks interact weakly with instantons (see Fig. 3). We get effective mass value in this case one order of magnitude smaller than for the light quark.

As a result, addition of a static antiquark to the light quark (transition to «gauge invariant propagator») leads only to its slight modifications. Moreover, even this small effective mass of a static quark is nearly compensated by attraction to the light one.

We have shown in the preceeding section that the resulting description of the correlation functions for B-type mesons is quite reasonable. The correlators are determined in much wider region compared to earlier analysis based on the operator product expansion, and the results are significantly improved.

If one tries to describe these correlators by standard «delta plus theta function» three parameter fit, he get better agreement with data than was obtained previously. The energy of lowest $P=-1$ state, as well as its splitting from the $P=+1$ state becomes much more reasonable. It is desirable to have experimental data on coupling constants of such mesons, because these parameters are defined by the correlators much more accurately than masses.

As a point for discussion let us consider a question raised in the Introduction: do quarks form bound state in the model considered? For mesons made of heavy and light quarks we may give the answer, and it is negative. And indeed, potential well has the depth of the order of static quark effective mass, which is less than 50 MeV. The width of this well is 0.5 fm or so, while effective mass of light quark is about 300 MeV. Standard quantum-mechanical calculation shows, that under such conditions no bound states exist: the potential is too shallow. It means, that if we are able to measure correlation functions at arbirtrary large distances, we have to observe two unbound quarks with their effective masses, not a meson.

If so, how can one understand good fit of our correlation functions and reasonable results for the mesonic masses? This is the same trick, as used in the «QCD sum rules» approach. Actually we discuss not the stationary mesons, but some virtual state, existing for short time period. For such virtual state our theory may turn to be quite correct: for example, one may indeed ignore confining forces. The point is that if we assume certain shape of the physical spectral function, we get reasonable results for the parameters involved.

4. Let us add one more remark to our discussion of mesonic correlation functions. It is an instructive example of a situation, in which smallness of corrections to the asymptotic freedom (which holds, say, up to distances about 1/2 fm) does not imply applicability of the OPE-type formulae. For example, the splitting of different parity channels is relatively small in this region, but still it is determined by the nonlocal quantity $\langle \bar{\psi}(x)\psi(0) \rangle$ which is much smaller than $\langle \bar{\psi}(0)\psi(0) \rangle$, as suggested by OPE. The lesson is, that in any OPE-based sum rules we actually deal with nonlocal quantities, treated as some effective local ones. Therefore, we may significantly underestimate the true values of local parameters.

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