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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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ASYMPTOTIC BEHAVIOUR OF EXCLUSIVE PROCESSES IN QCD

10. THE NUCLEON

PREPRINT 83-108

НОВОСИБИРСК

CONTENT

10.1. THE NUCLEON WAVE FUNCTIONS.

We use in this chapter the proton wave functions introduced in /2.8/ and determined by the matrix element of the three-local operator:

$$\langle 0| \epsilon^{ijk} u^i_\alpha(z_1)\, u^j_\beta(z_2)\, d^k_\gamma(z_3)|P\rangle = f_N \frac{1}{4}\Big\{ (\hat{P}C)_{\alpha\beta}(\gamma_5 N)_\gamma V(z_i p) + (\hat{P}\gamma_5 C)_{\alpha\beta} N_\gamma A(z_i p) - (\sigma_{\mu\nu} P_\nu C)_{\alpha\beta}(\gamma_\mu \gamma_5 N)_\gamma T(z_i p)\Big\}, \quad (10.1)$$

$$V(z_i p=0) = T(z_i p) = 1, \quad A(z_i p=0) = 0, \quad p_z \to \infty.$$

Here: N_γ -is the proton spinor, p-is the proton momentum, C-is the charge conjugation matrix, i, j, k-are color indices, and α, β, γ are spinor indices. As usually, the Fourier transform of dimensionless wave function $\left(d_3x = dx_1 dx_2 dx_3\, \delta(1-x_1-x_2-x_3) \right)$:

$$V(z_i p) = \int_0^1 d_3x \exp\Big\{-i \sum_1^3 x_i (z_i p)\Big\} V(x_i), \quad i=1,2,3 \quad (10.2)$$

(and analogously for $A(z_i p)$ and $T(z_i p)$) determines a distribution of three proton quarks in the longitudinal momemtum fractions $0 \le x_i \le 1$, $\sum x_i = 1$. The identity of two u-quarks leads to the symmetry properties:

$$V(1,2,3) = V(2,1,3), \quad T(1,2,3) = T(2,1,3), \quad A(1,2,3) = -A(2,1,3). \quad (10.3)$$

Requiring the total isospin of three quarks to be equal 1/2, one obtains the relation:

$$2T(1,2,3) = V(1,3,2) - A(1,3,2) + V(3,2,1) - A(3,2,1). \quad (10.4)$$

Therefore, there is only one independent nucleon wave function of the leading twist 3, say, $\varphi_N(x) = V(x) - A(x)$.

The above introduced wave functions correspond to the proton state of the form ($p_z \to \infty$,the arrows show spin projections onto the z-axis,for the neutron: u → d):

$$|P^{\uparrow}\rangle = \mathrm{const}\int_0^1 d_3x \left\{ \frac{V(x)-A(x)}{2}\,|u^{\uparrow}(x_1)u^{\downarrow}(x_2)d^{\uparrow}(x_3)\rangle + \right. \tag{10.5}$$

$$\left. \frac{V(x)+A(x)}{2}\,|u^{\downarrow}(x_1)u^{\uparrow}(x_2)d^{\uparrow}(x_3)\rangle - T(x)\,|u^{\uparrow}(x_1)u^{\uparrow}(x_2)d^{\downarrow}(x_3)\rangle \right\}.$$

The properties of the nucleon wave functions $V(x)$, $A(x)$ and $T(x)$ were investigated in detail in the paper /10.1/,using the QCD sum rules. We give below the main results only and refer to the paper /10.1/ for details. The dimensional constant f_N which determines the value of the nucleon wave function at the origin,has been found from QCD sum rules /1.48,10.1/ and is equal:

$$|f_N|_{\mu=1\,\mathrm{GeV}} = (5.2\pm0.3)\cdot10^{-3}\ \mathrm{GeV}^2. \tag{10.6}$$

The wave function moments are defined in the usual way:

$$\varphi^{(n_1n_2n_3)} = \int_0^1 d_3x\, x_1^{n_1}x_2^{n_2}x_3^{n_3}\,\varphi(x). \tag{10.7}$$

The moment values of the proton wave functions V(x), $A(x)$ and T(x) obtained in /10.1/ using the sorresponding QCD sum rules are presented in Table 10.1.* (Everywhere in this chapter the mean normalization point of the wave functions is $\bar{\mu}\simeq1\,\mathrm{GeV}$). The moment values of the asymptotic nucleon wave function: $V(x,\mu\to\infty)=T(x,\mu\to\infty)=\varphi_{as}(x)=120x_1x_2x_3$ are also given in Table 10.1 for comparison.

It is seen from Table 10.1 that the most characteristic fea-

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* Let us point that the values of the moments $T^{(n_1n_2n_3)}$ have been found in /10.1/ by two independent methods: a) using the values of the moments $\varphi_N^{(n_1n_2n_3)}$ and the relation (10.4); b) directly from the independent sum rules for $T^{(n_1n_2n_3)}$. The results agree well.

ture of the proton wave function is the large momentum fraction carried by first u-quark:

$$\varphi_N^{(100)}\Big/\varphi_N^{(010)} \simeq \varphi_N^{(100)}\Big/\varphi_N^{(001)} \simeq 4-5, \tag{10.8}$$

while these ratios are equal unity for $\varphi_{as}(x)$. These results show unambiguously that the proton wave function $\varphi_N(x,\mu\simeq1\,\mathrm{GeV})$ is very asymmetric,unlike $\varphi_{as}(x)$. The largest part of the proton momentum ($\simeq$ 65% at $p_z\to\infty$) is carried by one u-quark with the spin directed along the proton spin,while each of two rest quarks carries $\simeq$ 15%-20% of the total momentum.

Using the moment values from Table 10.1,the following model proton wave functions have been proposed in /10.1/:*

$$V(x_1,x_2,x_3)=\varphi_{as}(x)\left[11.35(x_1^2+x_2^2)+8.82x_3^2-1.68x_3-2.94\right],$$

$$A(x_1,x_2,x_3)=\varphi_{as}(x)\left[6.72(x_2^2-x_1^2)\right], \tag{10.9}$$

$$T(x_1,x_2,x_3)=\varphi_{as}(x)\left[13.44(x_1^2+x_2^2)+4.62x_3^2+0.84x_3-3.78\right].$$

The moment values of these model wave functions are also presented in Table 10.1 for comparison. The profiles of the wave functions $\varphi_N(x)=V(x)-A(x)$, $V(x)$, $\varphi_{as}(x)$ are shown at figs. 10.1a,10.1b and 10.1c respectively,using the Mandelstam plane $0\le x_i\le1$, $\sum x_i=1$.

The multiplicatively renormalized polinomials for the three-quark operators of the twist 3 are the Appel polinomials $P_n(x_{1,2,3})$:

$$\int_0^1 d_3x\,\varphi_{as}(x)\,P_n(x)\,P_m(x)=\delta_{nm}.$$

These orthogonality conditions do not determine,however,completely the explicit form of the Appel polinomials (unlike to two-particle operators). Therefore,either a standard computation

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* see the next page

of one-loop diagram divergences /2.8/, or the solution of the corresponding evolution equation /1.29/ is required. The explicit form of few lowest polinomials and values of the corresponding anomalous dimensions have been found for the first time in /1.29,2.8/, a more complete investigation can be found in /10.2/. We give here some results which will be used in the following:

$$f_N(M_2) = f_N(M_1)\left(\alpha_s(M_2)/\alpha_s(M_1)\right)^{2/3b_0},$$

$$\varphi_N(x,M_2) = \varphi_{as}(x)\left[1 + \frac{21}{2}a_1(M_1)\left(\frac{\alpha_s(M_2)}{\alpha_s(M_1)}\right)^{\frac{26}{9b_0}}(x_1 - x_3) + \right. \tag{10.10}$$

$$\left. \frac{7}{2}a_2(M_1)\left(\frac{\alpha_s(M_2)}{\alpha_s(M_1)}\right)^{\frac{30}{9b_0}}(1-3x_2) + \dots\right], \qquad b_0 = 11 - \frac{2}{3}n_{fe},$$

$$a_1(M) = \int_0^1 d_3x\,(x_1 - x_3)\varphi_N(x,M), \quad a_2(M) = \int_0^1 d_3x\,(1-3x_2)\varphi_N(x,M).$$

For the wave function (10.9):

$$a_1(M \simeq 1\,\mathrm{GeV}) \simeq 0.41, \quad a_2(M \simeq 1\,\mathrm{GeV}) \simeq 0.55. \tag{10.11}$$

As usual, the anomalous dimensions are not large, the dependence of $\alpha_s(M)$ on M is logarithmic only and hence, the dependence of $\varphi_N(x,M)$ on M is very weak. Therefore, because $\varphi_N(x, M \simeq 1\,\mathrm{GeV})$ differs greatly in its form from $\varphi_{as}(x)$, this difference will persist up to enormously large values of M.

10.2. THE NUCLEON ELECTROMAGNETIC FORM FACTORS.

These form factors are defined in a standard way:

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* (from the previous page)

Let us remind (see ch.1) that it is follows from the duality relations that $\varphi_N(x, M \sim 1\,\mathrm{GeV})$ has the same behaviour at $x_i \to 1$ as $\varphi_{as}(x)$, $i = 1,2,3$.

$$\langle P_2|J_\mu(0)|P_1\rangle = \bar N_2\left[\gamma_\mu F_1(q^2) - \frac{\sigma_{\mu\nu}q_\nu}{2M_N}F_2(q^2)\right]N_1, \quad q = P_2 - P_1,$$

$$F_1^p(0) = 1, \quad F_1^n(0) = 0, \quad F_2^p(0) \simeq 1.79, \quad F_2^n(0) \simeq -1.91,$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2), \quad G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_N^2}F_2(q^2), \tag{10.12}$$

$$G_M^p(0) \equiv \mu_p \simeq 2.79, \qquad G_M^n(0) \equiv \mu_n \simeq -1.91.$$

It is known that the experimental data for $G_M^{p,n}(q^2)$ /1.34, 10.3/ are approximately described by the famous dipole formula:

$$\frac{1}{\mu_p}G_M^p(q^2) \simeq \frac{1}{\mu_n}G_M^n(q^2) \simeq \left(1 - q^2/M_0^2\right)^{-2}, \quad M_0^2 = 0.71\,\mathrm{GeV}^2, \tag{10.13}$$

($0 \lesssim (-q^2) \lesssim 25\,\mathrm{GeV}^2$ for the proton, $0 \lesssim (-q^2) \lesssim 10\,\mathrm{GeV}^2$ for the neutron).

The asymptotic behaviour of the nucleon form factors in QCD has been studied in the papers /1.26,1.29,1.30,2.7,2.8,2.9,3.3, 10.4/. We consider below only the form factors $F_1^{p,n}(q^2)$. The form factors $F_2^{p,n}(q^2)$ have the additional suppression $\sim \mathrm{K}_\perp/q$ as compared with the behaviour $\sim 1/q^5$ expected from a dimensional counting, because a turn over of the quark spin projection onto the z-axis is required (in the nucleons c.m.s.). Thus: $F_2^{p,n}(q^2) \sim 1/q^6$ and $G_M^{p,n}(q^2) \to F_1^{p,n}(q^2)$ /1.9/.

The Born diagrams for the nucleon form factor are shown in Table 10.2 ("X" -is the electromagnetic current vertex, the lowest line at each diagram is the d-quark for the proton, u → d for the neutron). The contributions of the Born diagrams into $F_1^{p,n}(q^2)$ have been first calculated in /1.26/, supposing the SU(6)-symmetry for the nucleon wave functions, i.e. A(x)=0,

$T(x)=V(x)$ and $V(x)$ is symmetric in all its arguments. Nothing definite can be said both about the signs of $E_1^{p,n}$ and about the ratio E_1^n/E_1^p in this case, because different diagrams contribute with different signs, see Table 10.2. It has been emphasized in /2.7/ that the non-relativistic form of the nucleon wave functions: $A(x)=0$, $V(x)=T(x)=\delta(x_1-1/3)\,\delta(x_2-1/3)$ leads to the results: $E_1^p(q^2)<0$, $E_1^n(q^2)>0$, which contradict to the experimental data (10.13).

The contributions of the Born diagrams into $E_1^{p,n}(q^2)$ have been calculated in the papers /1.29,2.8/ without any assumptions on the possible form of the nucleon wave function $\varphi_N(x)$ (the results for $G_M^{p,n}(q^2)$ obtained in /1.29/ contain a trivial error in an overall sign). The expression for $E_1^p(q^2)$ is:

$$q^4 E_1^p(q^2) \to \frac{(4\pi\alpha_s)^2}{54}\,|f_N|^2 \int_0^1 d_3x \int_0^1 d_3y \left\{ 2\sum_1^7 e_i T_i(x,y) + \sum_8^{14} e_i T_i(x,y) \right\}, \quad (10.14)$$

where e_i -is the charge of those quark which interacts with the electromagnetic current in given diagram ($e_u=2/3$, $e_d=-1/3$), the expressions for $T_i(x,y)$ are given in Table 10.2.

When leading logarithmic corrections are taken into account, one can replace: $\varphi_N(x)\to\varphi_N(x,Q^2)$ and $\alpha_s\to\alpha_s(Q^2)$. Then one has from (10.10),(10.14) and Table 10.2 in the formal limit $Q^2\to\infty$ /1.29,2.8,2.9,3.3,10.4/:

$$q^4 E_1^n(q^2) \to \left(4\pi\alpha_s(Q^2)\right)^2 \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{4/3b_0} |f_N(\mu^2)|^2\,\frac{100}{3}\,\cdot \quad (10.15)$$

$$\left[1-\frac{49}{6}a_1(\mu^2)\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{\frac{20}{9b_0}} - \frac{35}{6}\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{\frac{24}{9b_0}} a_2(\mu^2)+\dots\right],$$

$$q^4 E_1^p(q^2) \to \left(4\pi\alpha_s(Q^2)\right)^2 \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{32/9b_0} |f_N(\mu^2)|^2\,\frac{100}{3}\,\frac{49}{6}\,a_1(\mu^2)+\dots$$

Therefore, $G_M^n(q^2)>0$ and $G_M^p(q^2)/G_M^n(q^2)\sim\left(\alpha_s(Q^2)\right)^{20/9b_0}\to 0$ at $Q^2\to\infty$. Using the expression (10.15) for $E_1^n(q^2)$ and a_1, a_2 from (10.11), one can calculate the corrections to the leading term in $E_1^n(q^2)$:

$$\left[1-3.3\left(\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right)^{\frac{20}{9b_0}} - 3.2\left(\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right)^{\frac{24}{9b_0}}+\dots\right], \quad \mu\simeq 1\,\text{GeV}. \quad (10.16)$$

Hence, the corrections to the leading term in $E_1^n(q^2)$ (10.15) are smaller than 30% only at enormously large Q^2: $\ln(Q^2/\Lambda^2)\gtrsim 10^4$ (at $b_0\simeq 7$). Moreover /1.48/, retaining only the leading term for $E_1^n(q^2)$ in (10.15) and substituting f_N from (10.6), one has: $\frac{1}{\mu_n}q^4 G_M^n(q^2)\simeq -0.2\cdot 10^{-2}\,\text{GeV}^4$, and this number is two orders smaller than the experimental data (10.13) (and has the opposite sign).

All this shows unambiguously that the formal results (10.15) have nothing to do with a real life. As it was pointed out before, the nucleon wave function $\varphi_N(x,\mu\simeq 1\,\text{GeV})$ differs greatly from $\varphi_{as}(x)$, the logarithmic dependence of $\varphi_N(x,\mu)$ on μ is very weak and as a result, the formal asymptotic behaviour (10.15) sets up at enormously large values of Q^2 only.

It is shown in /10.1/ that the mean normalization point $\bar\mu$ of $\varphi_N(x)$ in (10.14) at $Q^2\simeq(20-60)$ GeV^2 is $\bar\mu\simeq 1$ GeV (because the nucleon momentum is divided between quarks) and therefore, one can use the wave functions (10.9),(10.6). Substituting them into (10.14) and Table 10.2, one obtains* /10.1/ ($\bar\alpha_s\simeq 0.3$):

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* For $\varphi_N(x)=\varphi_{as}(x)=120x_1x_2x_3$: $G_M^p=0$, $\frac{1}{\mu_n}q^4 G_M^n(q^2)\simeq -0.2\cdot 10^{-2}\,\text{GeV}^4$,

For $\varphi_N(x)=\varphi_{nonrel}(x)=\delta(x_1-1/3)\,\delta(x_2-1/3)$:

$$\frac{1}{\mu_p}q^4 G_M^p(q^2) = -\frac{1}{\mu_n}q^4 G_M^n(q^2)\simeq -10^{-3}\,\text{GeV}^4.$$

$$\frac{1}{\mu_p} q^4 G_M^p(q^2) \simeq 0.4\ \mathrm{GeV}^4, \quad \frac{1}{\mu_n} q^4 G_M^n(q^2) \simeq 0.3\ \mathrm{GeV}^4,$$
$$G_M^n(q^2)/G_M^p(q^2) \simeq -0.5, \quad \mu_p \simeq 2.8, \ \mu_n \simeq -1.9, \tag{10.17}$$

and these results agree with the experiment /1.34,10.3/ both in signs and magnitudes, see fig. 10.2.

From the theoretical viewpoint therefore, the experimental data on the nucleon form factor confirm the main characteristic property of the nucleon wave function $\varphi_N(x, \mu \simeq 1\,\mathrm{GeV})$, predicted by the QCD sum rules: very uneven distribution of the nucleon longitudinal momentum (at $P_z \to \infty$) between three quarks. The wave functions for which the nucleon momentum is more or less equally distributed between the quarks give wrong signs for the nucleon form factors.

10.3. THE DECAYS $\psi \to \bar{P}P, \bar{n}n$ AND $\chi_2 \to \bar{N}N$

Let us consider the properties of $\psi \to \bar{N}N$, $N = p, n$ decays (we put below the z-axis to be along the nucleon momentum, the angular distribution is of the form: $(1+\cos^2\theta)$). The strong decay amplitude M_{str} is described by the diagrams like those shown at fig. 10.3 and it has been calculated in /2.2/ in terms of the nucleon wave function $\varphi_N(x)$. In our notation the result has the form:

$$\langle \bar{N}_2, N_1 | S | \psi(P) \rangle = i\,\delta(P - P_1 - P_2) M_{str},$$
$$M_{str} = (4\pi\bar{\alpha}_s)^3 \frac{f_\psi}{M_\psi} \left| \frac{120 f_N}{M_\psi^2} \right|^2 \frac{20}{81} M_0 \bar{N}_2 \hat{\psi} N_1, \tag{10.18}$$
$$M_0 = \int_0^1 d_3x \int_0^1 d_3y \left[\frac{\varphi_N(x)\varphi_N(y)\, y_1 x_3}{\varphi_{as}(x)\varphi_{as}(y)\, D_{13}} + 2\frac{T(x)T(y)\, y_1 x_2}{\varphi_{as}(x)\varphi_{as}(y)\, D_{12}} \right],$$
$$D_{13} = [1 - (2x_1 - 1)(2y_1 - 1)][1 - (2x_3 - 1)(2y_3 - 1)], \quad D_{12} = D_{13}(x_3, y_3 \to x_2, y_2),$$

where ψ_μ -is the ψ -meson polarization vector.

Substituting the wave functions (10.9) into (10.18), one obtains*:

$$M_0 \simeq 0.5. \tag{10.19}$$

Using (10.18), (9.6), one has:

$$Br\left(\frac{\psi \to \bar{P}P}{\psi \to 3g}\right) \simeq (\pi\bar{\alpha}_s)^3 \left| \frac{120 f_N}{M_\psi^2} \right|^2 \frac{128\pi^2}{(\pi^2 - 9)} M_0^2 \left(\frac{2|\vec{p}|}{M_\psi} \right), \tag{10.20}$$

where $\vec{p}$ is the nucleon momentum in the ψ -meson rest frame. Substituting (10.6), (10.19) into (10.20) and using $\Gamma(\psi \to 3g) \simeq 0.8\Gamma(\psi \to \mathrm{all})$, one obtains (at $\bar{\alpha}_s \simeq 0.3$):

$$Br\left(\frac{\psi \to \bar{P}P}{\psi \to \mathrm{all}}\right) \simeq (\pi\bar{\alpha}_s)^3 \cdot 0.35\% \simeq 0.3\%, \tag{10.21}$$

while the experimental value is /2.6/:

$$Br\left(\frac{\psi \to \bar{P}P}{\psi \to \mathrm{all}}\right) = (0.22 \pm 0.02)\%. \tag{10.22}$$

Besides, there are two types of the electromagnetic contributions into $\psi \to \bar{N}N$, figs. 10.4a and 10.4b, leading to $\Gamma(\psi \to \bar{P}P) \neq \Gamma(\psi \to \bar{n}n)$. The contribution of the fig. 10.4a diagram is:

$$\left[M_\gamma^{(p,n)}\right]_a = 4\pi\alpha \frac{2}{3} \frac{f_\psi}{M_\psi} G_M^{(p,n)}(M_\psi^2) \cdot \bar{N}_2 \hat{\psi} N_1, \tag{10.23}$$

where $G_M^{p,n} \simeq F_1^{p,n}$ is the nucleon magnetic form factor. The contribution of the fig. 10.4b diagram is:

$$\left[M_\gamma^{(p,n)}\right]_b = -\frac{4}{5} \frac{\alpha}{\bar{\alpha}_s} e_{p,n} M_{str}, \quad \alpha = 1/137, \tag{10.24}$$

where $e_p = 1$ and $e_n = 0$ are the electric charges of the proton and neutron. We want to emphasize that all relative signs for different contributions are determined unambiguously.

* For $\varphi_N(x) = \varphi_{as}(x)$: $M_0 \simeq 0.1$; for $\varphi_N(x) = \varphi_{nonrel}(x)$: $M_0 \simeq 2 \cdot 10^{-2}$.

Let us estimate now the ratio $r=\Gamma(\psi\to\bar{p}p)/\Gamma(\psi\to\bar{n}n)$:

$$r=\left(\frac{1+\delta^{p}}{1+\delta^{n}}\right)^{2},\quad \delta^{p,n}=\frac{\left[M_\gamma^{p,n}\right]_a+\left[M_\gamma^{p,n}\right]_b}{M_{str}}\equiv\delta_a^{p,n}+\delta_b^{p,n},$$

$$\begin{pmatrix}\delta_b^{p}\\ \delta_b^{n}\end{pmatrix}=-\frac{4}{5}\,\frac{\alpha}{\bar{\alpha}_s}\begin{pmatrix}-2.16\cdot10^{-2}\\ 0\end{pmatrix}\quad \text{at } \bar{\alpha}_s=0.27 \tag{10.25}$$

($\bar{\alpha}_s=0.27$ corresponds $\Gamma(\psi\to\bar{p}p)/\Gamma(\psi\to all)\simeq0.22\%$ in an absence of the electromagnetic corrections, see (10.20)-(10.22)),

$$\delta_a^{p,n}\equiv G_M^{p,n}(M_\psi^2)\,\delta_0,\quad \delta_0=\frac{\frac{2}{3}\,4\pi\alpha}{(4\pi\bar{\alpha}_s)^3\left|\frac{120 f_N}{M_\psi^2}\right|^2\frac{20}{81}M_0}\simeq3.25. \tag{10.26}$$

Let us take:

$$M_\psi^4\,G_M^{p}(M_\psi^2)\simeq1.1\ \text{GeV}^4,\quad M_\psi^4\,G_M^{n}(M_\psi^2)\simeq-0.55\ \text{GeV}^4. \tag{10.27}$$

Then: $\delta_a^{p}\simeq3.9\cdot10^{-2}$, $\delta^{p}=(\delta_a^{p}+\delta_b^{p})\simeq1.75\cdot10^{-2}$,

$$\delta^{n}=\delta_a^{n}\simeq-2.0\cdot10^{-2}, \tag{10.28}$$

$$(r-1)=\left[\frac{\Gamma(\psi\to\bar{p}p)}{\Gamma(\psi\to\bar{n}n)}-1\right]\simeq8\%.$$

The experimental data are /2.6/ (compare with (10.22):

$$Br\left(\frac{\psi\to\bar{n}n}{\psi\to all}\right)=(0.18\pm0.09)\%. \tag{10.29}$$

We have calculated also the strong decay amplitude $\chi_2\to\bar{N}N$ which is described by the diagrams like those shown at fig.10.5

(the decay $\chi_0\to\bar{N}N$ is suppressed: $\Gamma(\chi_0\to\bar{N}N)/\Gamma(\chi_2\to\bar{N}N)\sim1/M_c^2$):

$$M(\chi_2\to\bar{p}p)=I_0(4\pi\bar{\alpha}_s)^3\,\frac{2}{81}\,\frac{f_{\chi_2}}{\bar{M}^2}\left|\frac{120 f_N}{\bar{M}^2}\right|^2 M_2,\quad I_0=\bar{N}_2\gamma_\mu N_1\varepsilon_{\mu\nu}\Delta_\nu,$$

$$M_2=\int_0^1 d_3x\int_0^1 d_3y\left\{\left[\frac{\varphi_N(x)}{120}\,\frac{\varphi_N(y)}{120}+4\,\frac{T(x)}{120}\,\frac{T(y)}{120}\right](f_{12}+f_{13})+\right.$$

$$\left.2\left[\frac{V(x)}{120}\,\frac{V(y)}{120}+\frac{A(x)}{120}\,\frac{A(y)}{120}\right]f_{31}\right\}, \tag{10.30}$$

$$f_{ij}=N_i/D_{ij},\quad N_i=1-2y_i(y_i-x_i)/\varkappa_i,$$

$$\varkappa_i=1-(2x_i-1)(2y_i-1),$$

$$D_{ij}=\varkappa_i\left[x_i(1-x_i)x_j\,y_i(1-y_i)^2y_j\right],$$

where $\varepsilon_{\mu\nu}$ is the χ_2 meson polarization tensor, $\bar{M}\simeq2\bar{M}_c\simeq3$ GeV. Substituting the wave functions (10.9), we have*:

$$M_2\simeq9.0. \tag{10.31}$$

Using also

$$\Gamma(\chi_2\to2g)=\frac{2}{45\pi}(4\pi\bar{\alpha}_s)^2\,\frac{|f_{\chi_2}|^2}{\bar{M}}, \tag{10.32}$$

we have from (10.30),(10.32):

$$Br\left(\frac{\chi_2\to\bar{p}p}{\chi_2\to hadr}\right)=(\pi\bar{\alpha}_s)^4\,\frac{64}{729}\left|\frac{120 f_N}{\bar{M}^2}\right|^4 M_2^2\simeq \tag{10.33}$$

$$(\pi\bar{\alpha}_s)^4\,1.5\cdot10^{-2}\%\simeq10^{-2}\%.$$

The experiment gives at present the upper limit only/2.6/:

$$Br(\chi_2\to\bar{p}p)<0.1\%.$$

The cross section of the resonance production $\bar{p}p\to\chi_2(3555)$,

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* For $\varphi_N(x)=\varphi_{as}(x)$: $M_2\simeq1.3$; for $\varphi_N(x)=\varphi_{nonrel}(x)$: $M_2\simeq0.25$.

$\bar{p}p \to \chi_0(3415)$ has the form:

$$\sigma_J \simeq \sqrt{2\pi^{3/2}}\,(2J+1)\frac{1}{M_J^2}\;\frac{\Gamma(\chi_J \to \bar{p}p)}{\Delta M}\quad, \tag{10.34}$$

where M_J is the resonance mass, J is its spin, ΔM is the beam spread. For $\bar{p}p \to \chi_2(3555)$: $J=2$, $M_J^2 = 12.6\ \mathrm{GeV}^2$, $\Gamma(\chi_2 \to \bar{p}p) \simeq 10^{-4}\,\Gamma_{tot}(\chi_2) \simeq 2\cdot 10^{-4}$ MeV and

$$\sigma(\bar{p}p \to \chi_2) \simeq 2.5\cdot 10^{-32}\ \mathrm{cm}^2 \quad \text{at} \quad \Delta M \simeq 10\,\mathrm{MeV}. \tag{10.35}$$

10.4. CONCLUSIONS

1. The QCD sum rules predict unambiguously that the nucleon wave function $\varphi_N(x, M\simeq 1\,\mathrm{GeV})$ has very specific properties and is very unlike both the asymptotic wave function $\varphi_{as}(x) = 120x_1x_2x_3$ and the nonrelativistic wave function $\varphi_{nonrel}(x) = \delta(x_1 - 1/3)\,\delta(x_2 - 1/3)$. The most characteristic property of the nucleon wave function $\varphi_N(x, M\simeq 1\,\mathrm{GeV})$ is the following: about 65% of the nucleon momentum (at $P_z \to \infty$) is carried by one u-quark with the spin parallel to the nucleon spin, while each of two rest quarks carries $\simeq$ 15%-20% of the total momentum.

2. The signs and the absolute values of the proton and neutron magnetic form factors $G_M^{p,n}(q^2)$ are very sensitive to the precise form of the nucleon wave function. Just the wave function with the above described properties leads to the predictions for $G_M^{p,n}(q^2)$ which agree with the experiment both in signs and absolute values. At the same time, any wave function (like $\varphi_{as}(x)$, $\varphi_{nonrel}(x)$, ...) which corresponds to more or less equal distribution of the nucleon momentum between three quarks, leads to wrong predictions for the signs of both magnetic form factors.

It was shown in /1.30/ that there are "anomalous" logarithmic contributions into the nucleon form factor, which appear first at the two-loop level and are not described by the renormalization group. The hard kernel for the sum of such two-loop contributions into $G_M^{p,n}(q^2)$ has been calculated explicitly in the paper /3.6 /. Integrating this kernel with the nucleon wave functions (10.9) one obtains that the total "anomalous" contribution is smaller by a factor $\approx \frac{\alpha_s^2(Q^2)}{\pi^2}\ln\frac{Q^2}{\mu^2} \sim 10^{-2}$ in comparison with the above calculated Born contributions and is, therefore, negligble.

3. Using the same wave function, one can obtain the predictions for $\Psi \to \bar{p}p$ and $\Psi \to \bar{n}n$ decay widths in agreement with the experiment, while the wave function $\varphi_{as}(x)$, for instance, gives the result $\simeq$ 25 times smaller.

4. The ep- and en deep inelastic structure functions $F_2^p(x)$ and $F_2^n(x)$ are dominated at $x \to 1$ by the three-quark component of the total nucleon wave functions. Hence, the behaviour of $F_2^{p,n}(x)$ at $x \to 1$ can be expressed through corresponding integrals of the nucleon wave function $\varphi_N(x)$. Because $\simeq$65% of the proton momentum is carried by one u-quark (for the neutron by d-quark), we can accept in the first approximation that the whole nucleon momentum is carried by one quark, while the rest two quarks are "wee" and can be ignored. In this case/10.1/:

$$r = \frac{F_2^n(x)}{F_2^p(x)} \simeq \frac{e_d^2}{e_u^2} = \frac{1}{4} \qquad \text{at} \quad x \to 1. \tag{10.36}$$

It is reasonable to expect that the true value of "r" is near 0.25 and, at least, much smaller than the value $r = (3/7) \simeq 0.43$ which is predicted for the SU(6)-symmetric nucleon wave function /1.17/. The experimental data are at present /10.5/: $r = (0.20 \pm 0.10)$ at $x \simeq 0.65$.

Analogously (in the same approximation), one can argue that the observation of the leading proton (neutron) in the reaction $e^+e^- \to$ hadrons indicates that the photon has transformed into the pair of u(d)-quarks. Thus, one should expect that the number of leading protons in the e^+e^--annihilation is $\simeq 4$ times larger than of leading neutrons. Besides, the presence of the leading proton in one of two jets indicates that the second jet has the parent $\bar{u}$-quark and so the particles ($\bar{P}, \pi^-, K^-$) rather than ($\bar{n}, \pi^+, \bar{K}^0$) will be the leading ones in this jet.

11. SUMMARY

The investigation of the asymptotic behaviour of hadronic exclusive processes has its own history. It is interesting to compare our understanding of these problems at present and in the past . We can distinguish two periods-before and after the 1977, when the modern approach to these problems started.

The main results before the 1977 were the "dimensional counting rules" and the pinch contributions in scattering amplitudes. The "dimensional counting rules" gave a large number of predictions which agreed with the experimental data. Hence, it was a great success, from a phenomenological viewpoint.

However, the above results had a little specific to QCD and, besides, a number of important questions remained without answers, such as: the dependence of the asymptotic behaviour on hadron's quantum numbers, the absolute values of exclusive amplitudes, the role of higher order perturbation theory contributions, etc.

The main ideas and methods of the modern approach to an investigation of exclusive processes within the QCD framework have been formulated in 1977. The new operator expansions were proposed which gave a possibility to calculate a power behaviour of exclusive amplitudes. As a result, the power dependence of the asymptotic behaviour on the hadron's quantum numbers was obtained, while logarithmic corrections were neglected at this stage. Besides, the hadron's wave functions were introduced as the matrix elements of multilocal gauge-invariant operators. The above operator expansions gave, in principle, a possibility to calculate the absolute normalization of exclusive amplitudes, provided these nonperturbative hadron's wave functions were known.

Besides , the properties of logarithmic corrections were investigated in the QCD perturbation theory. The cancelation of double logarithmic contributions has been pointed out. Based on the explicit calculation of one- and two-loop logarithmic corrections, the operator expansion for the sum of logarithmic corrections has been proposed. Finally, the asymptotic behaviour of the pion form factor was calculated explicitly in the formal limit $Q^2 \to \infty$

Further investigations developed into the perturbative and nonperturbative directions.

a) The perturbative direction. The more and more rigorous formal proofs of the operator expansions were proposed. The simple equivalent approach to the summation of logarithmic corrections using a "physical" gauge and the Bethe-Salpeter equation has been described.

It was pointed out that there are the logarithmic corrections which are not described by the renormalization group. It was elusidated how the Sudakov effects suppress such contributions.

The properties of the quark-gluon mixing were investigated in detail for the flavour singlet channels. The properties of nonleading logarithms, higher twist processes and power corrections were considered.

b) The nonperturbative direction. The QCD sum rules have been used for the investigation of hadron's nonperturbative wave functions. Specific properties of various light hadron wave functions were elucidated. The properties of the SU(3)-symmetry breaking effects in the hadron's wave functions, the wave functions of the mesons which contain heavy quarks, etc., were considered. It has been demonstrated that, in general, the form of nonperturbative wave functions $\varphi(x, \mu \sim 1\,GeV)$ differs greatly both from

the non-relativistic and the asymptotic (perturbative) forms of wave functions.

A large number of applications of the general methods has been considered. The main result is that it is possible to obtain the selfconsistent description of all considered exclusive processes in a reasonable agreement with the experiment. The selfconsistency means that the same wave function sumiltaneously fulfills the QCD sum rules and is used for a description of all exclusive processes in which the given hadron participates.

Probabilities of the exclusive processes are very sensitive to the form of hadron's wave functions. When the form of the wave function is varied considerably, the probabilities change by two orders of magnitude. Therefore, the agreement of theoretical predictions with experimental data for a large number of various exclusive processes can hardly be a mere coincidence, but ruther confirms a correctness of the whole approach. Note that the theoretical formulae for exclusive amplitudes contain no free adjustable parameters.

On the whole, it seems that we understand quite completely at present most characterisric properties of exclusive processes in QCD. We can also calculate their probabilities with a "reasonable" accuracy. This "reasonable" accuracy is not high, however (typically within the factor $\simeq 2$ in probabilities). From our viewpoint, just the problems connected with the increasing of an accuracy become the most important now.

ACKNOWLEDGMENTS

We are grateful to L.N.Lipatov, V.G.Serbo, E.V.Shuryak, M.V. Terent'ev, A.I.Vainshtein and I.R.Zhitnitsky for numerous fruitful discussions. We thank also B.L.Ioffe, I.B Khriplovich, L.B.Okun, M.A.Shifman and V.I.Zakharov for useful discussions and critical remarks.

APPENDIX A. UNITS AND NOTATIONS

1. $\hbar = c = 1\,, \quad (1\,fm)^{-1} = 197.3\ MeV\,, \quad m_p^{-1} = 2.1\cdot 10^{-14}\ sm,$

where m_p is the proton mass.

The metric tensor has the standard form:

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1 .$$

2. The Dirac matrices γ_μ:

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}\,, \quad \hat{a} = a_\mu\cdot\gamma_\mu = a_0\gamma_0 - \bar{a}\cdot\bar{\gamma}$$

$$\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]\,, \quad \gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

$$4I_{\alpha\tau}\cdot I_{\sigma\beta} = I_{\alpha\beta}\cdot I_{\sigma\tau} + (\gamma_5)_{\alpha\beta}\cdot(\gamma_5)_{\sigma\tau} + (\gamma_\mu)_{\alpha\beta}\cdot(\gamma_\mu)_{\sigma\tau} - (\gamma_\mu\gamma_5)_{\alpha\beta}\cdot(\gamma_\mu\gamma_5)_{\sigma\tau} - \frac{1}{2}(\sigma_{\mu\nu})_{\alpha\beta}\cdot(\sigma_{\mu\nu})_{\sigma\tau}\,,$$

$$\gamma_\mu\gamma_\nu\gamma_\lambda = g_{\mu\nu}\gamma_\lambda - g_{\mu\lambda}\gamma_\nu + g_{\nu\lambda}\gamma_\mu - i\epsilon_{\mu\nu\lambda\sigma}\gamma_\sigma\gamma_5 .$$

3. The SU(3)-matrices λ^a:

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = if^{abc}\frac{\lambda^c}{2}\,, \quad Sp\left(\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right) = \frac{1}{2}\delta^{ab}, \ a,b,c = 1,8,$$

$$\left(\frac{\lambda^a}{2}\frac{\lambda^a}{2}\right)_{ij} = \frac{4}{3}\delta_{ij}\,, \quad i,j = 1,2,3\,; \ \lambda^a\lambda^b = if^{abc}\lambda^c + d^{abc}\lambda^c + \frac{2}{3}\delta^{ab},$$

$$\left(\frac{\lambda^a}{2}\right)_{ij}\left(\frac{\lambda^a}{2}\right)_{kl} = \frac{1}{2}\left[\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl}\right],$$

$$\bar{\psi}_j\psi_i = \frac{(\bar{\psi}\psi)}{3}\delta_{ji} + (\lambda^a)_{ij}(\bar{\psi}\lambda^a\psi)\frac{1}{2} .$$

5. Probabilities have the form:

The decay $1 \to 2$:

$$dW = \frac{1}{8\pi\mu}|M|^2|\vec{p}|\frac{d\Omega}{4\pi}\,,$$

where μ is the initial mass, $\vec{p}$ is the final momentum in the c.m.s. At $\mu \gg m_{1,2}$:

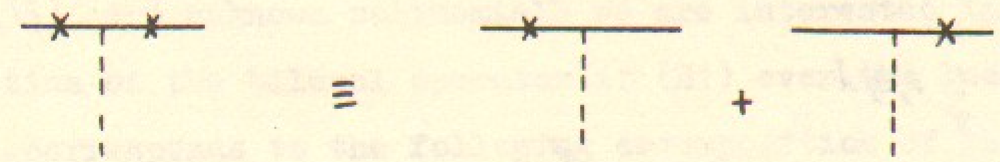
$$W = \frac{1}{16\pi\mu}|M|^2 .$$

The elastic scattering:

$$d\sigma = \frac{1}{64\pi s}|M|^2\frac{dt}{I^2}\frac{d\varphi}{2\pi}\,, \quad I^2 = (p_1p_2)^2 - m_1^2m_2^2\,,$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2\,, \quad t = (p_1-p_3)^2 = (p_2-p_4)^2 .$$

6. Dotted lines denote gluons, solid lines-quarks, wavy lines-photons (or W-bosons), "x"-denotes the external current vertex:



4. The Feynman rules are:

$$L = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_f \bar{\psi}_f (i\hat{D} - m)\psi_f + \Delta L \, ,$$

$$iD^{ij}_\mu = i\partial_\mu \delta^{ij} + g B^a_\mu \left(\frac{\lambda^a}{2}\right)^{ij} \, ,$$

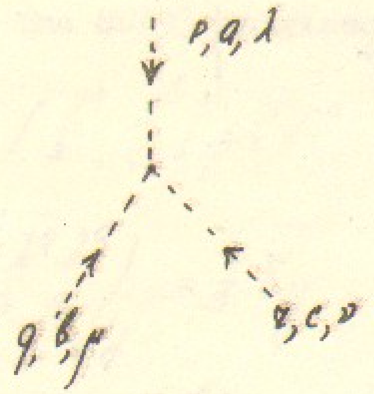
$$G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g f^{abc} B^b_\mu B^c_\nu \, ,$$

$$(D_\mu G_{\mu\nu})^a = -g \sum_f \bar{\psi} \gamma_\nu \frac{\lambda^a}{2} \psi \, , \quad \tilde{G}^a_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\lambda\sigma} G^a_{\lambda\sigma} \, ,$$

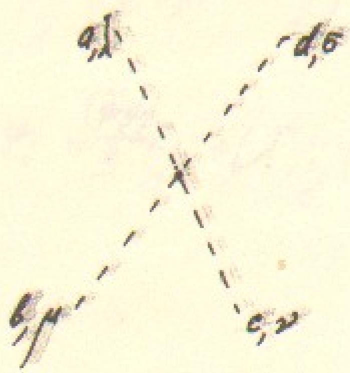
$$\Delta L = -\frac{1}{2\xi} (\partial_\mu B^a_\mu)^2 + (\partial_\mu \varphi^{*a})(\partial_\mu \varphi^a + g f^{abc} B^b_\mu \varphi^c) \, ,$$

$$i g \gamma_\mu \frac{\lambda^a}{2} \, ,$$

$$-i \frac{\delta^{ab}}{k^2} \left[g_{\mu\nu} - (1-\xi) \frac{k_\mu k_\nu}{k^2} \right] ,$$



$$+ g f^{abc} \left[(p-q)_\nu g_{\lambda\mu} + (q-k)_\lambda g_{\mu\nu} + (k-p)_\mu g_{\nu\lambda} \right] ,$$



$$-i g^2 f^{abl} f^{cdl} (g_{\lambda\nu} g_{\mu\sigma} - g_{\lambda\sigma} g_{\mu\nu})$$
$$-i g^2 f^{acl} f^{bdl} (g_{\lambda\mu} g_{\nu\sigma} - g_{\lambda\sigma} g_{\mu\nu})$$
$$-i g^2 f^{adl} f^{cbl} (g_{\lambda\nu} g_{\mu\sigma} - g_{\lambda\mu} g_{\sigma\nu}) .$$

APPENDIX B. THE FORM OF ASYMPTOTIC WAVE FUNCTIONS

The purpose of this appendix is to describe the simple method for finding the asymptotic form of various wave functions /6.1/, $\varphi_{as}(x) = \varphi(x, \mu \to \infty)$. We want to emphasize that the form of $\varphi_{as}(x)$ has nothing to do with hadron properties. The asymptotic wave functions are pure perturbative objects and so, they can be found by computing simplest Feynman diagrams.

To illustrate the method, consider the leading twist pion wave function $\varphi^A_\pi(\xi, \mu)$ defined through the matrix element of the bilocal operator:

$$\langle 0 | \bar{d}(z) \hat{z} \gamma_5 \exp\{ ig \int_{-z}^{z} d\sigma_\nu B_\nu(\sigma) \} u(-z) | \pi(q) \rangle_\mu = i f_\pi (zq) \int_{-1}^{1} d\xi \, e^{i\xi(zp)} \varphi^A_\pi(\xi, \mu), \quad \text{(B1)}$$

$$z^2 = 0 \, ,$$

where μ is the renormalization point.

Let us introduce now the system $\{O_n\}$ of multiplicatively renormalized operators, $O_n(0) = \bar{d}(0) \hat{z} \gamma_5 P_n(iz\overleftrightarrow{D}/zq) u(0)$, where $P_n(\xi)$ are unknown polinomials we are interested in. The decomposition of the bilocal operator in (B1) over the local ones, O_n, corresponds to the following decomposition of the wave function: $\varphi^A_\pi(\xi, \mu) = \varphi_{as}(\xi) \sum_{n=0}^{\infty} f^n_\pi(\mu) P_n(\xi)$,

$$\int_{-1}^{1} d\xi \, \varphi_{as}(\xi) P_n(\xi) P_m(\xi) = \delta_{nm}, \quad f^n_\pi(\mu) = \int_{-1}^{1} d\xi \, \varphi^A_\pi(\xi, \mu) P_n(\xi), \quad \text{(B2)}$$

$$f^n_\pi(\mu_2) = f^n_\pi(\mu_1) \exp\left\{ -\int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)} \frac{d\alpha}{\beta(\alpha)} \gamma_n(\alpha) \right\} ,$$

where $\gamma_n(\alpha)$ are the corresponding anomalous dimensions, $\gamma_{n+1} > \gamma_n$ and $\beta(\alpha)$ is the Gell-Mann-Low function. The system of polinomials $\{P_n(\xi)\}$ and the asymptotic wave function $\varphi_{as}(\xi)$ are determined completely by the perturbation theory properties, while the properties of the hadron determine the values of the constants f_π^n in (B2).

To find the form of $\varphi_{as}(\zeta)$, it is convenient to use the conformal invariance of QCD in the small distance region. Let us restrict ourselves at first with the Born approximation, put the quark masses equal zero and take two local operators O_1 and O_2 belonging to different representations of the conformal group. Then the two-point correlator

$$T_{12} = i\int dx\, e^{iqx} \langle 0 | T\, O_1(x)\, O_2(0) | 0 \rangle = 0 \tag{B3}$$

equals zero due to the conformal invariance. When the quark masses and the non-perturbative corrections which break the conformal symmetry are taken into account (but all logarithmic corrections are still neglected), the formula (B3) remains true at $|q^2| \to \infty$, because all symmetry breaking corrections die off at large q^2. It has been pointed out in /3.10/ that the conformal spin is still conserved when leading logarithmic corrections are accounted for. This implies that the conformal operators do not mix with each other in this approximation and so, they are multiplicatively renormalized. Therefore, the formula (B3) remains true in the leading logarithmic approximation also. We now use it to determine the form of $\varphi_{as}(\zeta)$ and $\{P_n(\zeta)\}$.

Let us choose in (B3): $O_1 = \bar d \hat z \gamma_5 (i z \overleftrightarrow{D})^n u$, $O_2 = \bar u \hat z \gamma_5 d$, and calculate the correlator in the Born approximation:

$$T_n(q,z) = i\int dx\, e^{iqx} \langle 0 | T\, \bar d(x) \hat z \gamma_5 (i z\overleftrightarrow{D})^n u(x)\, \bar u(0) \hat z \gamma_5 d(0) | 0 \rangle =$$

$$(zq)^{n+2}\, T_n(q^2), \quad z^2 = 0, \tag{B4}$$

$$T_n(q^2) = -\frac{1}{4\pi^2} \ln(-q^2) \cdot \int_{-1}^{1} d\zeta\, \frac{3}{4}(1-\zeta^2)\zeta^n .$$

It follows now from (B4) and (B3):

a) the asymptotic wave function is: $\varphi_{as}(\zeta) \sim (1-\zeta^2)$,

b) the system of multiplicatively renormalized operators is: $\{O_n\} = \{\bar d \hat z \gamma_5 C_n^{3/2}(z\overleftrightarrow{D}/z\partial) u\}$, where $C_n^{3/2}(\zeta)$ are the Gegenbauer polinomials. The reason is that just these polinomials are orthogonal with the measure $(1-\zeta^2)$:

$$\int_{-1}^{1} d\zeta\, (1-\zeta^2)\, C_n^{3/2}(\zeta)\, C_m^{3/2}(\zeta) \sim \delta_{nm} .$$

That is, the explicit form of $\varphi_{as}(\zeta)$ and the orthogonality conditions (B3) determine uniquely the system of polinomials corresponding to multiplicatively renormalized two-particle operators.

On the whole, we see that it is sufficient to calculate the corresponding correlators in the Born approximation (i.e. loop diagrams with free massless quarks and gluons) to find $\varphi_{as}(\zeta)$ and $\{P_n(\zeta)\}$*.

Below some examples are presented ($\zeta = x_1 - x_2$, $x_1 + x_2 = 1$):

$$\begin{aligned}
&a)\ \bar\psi(1\pm\gamma_5)\psi, \quad \varphi_{as} \sim 1 \to C_n^{1/2}(\zeta) \quad /9.1/\\
&b)\ \bar\psi\gamma_\mu(1\pm\gamma_5)\psi,\ \bar\psi\sigma_{\mu\nu}\psi, \quad \varphi_{as} \sim (1-\zeta^2) \to C_n^{3/2}(\zeta) \quad /1.26, 1.28, 1.29/\\
&c)\ G_{\mu\nu}G_{\alpha\beta}, \quad \varphi_{as} \sim (1-\zeta^2)^2 \to C_n^{5/2}(\zeta) \quad /3.14/\\
&d)\ \bar\psi\gamma_\mu(D_\perp^2)^k\psi, \quad \varphi_{as} \sim (1-\zeta^2)^{2k+1} \to C_n^{2k+3/2}(\zeta) \quad /6.1/\\
&e)\ \bar\psi(D_\perp^2)^k\psi, \quad \varphi_{as} \sim (1-\zeta^2)^{2k} \to C_n^{2k+1/2}(\zeta) \quad /6.1/.
\end{aligned} \tag{B5}$$

---

* It is even simpler to calculate the discontinuity of the correlator.

The asymptotic wave functions $\varphi_{as}(x_1, x_2, x_3)$ for three-particle operators can be found analogously, but the orthogonality conditions are unsufficient in this case to determine uniquely the corresponding polinomials.

Calculating the three-particle Born diagrams with free quarks and gluons, one obtains:

$$
\begin{aligned}
&f)\ \psi C\gamma_\mu(1\pm\gamma_5)\psi\cdot\psi\,,\quad \psi C\sigma_{\mu\nu}\psi\cdot\psi\,,\quad \varphi_{as}\sim x_1x_2x_3 \quad /1.29,2.8/\\
&g)\ \bar\psi\gamma_\mu(1\pm\gamma_5)\psi\, G_{\alpha\beta}\,,\qquad \varphi_{as}\sim x_1x_2x_3^2 \quad /6.1/\\
&h)\ G_{\mu\nu}G_{\lambda\sigma}G_{\alpha\beta}\,,\qquad \varphi_{as}\sim x_1^2x_2^2x_3^2 \quad /6.1/.
\end{aligned}
\tag{B6}
$$

The results (B5) and (B6) refer to the kinematical structures of the leading twist for each given operator. Analogously, by separating in the correlators the kinematical structures of the nonleading twist (for instance, the twist 3 for two-particle operators, the twist 4 for three-particle ones, etc.), one can find the form of corresponding asymptotic wave functions. In particular, all two-particle wave functions of the twist 3 used above in the text can be found in this way.

Let us consider as an example the $\rho_\perp$ -meson wave function of the twist three (see ch.9): $\varphi_\rho^{V,\perp}(\xi, \mu\to\infty) = \frac{3}{8}(1+\xi^2)$. Because $(1+\xi^2) = 2-(1-\xi^2)$, the corresponding polinomials have the form: $P_n(\xi) = a_n C_n^{3/2}(\xi) + b_n C_n^{1/2}(\xi)$. The coefficients a_n and b_n are determined by the orthogonality conditions:

$$\int_{-1}^{1} d\xi\, \frac{3}{8}(1+\xi^2)\, P_n(\xi) P_m(\xi) = \delta_{nm}.$$

As the mixing matrix is triangular, the anomalous dimension of the local operator O_n corresponding to $P_n(\xi)$ is:

$$\varepsilon_n = C_F\left[1 - \frac{2}{(n+1)(n+2)} + 4\sum_2^{n+1}\frac{1}{j}\right],$$

(before a mixing with three-particle operators).

APPENDIX C. <u>HEAVY QUARKONIUM WAVE FUNCTIONS</u> /1.44/

These wave functions are defined in a standard way:

$$\Phi_{\alpha\beta}(z,p) = \langle 0|\bar Q_\beta(z)\exp\{ig\int_{-z}^{z} d\sigma\, B(\sigma)\}Q_\alpha(-z)|M(p)\rangle, \tag{C1}$$

where Q is the heavy quark field, $|M(p)\rangle$ is the quarkonium state with the momentum p. The expansion of (C1) into z-series is at the same time an expansion in powers of quark relative momentum. The relative momentum is small in the non-relativistic case: $P_{rel}\sim (v/c) M_Q \ll M_Q$ (M_Q is the heavy quark mass). Hence, one can restrict himself by the first nontrivial order in z in (C1). In the same non-relativistic approximation one can consider the quarks in (C1) as free ones. Using then the Dirac equation it is not difficult to find the explicit form of the wave functions. The results are as follows:

1) $^3P_0$ state:

$$\Phi_{\alpha\beta}(z,p) = \frac{i}{4} f_{\chi_0}\left[\hat p(zp) - 4M_Q^2\hat z + 6iM_Q - 2M_Q\sigma_{\mu\nu}p_\mu z_\nu\right]_{\alpha\beta}, \tag{C2}$$

2) $^3P_2$ state:

$$\Phi_{\alpha\beta}(z,p) = \frac{1}{4} 2M_Q f_{\chi_2}\left[\hat\rho\, 2M_Q + \sigma_{\mu\nu}p_\mu \rho_\nu\right]_{\alpha\beta}, \tag{C3}$$

where $\rho_\mu = \varepsilon_{\mu\nu} z_\nu$ and $\varepsilon_{\mu\nu}$ is the $^3P_2$ meson polarization tensor,

3) $^3S_1$ state:

$$\Phi_{\alpha\beta}(z,p) = \frac{1}{4} f_\psi\left[\hat\varepsilon\, 2M_Q + \sigma_{\mu\nu}p_\mu\varepsilon_\nu\right]_{\alpha\beta}, \tag{C4}$$

where ε_μ is the $^3S_1$ meson polarization vector,

4) $^1S_0$ state:

$$\Phi_{\alpha\beta}(z,p) = \frac{i}{4} f_{\eta_c}\left[\hat p\gamma_5 + 2M_Q\gamma_5\right]_{\alpha\beta}. \tag{C5}$$

The constants f_i in (C2)-(C5) have the dimension of the mass and determine the wave function values at the origin. They are connected with the non-relativistic wave functions $R_S(0)$ and $R'_P(0)$ /1.40/ as follows:

$$f_{\eta_c}^2 = f_\psi^2 = \frac{3}{2}\left|R_{1S}(0)\right|^2/\pi M_Q ,$$

$$f_{\chi_0}^2 = \frac{81}{32}\left|R'_P(0)\right|^2/\pi M_Q^3 , \quad f_{\chi_2}^2 = \frac{243}{32}\left|R'_P(0)\right|^2/\pi M_Q^3 .$$

At large M_Q : $f_i \sim M_Q$. For the charmonium: $f_\psi \simeq 400$ MeV (compare with $f_\pi \simeq 133$ MeV, $f_\rho \simeq 200$ MeV).

APPENDIX D. QUARK VACUUM CONDENSATES AND CHIRAL SYMMETRY BREAKING

It has been shown in ch.6 that a selfconsistency of various sum rules requires unambiguously

$$\left|\langle 0|\bar{u}u|0\rangle\right| > \left|\langle 0|\bar{d}d|0\rangle\right| > \left|\langle 0|\bar{s}s|0\rangle\right| . \qquad (D1)$$

Besides, the numerical estimates were obtained therein (the expected accuracy is $\simeq 20\%$):

$$\frac{\langle 0|\bar{u}u-\bar{s}s|0\rangle}{\langle 0|\bar{s}s|0\rangle} \simeq (m_s - m_u)\frac{m_K^2}{(m_s+m_u)M_{K^*}^2} \simeq 0.3 ,$$

$$\frac{\langle 0|\bar{u}u-\bar{d}d|0\rangle}{\langle 0|\bar{d}d|0\rangle} \simeq (m_d - m_u)\frac{m_\pi^2}{(m_d+m_u)M_\rho^2} \simeq 0.8\cdot 10^{-2} . \qquad (D2)$$

There is at present a number of papers /6.2,6.4,6.5/ in which the signs and the absolute values of the differenses $\langle \bar{u}u-\bar{s}s\rangle$ and $\langle \bar{u}u-\bar{d}d\rangle$ were determined with a help of the QCD sum rules for meson and baryon mass differences or by the different method /6.3/. All results agree with each other and with (D1) and (D2). Therefore, there is no doubt at present that the heavier is the quark, the smaller is the absolute value of its vacuum condensate $\langle 0|\bar{\psi}\psi|0\rangle$*

The purpose of this appendix is to elucidate the relationship between the formulae (D1),(D2) and the chiral SU(2)xSU(2) and SU(3)xSU(3) perturbation theory / 5.1,6.1,6.28/. It seems at the first sight that the unequalities D(1) contradict the predictions of the chiral perturbation theory. Consider first the chi-

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* This is trivial for heavy quarks, because when the quark mass m_q is much larger than the inverse confinement radius /5.1/: $\langle 0|\bar{q}q|0\rangle \simeq -\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle/12 m_q$

ral SU(2)xSU(2) limit. Then:

$$\langle 0|\bar{u}u-\bar{d}d|0\rangle \simeq \frac{i}{2}(m_d-m_u)\int dx\langle 0|T\{\bar{u}(x)u(x)- \tag{D3}$$
$$-\bar{d}(x)d(x),\ \bar{u}(0)u(0)-\bar{d}(0)d(0)\}|0\rangle + O(m_u^2, m_d^2).$$

It seems that the spectral density in (D3) is positive definite and so $\langle 0|\bar{u}u-\bar{d}d|0\rangle \simeq (m_d-m_u)|C_0|>0$,in contradiction with (D1). The loophole is the following. The quark condensates $\langle 0|\bar{u}u|0\rangle, \langle 0|\bar{d}d|0\rangle$ and $\langle 0|\bar{s}s|0\rangle$ are so defined that the perturbation theory contributions are subtracted out of them (and are accounted separately in sum rules). Therefore, the perturbation theory contribution should be subtracted out of (D3),and the dispersion relation has really the form:

$$\langle 0|\bar{u}u-\bar{d}d|0\rangle \simeq \frac{m_d-m_u}{2}\,\frac{1}{\pi}\int_0^\infty \frac{ds}{s}\left[\mathrm{Im}\,T(s)-\mathrm{Im}\,T^{\text{pert.th}}(s)\right] \tag{D4}$$
$$\frac{1}{\pi}\mathrm{Im}\,T^{\text{pert.th.}}=\frac{3}{4\pi^2}s.$$

Hence, the answer can have, in principle, any sign.

The logarithmic two-Goldstone meson contribution is the dominant one in $\mathrm{Im}\,T(s)$ in the chiral limit. One can believe, more or less, to the chiral SU(2)xSU(2) perturbation theory results, because the pion mass is sufficiently small (the two pion threshold is: $4m_\pi^2 \simeq 0.075\,\text{GeV}^2$). But there is no two pion contribution into $\langle 0|\bar{u}u-\bar{d}d|0\rangle$.

There are K^+K^- and $\bar{K}^0K^0$ contributions into (D4) and these can easily be calculated in the chiral SU(3)xSU(3) limit:

$$\frac{1}{\pi}\int_0^\infty \frac{ds}{s}\,\mathrm{Im}\,T(s)\Big|_{KK} \simeq \frac{1}{2\pi^2}\left(\frac{\langle 0|\bar{u}u|0\rangle}{f_K^2}\right)^2 \ln\frac{M_0^2}{4m_K^2},$$

where M_0 is the characteristic mass, $M_0\simeq M_\rho - 1\,\text{GeV}$. Analogously,

$$\langle 0|\bar{u}u-\bar{s}s|0\rangle \simeq \frac{5}{12}m_s\left(\frac{\langle 0|\bar{u}u|0\rangle}{f_K^2}\right)^2 \ln\frac{M_0^2}{4m_K^2} + O\left(m_s^2,\ m_u \ln\frac{M_0^2}{4m_\pi^2}\right).$$

These contributions are the dominant ones in the formal chiral SU(3)xSU(3) limit $m_s \sim m_K^2 \to 0$, and their signs are opposite to (D1). But the experience shows that the real spectral densities and the results of the formal chiral SU(3)xSU(3) symmetry limit can differ essentially (the real threshold is $4m_K^2 \simeq 1\,\text{GeV}^2$!).

It has been shown in / 5.1/ using the model spectral density, that even in the correlator

$$i\int dx\langle 0|T\{(\bar{u}u+\bar{d}d)_x\,(\bar{u}u+\bar{d}d)_0\}|0\rangle - (\text{pert. th.}),$$

which includes the two-pion contribution, the role of the logarithmic enhancement is very mild:

$$\langle 0|\bar{u}u+\bar{d}d|0\rangle - \langle 0|\bar{u}u+\bar{d}d|0\rangle_{m_u=m_d=0} \simeq \tag{D5}$$
$$\simeq -\frac{m_d+m_u}{2}\,\frac{3}{2\pi^2}\left(\frac{\langle 0|\bar{u}u|0\rangle}{f_\pi^2}\right)^2\left[\ln\left(\frac{M_0^2}{4m_\pi^2}\right)-1\right],\quad M_0\simeq 1\,\text{GeV},$$

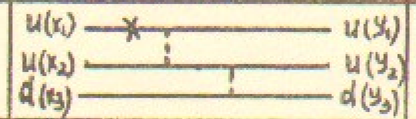
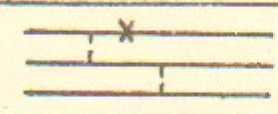
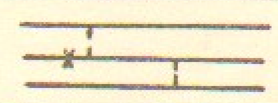
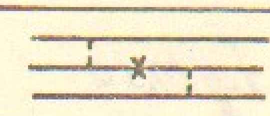
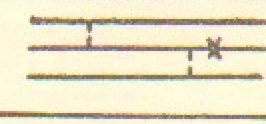
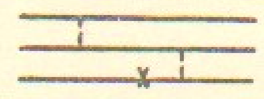
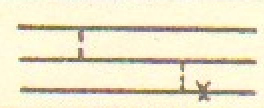
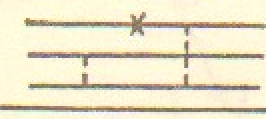
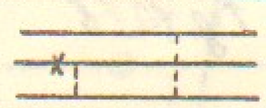
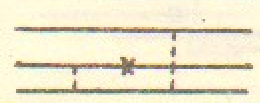
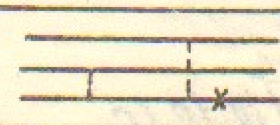
and the sign of the r.h.s. (D5) changes with increasing of m_π at $m_\pi \simeq 300\,\text{MeV}$. This shows once more that the result obtained in the formal SU(3)xSU(3) chiral limit can be misleading.

We conclude that there are no serious reasons to doubt the correctness of (D1),(D2).

Table 10.1

$n_1 n_2 n_3$	$V^{(n_1 n_2 n_3)}$ sum rules	$V^{(n_1 n_2 n_3)}$ model	$\varphi_N^{(n_1 n_2 n_3)}$ sum rules	$\varphi_N^{(n_1 n_2 n_3)}$ model	$T^{(n_1 n_2 n_3)}$ sum rules	$T^{(n_1 n_2 n_3)}$ model	$\varphi_{as}(x) = 120 x_1 x_2 x_3$
000	1	1	1	1	1	1	1
I00	0.38÷0.42	0.39	0,6÷0.75	0.63	0.4÷0.5	0.425	I/3≈0.33
0I0	"	"	0.09÷0.I6	0.I5	0.40÷0,50	0.425	I/3≈0.33
00I	0,I8÷0.24	0.22	0,I8÷0.24	0.22	0.05÷0.I6	0.I5	I/3≈0.33
200	0,I8÷0.25	0.2I	0.25÷0.4	0.40	0.22÷0.28	0.26	I/7≈0.I4
020	"	"	0.03÷0.08	0.025	"	"	I/7≈0.I4
002	0.08÷0.I2	0.08	0.08÷0.I2	0.08	0.02÷0.06	0.02	I/7≈0.I4
II0	0,07÷0.I2	0.II	0.07÷0.I2	0.II	0.09÷0.I3	0.I	2/2I≈0.I
I0I	0,04÷0.08	0.07	0.09÷0.I4	0.I23	0.04÷0.08	0.063	2/2I≈0.I
0II	0.04÷0.08	0.07	−0.03÷+0.03	0.027	0.04÷0.08	0.063	2/2I≈0.I

Table 10.2

i	Diagram	$T_i(x,y)$
1		$\dfrac{\varphi_N(x)\varphi_N(y) + 4T(x)T(y)}{(1-x_1)^2 x_3 (1-y_1)^2 y_3}$
2		0
3		$\dfrac{-4T(x)T(y)}{x_1(1-x_2)x_3\, y_1(1-y_1)y_3}$
4		$\dfrac{\varphi_N(x)\ \varphi_N(y)}{x_1 x_3 (1-x_2)\, y_1(1-y_1)y_3}$
5		$\dfrac{-\varphi_N(x)\ \varphi_N(y)}{x_2 x_3 (1-x_3)\, y_2(1-y_1)y_3}$
6		0
7		$\dfrac{\varphi_N(x)\varphi_N(y)}{x_1(1-x_3)^2 y_1(1-y_3)^2} + \dfrac{\varphi_N(x)\varphi_N(y)}{x_2(1-x_3)^2 y_2(1-y_3)^2}$
8		0
9		$\dfrac{\varphi_N(x)\varphi_N(y) + 4T(x)T(y)}{x_2(1-x_1)^2 y_2(1-y_1)^2}$
10		$\dfrac{\varphi_N(x)\varphi_N(y) + 4T(x)T(y)}{x_2(1-x_1^2) y_2(1-y_1)^2}$
11		0
12		$\dfrac{-\varphi_N(x)\varphi_N(y)}{x_1 x_2 (1-x_3)(1-y_1) y_1 y_2}$
13		$\dfrac{4T(x)T(y)}{x_1 x_2 (1-x_1) y_1 y_2 (1-y_2)}$
14		$\dfrac{-\varphi_N(x)\ \varphi_N(y)}{x_1 x_2 (1-x_1) y_1 y_2 (1-y_3)}$

$$\varphi_N(x) = V(x_1,x_2,x_3) - A(x_1,x_2,x_3),$$

$$2T(x_1,x_2,x_3) = \varphi_N(x_1,x_3,x_2) + \varphi_N(x_2,x_3,x_1).$$

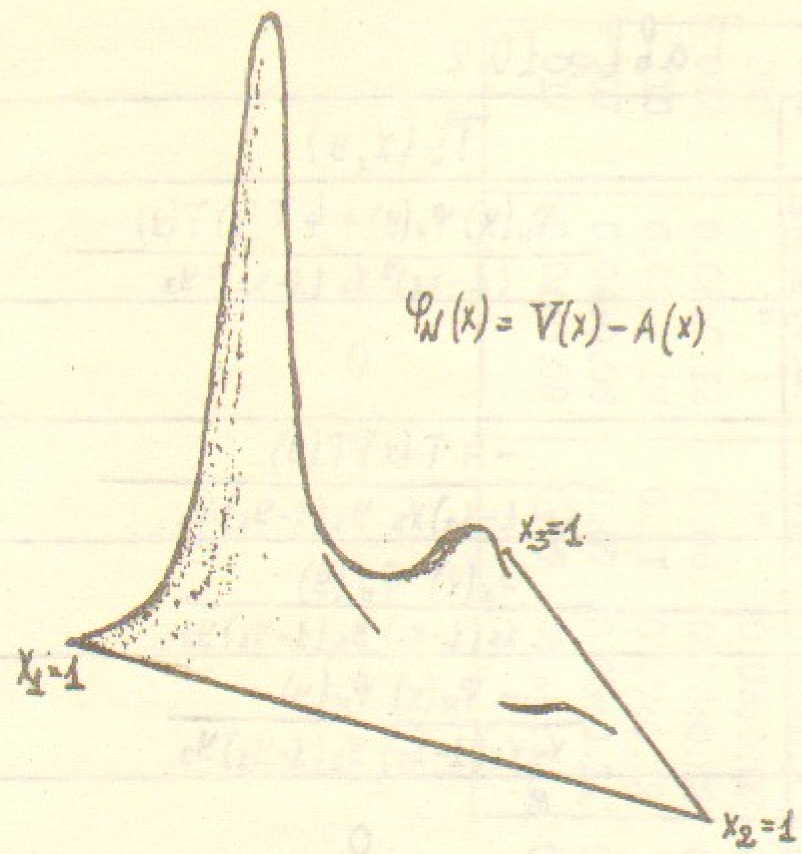


fig 10.1a

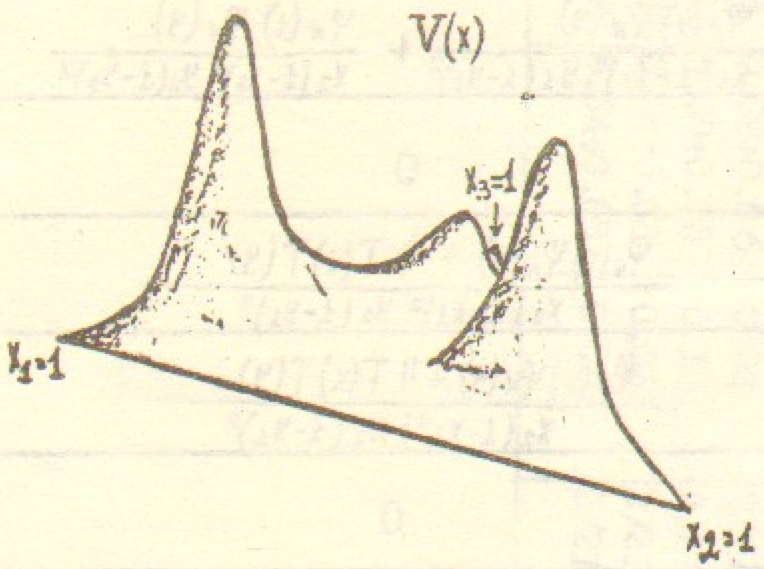


fig. 10.1b

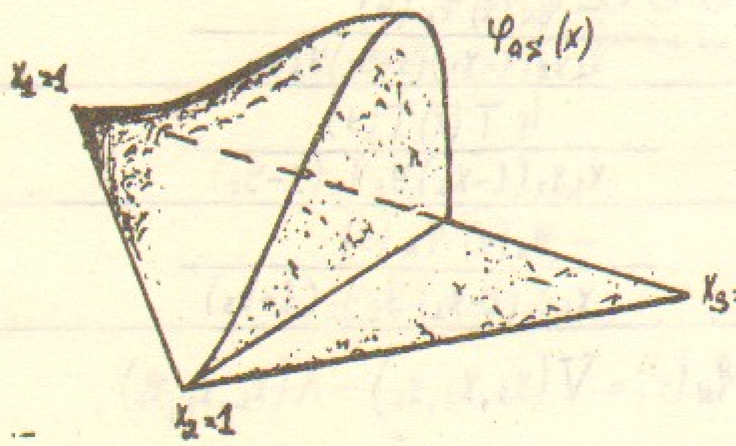
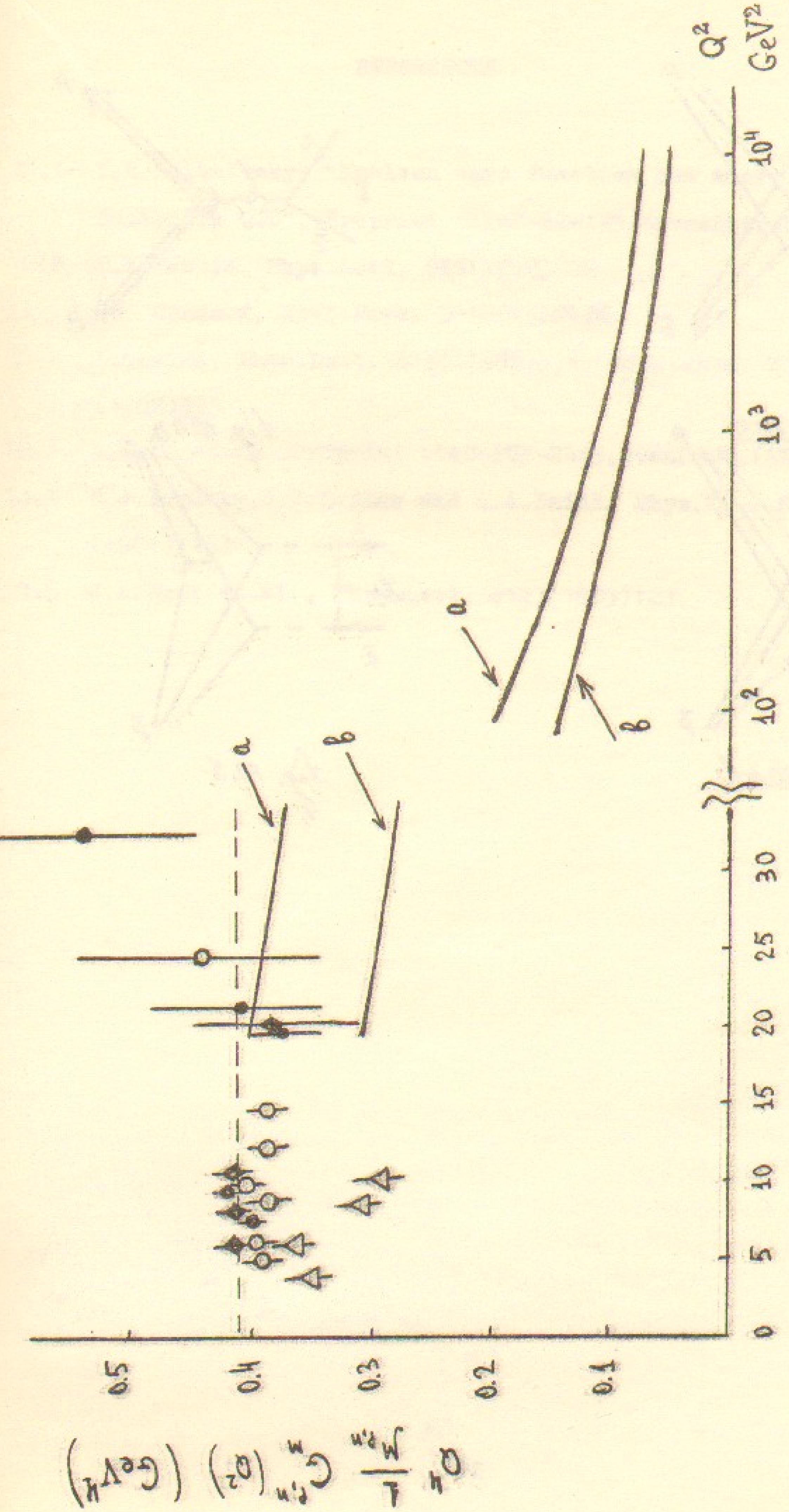


fig. 10.1c

Fig. 10.2



proton data ,

neutron data ,

a: $Q^4 \frac{1}{\mu_p} G_M^p(Q^2)$ - theory , $\mu_p = 2.79$

b: $Q^4 \frac{1}{\mu_n} G_M^n(Q^2)$ - theory , $\mu_n = -1.9$

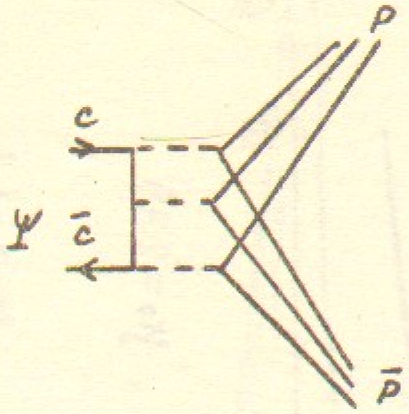


fig. 10.3

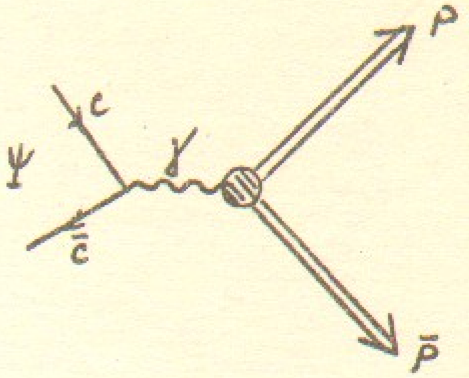


fig. 10.4a.

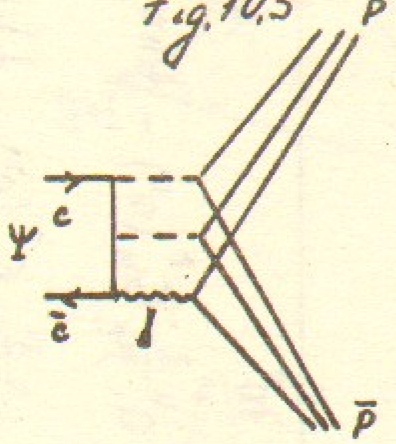


fig. 10.4b.

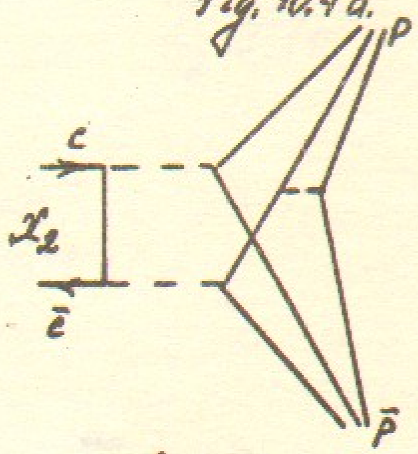


fig. 10.5

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АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ ЭКСКЛЮЗИВНЫХ
ПРОЦЕССОВ В КХД

IO. Нуклон.

Препринт
№83-I08

Работа поступила - 28 июня I983 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати I5.09-I983 г. МН 03356
Формат бумаги 60x90 I/I6 Усл.2,5 печ.л., 2,0 учетно-изд.л.
Тираж 290 экз. Бесплатно. Заказ № I08.

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90