ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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ASYMPTOTIC BEHAVIOUR OF EXCLUSIVE PROCESSES IN QCD

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3. LOGARITHMIC CORRECTIONS

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НОВОСИБИРСК

CONTENT

3.1. General properties.

The properties of logarithmic corrections are investigated in great details at present and are described in the literature /1.25,1.26,1.28-1.32/. For this reason,we describe below various formal results in short and put the main attention to the physical meaning of results. Besides,some new results are presented.

The main idea of the whole approach has been described in ch.1,that is the separation of the contributions into the amplitude which are due to the small ($\sim 1/Q$) and large ($\sim 1/\mu$) distance interactions,and the derivation of the corresponding operator expansions. It is clear from the physical reasoning, however,that the quark-antiquark pair $\bar{q}q$ produced in the small vicinity ($\sim 1/Q$) of the point "0" with $\bar{q}$ and q having the virtualities $\sim Q^2$, resembles very little the final meson. Before the final meson will be formed, q and $\bar{q}$ will interact by exchanging gluons between themselves and decreasing gradually their virtualities from $\sim Q^2$ down to $\sim \mu^2$. Just this evolution of the $\bar{q}q$-pair is described by the perturbation theory loop corrections.

The situation here is analogous to that in the deep-inelastic ep-scattering,fig.3.1. The quark and antiquark produced in the small vicinity ($\sim 1/Q^2$) of the point "0" interact then with each other and diminish their virtualities from $\sim Q^2$ down to $\sim \mu^2$. This final state interaction gives loop logarithmic corrections to the Born amplitude $M(x)$,so that $M(x) \to M(x, Q^2/\mu^2)$. These effects are described by accounting for a dependence of the bilocal operator $\langle p | T\, \bar{\Psi}(z) \exp\{ig\int_0^z d\sigma_\nu B_\nu(\sigma)\} \Psi(0) | p\rangle_{\mu_{max}}$

on the normalization point $\mu^2_{max} \sim (1/\bar{z}^2) \sim Q^2$.

It is natural, therefore, to suppose that analogous result will be correct for the form factor as well. That is, the initial and final state interaction effects can be accounted for by a dependence of the wave functions $\varphi_i(x)$ on the maximal quark virtualities, $\varphi_i(x) \to \varphi_i(x, \mu^2_{max} \sim Q^2)$, i.e. by a dependence of the bilocal operator $\langle 0 | T \bar{\Psi}(z) \exp\{ig \int_{-z}^{z} d\sigma_\nu B_\nu(\sigma)\} \times \Psi(-z) | P \rangle_{\mu^2_{max} \sim Q^2}$ on the normalization point $\mu^2_{max} \sim (1/\bar{z}^2) \sim Q^2$.

In order to calculate in an explicit form the dependence of the bilocal operator on the normalization point μ^2_{max} (μ^2_{max} is the upper cut off in loop integrals entering the matrix elements of this bilocal operator), one should expand it into a series of local operators O_n. The dependence of O_n on μ^2_{max} is determined by the renormalization group:

$$\langle O_n \rangle_{Q^2} = \langle O_n \rangle_{\mu^2} \exp\left\{ - \int_{\alpha_s(\mu)}^{\alpha_s(Q)} \frac{d\alpha}{\beta(\alpha)} \gamma_n(\alpha) \right\}, \tag{3.1}$$

where O_n is a multiplicatively renormalized local operator, γ_n is its anomalous dimensionality, $\beta(\alpha)$ is the Gell-Mann-Low function. After this, we will have instead of (1.5) the expression:

$$Q^2 F_\pi(Q^2) \to \sum_{n_1 n_2} \langle P_2 | O_{n_2}(0) | 0 \rangle_\mu \, C_{n_2 n_1}\left(\frac{Q^2}{\mu^2}, \alpha_s\right) \langle 0 | O_{n_1}(0) | P_1 \rangle_\mu , \tag{3.2}$$

$$C_{n_2 n_1}\left(\frac{Q^2}{\mu^2}, \alpha_s\right) = C_{n_2 n_1}\left(1, \alpha_s(Q^2)\right) \exp\left\{ - \int_{\alpha_s(\mu)}^{\alpha_s(Q)} \frac{d\alpha}{\beta(\alpha)} \left(\gamma_{n_1}(\alpha) + \gamma_{n_2}(\alpha)\right) \right\},$$

where $C_{n_2 n_1}$ are the corresponding expansion coefficients. It will be shown that the operator expansion of the type (3.2) is indeed true within the QCD perturbation theory (at least, for the form factors of the mesons with $\lambda_1 = \lambda_2 = 0$ and for a number of other exclusive processes).

The problem is, of course, to prove the validity (or non-validity) of the operator expansion (3.2) in the QCD perturbation theory (pert. th.).

It is much more difficult to calculate and to sum logarithmic corrections to the Born diagram for a hadron form factor than for the deep-inelastic scattering (in a covariant gauge). Each loop gives no more than one large logarithm $\sim \alpha_s \ln Q^2/\mu^2$ in the latter case (the summation of these logarithms was performed by V.N.Gribov and L.N.Lipatov for the first time /3.1/). In the case of a composite state form factor we encounter immediatly the diagrams which correspond to the form factor of elementary constituents, fig.3.2. Such loops give the double logarithmic contributions $\sim \alpha_s \ln^2 Q^2/\mu^2$ a là Sudakov, i.e. there are infrared logarithms in addition to usual "collinear" logarithms.

It has been shown in /3.2/ that leading contributions $\sim (\alpha_s \ln^2 Q^2)^n$ in n-th order of the pert.th. cancel in a sum of diagrams for a colorless composite state. Therefore, the problem is to see whether all terms from $\sim \alpha_s^n \ln^{2n-1} Q^2$ down to $\sim \alpha_s^n \ln^{n+1} Q^2$ cancel out also and, if so, to what function sum all rest terms $\sim \alpha_s^n \ln^n Q^2$ * ; agrees their sum with the operator expansion (3.2) or not.

The simplest approaches to the problem have, evidently, been used in the first papers on this subject, which appeared in 1977 /1.25, 1.26/. The verification of the operator expansion (3.2) has been performed in /1.25/ as follows. If one calculates all quantities entering (3.2)-coefficients $C_{n_2 n_1}$, anomalous dimensions γ_n, matrix elements $\langle 0 | O_n(0) | P \rangle$, etc., in the lowest nontrivial order and then expands (3.2) into a power series in α_s, then one obtains the predictions for the sum of

* see the next page

leading logarithmic corrections in all orders of the pert.th. From the other hand, these logarithmic corrections can directly be calculated from the Feynman diagrams in few lowest orders of the pert.th. If the results agree, this is a strong argument in favour of a correctness of the operator expansion (3.2). Such a verification has been carried out for the pion form factor $F_\pi(Q^2)$ at the two-loop level. That is:

a) it has been shown by direct calculation of the Feynman diagrams that the contributions $\sim g^2 \ln^2 Q^2$ and $\sim g^4 \ln^4 Q^2$, $g^4 \ln^3 Q^2$ cansel and the rest contributions $\sim g^2 \ln Q^2$ and $\sim g^4 \ln^2 Q^2$ have been calculated explicitly;

b) the terms $\sim g^2 \ln Q^2$ and $\sim g^4 \ln^2 Q^2$ in the expansion of (3.2) into a power series in $(g^2)^n$ have been calculated and shown to agree with the direct diagram calculation (see the sect.3.2 for details).

The ladder diagram contributions into $F_\pi(Q^2)$ have been summed by D.R.Jackson /1.26/ using the Feynman gauge. The ladder diagrams, fig.3.3, have no double logarithmic contributions in this gauge. The selection of ladder diagrams allows one to write the Bethe-Salpeter equation for the pion wave function. Using the solution of this equation, one can easily obtain then the "asymptotic behaviour" of the form factor. The sum of ladder diagrams, fig.3.3, corresponds, evidently, to the operator expansion of the type (3.2), but with gauge non-invariant wave functions $\sim \langle 0 | \bar{\Psi}_\beta(z) \Psi_\alpha(-z) | P \rangle$ (i.e. without the gluonic string between the quarks).

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* (from the previous page)

The renormalization group formulae (3.1), (3.2) describe, evidently, the situation when the leading terms in the n-th order of the pert. th. are $\sim \alpha_s^n \ln^n Q^2$.

The formal methods for a summation of both leading and nonleading logarithmic corrections in all orders of the pert.th. (in the Feynman gauge) have been presented in the papers /1.28, 1.30, 3.3/, see also the review /1.32/. All the authors agree that the operator expansion (3.2) indeed takes place for $F_\pi(Q^2)$.

Useful contribution to the subject has been made in /1.29/. It has been pointed that analogously to the deep-inelastic scattering and to the Drell-Yan process /3.4, 3.5/, the use of the "physycal" gauge simplifies considerably the problem, because all leading logarithmic corrections are given in this gauge by ladder diagrams only (but with an account of self-energy corrections), fig.3.4. The problem can again be reduced to a solution of the Bethe-Salpeter equation, but the solution describes in this case the true asymptotic behaviour (see the sect.3.3). The dominance of ladder diagrams makes selfevident a correctness of the operator expansion (3.2) and gives a simple and beautiful picture of the process.

It has been pointed in /1.30/ that one encounters new type of logarithmic corrections when calculating the Feynman diagrams for the nucleon form factor. These appear first at the two loop level, fig.3.5, and are not described by the renormalization group. For instance, the regions $k_i^2 \sim q^2$, i=1,2,3,4, $\mu^2 \ll k_j^2 \lesssim q^2$, j=5,....10 and $k_i^2 \sim q^2$ i=5,6,7,8, $\mu^2 \ll k_j^2 \lesssim q^2$, j=1,2,3, 4,9,10 in the fig. 3.5 diagram give the usual renormalization group contribution $\sim \alpha_s^4 \ln^2 q^2$. All the masses can be put equal zero in this case. These contributions are connected, evidently, with anomalous dimensions of three-particle operators, fig.3.6, and are described by the operator expansion (3.2). It is shown in /1.30/ that there is also the contribution $\sim \alpha_s^4 \ln q^2$ from the region: $k_9^2 \sim k_{10}^2 \sim m^2$, $k_1^2, k_5^2 \ll q^2$, $k_1^2 k_5^2 \sim m^2 q^2$.

This contribution is tightly connected with the quark mass m and is absent if m is taken zero beforehand. It has also been pointed in /1.30/ that (in the abelian case) higher order corrections suppress such contributions.

All two-loop contributions of this kind into the nucleon form factor have been summed in QCD in /3.6/, and the explicit form of the corresponding hard kernel has been obtained there. Besides, it has been shown in /3.7/ by explicitly calculating three-loop diagrams, that analogous suppression mechanism works in QCD as well.*

The physical meaning of this suppression is simple. The corrections like those from the fig. 3.7 diagram lead to an appearance of double logarithmic contributions which sum into the Sudakov form factor $\sim \exp\left\{-\frac{\alpha_s}{2\pi}\ln\frac{Q^2}{K_1^2}\ln\frac{Q^2}{K_5^2}\right\}$, multiplying the external current vertex. There is a large leap of virtualness at the vertex for $K_1^2 \sim K_5^2 \sim \sqrt{m^2Q^2} \ll Q^2$ (see above), and so the Sudakov form factor suppresses such additional contributions.

Analogous, but more complicated situation takes place when one considers logarithmic corrections to the threshold behaviour of inclusive structure functions and to large angle scattering amplitudes /1.30, 1.32, 2.1/ (see the sect. 3.8).

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* The contributions under considerations, fig. 3.5, play no practical role, because they appear first at a two-loop level and include the additional small factor $\simeq \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 \lesssim 10^{-2}$.

Those readers who are not interested in details of logarithmic corrections calculations, but would like to see the results, can confine ourselves to the sects. 3.4, 3.6–3.8 only.

3.2. The verification of the operator expansion. One- and two-loop corrections.

Consider the pion form factor $F_\pi(Q^2)$, $Q^2 = -q^2$. The Born expression (1.8) can be written in the form ($\xi = x_1 - x_2$, $e_u = 2/3$, $e_d = -1/3$, $\varphi^A_\pi(\xi) = \varphi^A_\pi(-\xi)$ due to the negative pion G-parity):

$$\Phi(Q^2) \equiv \frac{9\,Q^2}{32\pi|f_\pi|^2} F_\pi(Q^2) \to \alpha_s I_i^{Born} \overset{*}{I}{}_f^{Born}, \quad I_i^{Born} = \int_{-1}^{1} \frac{d\xi}{1-\xi^2}\varphi^A_\pi(\xi). \qquad (3.3)$$

The method used in /1.25/ for a verification of the operator expansion (3.2) is the following. The initial pion is substituted by two free massless quarks with the longitudinal momentum fractions: $x_1 = x_0$, $x_2 = 1 - x_0$ ($y_1 = y_0$, $y_2 = 1 - y_0$ for the final pion), that corresponds to the wave function $\varphi_{model}(x_{1,2}) = \frac{1}{4}\left(\delta(x_1 - x_0) + \delta(x_2 - x_0)\right)$ in (3.3). Then the Born approximation has the form:

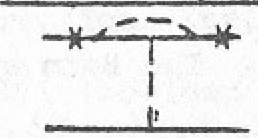
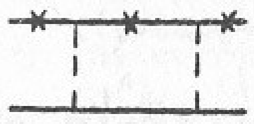
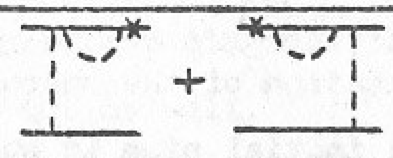
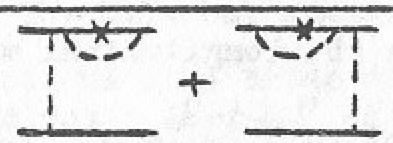
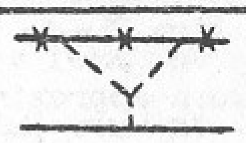
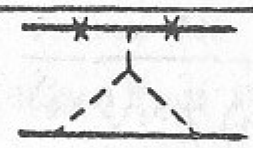
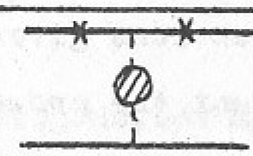
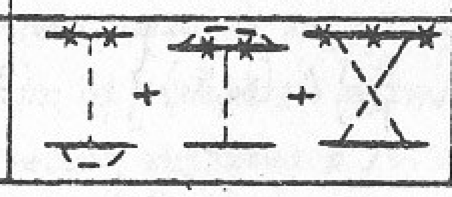
$$I_i^{Born} = \frac{1}{4x_0(1-x_0)}, \qquad \Phi^{Born}(Q^2) = \frac{\alpha_s}{4x_0(1-x_0)\,4y_0(1-y_0)}. \qquad (3.4)$$

Let us calculate now the logarithmic corrections given by one- and two-loop Feynman diagrams and represent the answer in the form ($\alpha_s \equiv \alpha_s(\mu^2)$):

$$\Phi(Q^2) = \Phi_{Born}\left\{1 - \left(\frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}\right)\Delta_1(x_0,y_0) + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}\right)^2\Delta_2(x_0,y_0)\right\}. \qquad (3.5)$$

The contributions of one-loop diagrams into $\Delta_1(x_0, y_0)$ are presented in the Table 3.1. The Feynman gauge is used everywhere in this paper, except for the sect. 3.3. The regularization of infrared singularities in separate diagrams is performed here and below by introducing the gluon mass " μ ", the quarks

Table 3.1

N	Diagrams	Δ_I^i
1		$\left(C_F-\frac{1}{2}C_A\right)\left(\frac{1-x}{2}\ln\frac{1}{1-x}+\frac{x}{2}\ln\frac{1}{x}-\frac{1}{4}\right)+(x\to y)$
2		$C_F\left[-(1-x)\ln\frac{1}{1-x}-x\ln\frac{1}{x}\right]+(x\to y)$
3		$-\frac{1}{2}C_F$
4		$+\frac{1}{2}C_F$
5		$\frac{1}{2}C_A\left(\ln\frac{1}{1-x}+\ln\frac{1}{x}-\frac{1}{4}\right)+(x\to y)$
6		$-\frac{1}{4}C_A$
7		$-\frac{1}{2}\left(-\frac{5}{3}C_A+\frac{2}{3}m\right)$
8		$\left(C_F-\frac{1}{2}C_A\right)\left(\frac{1+x}{2}\ln\frac{1}{1-x}+\frac{2-x}{2}\ln\frac{1}{x}-\frac{5}{4}\right)+(x\to y)$

$$\Delta_I=\sum_{i=1}^{8}\Delta_I^i=C_F\left[x\ln\frac{1}{1-x}+(1-x)\ln\frac{1}{x}-\frac{3}{2}+(x\to y)\right]+\frac{1}{2}\left(\frac{11}{3}n-\frac{2}{3}m\right);$$

$$C_F=\frac{n^2-1}{2n};\quad C_A=n.$$

are massless and on the mass shell. Note that each of seven diagrams N°8 in Table 3.1 contains the Sudakov double-logarithmic contribution $\sim\alpha_s\ln^2 Q^2$,but all such contributions cansel in the sum of the diagrams N°8. As a result:

$$\Delta_1=\left\{C_F\left(x_0\ln\frac{1}{1-x_0}+(1-x_0)\ln\frac{1}{x_0}-\frac{3}{2}\right)+(x_0\to y_0)\right\}+\frac{1}{2}\left(\frac{11}{3}C_A-\frac{2}{3}n_{fe}\right). \quad (3.6)$$

The number of two-loop diagrams is very large ($\sim 10^3$). For this reason (and taking into account that the one-loop results agree,see below),only leading at $(1-x_0)\ll 1, (1-y_0)\ll 1$ contributions have been calculated (there are still $\simeq 10^2$ diagrams). The result is*:

$$\Delta_2=C_F^2\left(\ln\frac{1}{1-x_0}+\ln\frac{1}{1-y_0}\right)^2\left[1+O\left(\ln^{-1}(1-x_0)\right)\right]. \quad (3.7)$$

On the whole,one can represent the results of the one- and two-loop calculations in the form:

$$\Phi^{p.th.}\left(\frac{Q^2}{\mu^2}\right)=\Phi_{Born}\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\Upsilon_i^{p.th.}\left(\frac{Q^2}{\mu^2}\right)\Upsilon_f^{p.th.}\left(\frac{Q^2}{\mu^2}\right), \quad (3.8)$$

$$\Upsilon_i^{p.th.}\left(\frac{Q^2}{\mu^2}\right)=1-\left(\frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}\right)f_1^{p.th.}(x_0)+\frac{1}{2}\left(\frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}\right)^2 f_2^{p.th.}(x_0),$$

$$f_1^{p.th.}(x_0)=C_F\left(x_0\ln\frac{1}{1-x_0}+(1-x_0)\ln\frac{1}{x_0}-\frac{3}{2}\right), \quad (3.9)$$

$$f_2^{p.th.}(x_0)=C_F^2\ln^2\frac{1}{1-x_0}\left[1+O\left(\ln^{-1}(1-x_0)\right)\right],$$

$$\alpha_s(Q^2)/\alpha_s(\mu^2)=1-\frac{\alpha_s}{4\pi}\left(\frac{11}{3}C_A-\frac{2}{3}n_{fe}\right)\ln\frac{Q^2}{\mu^2}.$$

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* Separate diagrams have the terms $\sim\alpha_s^2\ln^4 Q^2$ and $\sim\alpha_s^2\ln^3 Q^2$, but all such contributions cansel in the sum of diagrams, see /1.25/ for details.

Let us compare now (3.8),(3.9) with the operator expansion (3.2). In the leading logarithm approximation the operator expansion (3.2) is equivalent to the replacement in (3.3):

$$\alpha_s \to \alpha_s(Q^2), \quad I^{Born} \to I(Q^2) = \int_{-1}^{1} \frac{d\zeta}{1-\zeta^2}\, \varphi_\pi^A(\zeta, Q^2),$$

where the wave function $\varphi_\pi^A(\zeta, Q^2)$ is the pion matrix element of the gauge invariant bilocal operator normalized at the point "Q^2". Decompose $\varphi_\pi^A(\zeta, Q^2/\mu^2)$ into a series of matrix elements of multiplicatively renormalizable local operators (see the appendix B):

$$\varphi_\pi^A\left(\zeta, \frac{Q^2}{\mu^2}\right) = \varphi_{as}(\zeta) \sum_{n=0}^{\infty} f_\pi^{(n)}(Q^2)\, C_n^{3/2}(\zeta), \quad \varphi_{as}(\zeta) = \frac{3}{4}(1-\zeta^2), \tag{3.10}$$

where $C_n^{3/2}(\zeta)$ are the Gegenbauer polinomials and $f_\pi^{(n)}(Q^2)$ are the corresponding matrix elements:

$$\langle 0 |\, O_n = \bar{d}(0) \frac{\hat{z}\gamma_5}{i f_\pi (zp)} C_n^{3/2}\left(i z \overleftrightarrow{D}/zp\right) u(0) \,|\, \pi(p)\rangle_Q = f_\pi^{(n)}(Q^2), \quad z^2 = 0,$$

$$f_\pi^{(n)}(Q^2) = f_\pi^{(n)}(\mu^2) \exp\left\{ -\int_{\alpha_s(\mu)}^{\alpha_s(Q)} \frac{d\alpha}{\beta(\alpha)} \gamma_n(\alpha) \right\} \equiv f_\pi^{(n)}(\mu^2)\, e^{-\varepsilon_n \tau},$$

$$\varepsilon_n = C_F\left[1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1} \frac{1}{j}\right], \tag{3.11}$$

$$\tau = \frac{1}{b_0} \ln\left(\frac{\alpha_s(\mu)}{\alpha_s(Q)}\right), \quad b_0 = \frac{11}{3} C_A - \frac{2}{3} n_{fe}.$$

Substituting (3.10),(3.11) into (3.3),one has:

$$I_i^{ren.gr.}(Q^2) = \frac{3}{4} \sum_{n=0} \left(1 + (-1)^n\right) f_\pi^{(n)}(\mu^2)\, e^{-\varepsilon_n \tau} \equiv I_i^{Born}\, \Upsilon_i^{ren.gr}(Q^2). \tag{3.12}$$

For comparison with the result of the pert.th. calculations,

one should expand (3.12) into the $\alpha_s \equiv \alpha_s(\mu^2)$ series:

$$\Upsilon_i^{ren.gr.}(Q^2) = 1 - \left(\frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2}\right) f_1^{ren.gr.} + \frac{1}{2}\left(\frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2}\right)^2 f_2^{ren.gr}. \tag{3.13}$$

For the used wave function $\varphi_{model}(\zeta) = \frac{1}{2}\left[\delta(\zeta-\zeta_0) + \delta(\zeta+\zeta_0)\right]$:

$$f_\pi^{(n)} = \int_{-1}^{1} d\zeta\, \varphi_{model}(\zeta)\, C_n^{3/2}(\zeta) = \frac{1}{2}\left[1 + (-1)^n\right] C_n^{3/2}(\zeta_0). \tag{3.14}$$

To calculate one-loop corrections,i.e. $f_1^{ren.gr.}$ in (3.13), it is sufficient to confine ourselves by the first term in the expansion of $\exp\{-\varepsilon_n \tau\}$:

$$e^{-\varepsilon_n \tau} = \left(\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right)^{\varepsilon_n/b_0} \simeq \left(1 - \frac{\alpha_s}{4\pi} b_0 \ln\frac{Q^2}{\mu^2}\right)^{\varepsilon_n/b_0} \simeq 1 - \frac{\alpha_s}{4\pi} \varepsilon_n \ln\frac{Q^2}{\mu^2}. \tag{3.15}$$

Substituting (3.14),(3.15) into (3.12),(3.13),one has:

$$f_1^{ren.gr.} = \left\{ x_0 \ln\frac{1}{1-x_0} + (1-x_0) \ln\frac{1}{x_0} - \frac{3}{2} \right\} C_F, \tag{3.16}$$

that coincides with (3.9). It is worth noting that all characteristic features of the non-abelian gauge theory show up at the one-loop level already,and so the above described verification of the operator expansion (3.2) is highly nontrivial.

Expanding in (3.15) $e^{-\varepsilon_n \tau}$ up to the terms $\sim \alpha_s^2$, substituting them into (3.12) and keeping only leading at $(1-x_0) \ll 1$ terms,one obtains (it is sufficient to put $\varepsilon_n \simeq 4 C_F \ln(n)$ in this limit):

$$f_2^{ren.gr.} = C_F^2 \ln^2\frac{1}{1-x_0}\left[1 + O\left(\ln^{-1}(1-x_0)\right)\right], \tag{3.17}$$

that agrees with (3.9).

On the whole,the verification of the operator expansion (3.2) at the two-loop level confirms its correctness.

3.3. THE "PHYSICAL" GAUGE.

Although the direct calculation in a covariant gauge confirms the correctness of the operator expansion (3.2), the use of a covariant gauge has the great disadvantage. Indeed (see the Table 3.1), leading logarithmic contributions give in a covariant gauge not only the diagrams like N°2, but N°8 as well, in which gluons connect the initial and final states. Hence, the fact that the sum of all logarithmic corrections factorizes like (3.2) is not selfevident beforehand, and the result looks like a trick.

The answer in an arbitrary gauge includes the matrix element of the bilocal operator (i.e. the wave function) of the form: $\langle 0|\bar{\Psi}(z)\exp\{ig\int_{-z}^{z} d\sigma_\nu B_\nu(\sigma)\}\psi(-z)|P\rangle$. It is just the gluonic string in this operator which corresponds to non-ladder diagrams in the QCD pert. th. The question is reduced to the following: can we choose such a gauge that the gluonic string operators can be put equal to unity for the initial and final hadrons (at least, in the leading logarithm approximation, LLA)? If so, then the ladder-like diagrams only will be the leading ones, and the whole problem simplifies considerably.

An expansion of the gluonic string operator into a power series leads to an appearance of diagrams like those shown at fig.3.8. If a suitable axial-like gauge is used (for instance, the planar gauge / 3.5 /: $n_\mu B_\mu=0$, $n_\mu=a p_\mu+b p'_\mu$), the fig.3.8 diagram gives no logarithmic contributions into initial and final states. This shows that the gluonic string operator gives no leading logarithm contributions in this gauge and so, it can be replaced by the unity operator in the LLA.

At the same time, the fig.3.9 diagrams do give logarithmic contributions, and all this shows that only the ladder-like dia-

grams (with self-energy corrections) are of importance in this gauge in the LLA, fig.3.4.

This can also be seen directly from the QCD pert.th. The properties of the QCD pert.th. diagrams for collinear (or nearly collinear) processes in axial-like gauges are well known since 1978, when the paper / 3.5 / appeared. It is shown in / 3.5 / that the ladder-like diagrams only are of importance for such processes in the LLA.* The ladder character of leading diagrams allows one to sum them easily by solving the Bethe-Salpeter equation.

The use of the axial gauge for a calculation of the hadron form factor asymptotic behaviour within the QCD pert.th. was first described by G.P.Lepage and S.J.Brodsky /1.29/. Form factors also belong to collinear processes, and so all the machinery described in / 3.5 / is directly applicable here as well. The case of the $\gamma\gamma\pi^0$ -form factor considered in /1.29/ in detail, fig.3.10, is much like to the deep-inelastic scattering / 3.4 /, fig.3.1, while the case of the meson form factor, fig. 3.4, is much like to the fig.3.11 process, the absorptive part of which is the Drell-Yan process.

The ladder-like character of the leading diagrams in the LLA allows one immediatly to write the form factor in the form (see fig.3.4):

$$F_\pi(Q^2)=\int_0^1 d_2x\int_0^1 d_2y\,\overset{*}{\varphi}(y_i,Q)\,T_H(x_i,y_i,Q,\alpha_s(Q))\,\varphi(x_i,Q), \qquad (3.18)$$

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* It seems, that there is no such a gauge which leads to the dominance of the ladder-like diagrams in large-angle scattering amplitudes in the LLA.

where T_H is the hard kernel of the process and the pion wave function $\varphi(x,Q)$ fulfills the Bethe-Salpeter equation. The wave function $\Psi(x_i,k_\perp)$ is used also in /1.29/, and it is connected with $\varphi(x,Q)$ as follows:

$$\varphi_{\alpha\beta}(zp,Q^2)=\langle 0|T\,u_\alpha(-z)\,\bar d_\beta(z)|\pi(p)\rangle_{Q,\,z_+=0}\sim \tag{3.19}$$

$$\int_0^1 d_2x\, e^{i(x_1-x_2)z_-p_+}\left\{d_F^{-1}(Q^2)\int_0^{\sim Q}\frac{d^2k_\perp}{16\pi^3}\Psi_{\alpha\beta}(x_{1,2},k_\perp)\right\},\quad p_+\to\infty,$$

$$k_1=x_1p_++k_\perp+O(k_\perp^2/p_+),\quad k_2=x_2p_+-k_\perp+O(k_\perp^2/p_+),$$

where k_1 and k_2 are the quark momenta, $d_F(Q^2)=\left(\ln\frac{Q^2}{\mu^2}\right)^{\gamma_F/b_0}$ is the quark propagator and γ_F is its anomalous dimensionality. The function $\Psi_{\alpha\beta}(x_i,k_\perp)$ has the meaning /1.29/ of the probability amplitude for finding two quarks in the pion with the longitudinal momentum fractions x_1 and x_2, with the transverse momenta $\pm\vec{k}_\perp$ and collinear up to a scale $\sim Q$ (at $p_+\to\infty$).

The one gluon excange contribution serves as a kernel of the Bethe-Salpeter equation in the LLA, and this allows one to write the evolution equation for the wave function, like that of the Lipatov-Altarelli-Parisi evolution equation /3.4,3.8/. This equation has the form /1.29/:

$$x_1x_2Q^2\frac{\partial}{\partial Q^2}\tilde\Phi(x,Q)=C_F\frac{\alpha_s(Q^2)}{4\pi}\left\{\int_0^1 d_2y\,V(x,y)\tilde\Phi(y,Q)-x_1x_2\tilde\Phi(x,Q)\right\}, \tag{3.20}$$

where $\tilde\Phi(x,Q)=\varphi(x,Q)/x_1x_2$, and the kernel $V(x,y)$ is:

$$V(x,y)=2\left\{x_1y_2\,\theta(y_1-x_1)\left(1+\frac{\Delta}{y_1-x_1}\right)+(1\leftrightarrow 2)\right\}, \tag{3.21}$$

$$\Delta\tilde\Phi(y,Q)\equiv\tilde\Phi(y,Q)-\tilde\Phi(x,Q),$$
$$d_2y=dy_1dy_2\,\delta(1-y_1-y_2).$$

The Sudakov double logarithmic contributions manifest itselves in this gauge as the poles $1/y_i-x_i$ in the ladder diagram contributions and as analogous poles in the quark propagator d_F (in γ_F). They cancel in the total expression (3.21) (i.e. the term Δ/y_i-x_i is regular), and this confirms that there are no double logs in the sum of diagrams.

The integral equation (3.20),(3.21) can be solved by expanding $\tilde\Phi(x,Q)$ over the kernel eigenfunctions. These eigenfunctions are the Gegenbauer polinomials $C_n^{3/2}(x_1-x_2)$, and the corresponding eigenvalues are the anomalous dimensions ε_n. As a result, one has the solution of the form (3.10), which demonstrates an equivalence of this approach and the method of operator expansions.

The many particle components, $\Psi_n(x_i,k_\perp^i,s)$, of the total hadron wave function are introduced by analogy in /1.29/. The quantity $|\Psi_n(x_i,k_\perp^i,s)|^2$ is interpreted as the probability density to find in the hadron (at $P_z\to\infty$) just n constituents with the longitudinal momentum fractions x_i, the transverse momenta $k_\perp^i$ and with the definite spin structure "S". The normalization condition is then, by definition:

$$\sum_n\int_0^1 d_nx\int_0^\infty\prod^n\frac{d^2k_\perp}{16\pi^3}\sum_s\left|\Psi_n(x_i,k_\perp^i,s)\right|^2=1, \tag{3.22}$$

and the structure function (i.e. the one-particle probability distribution) is:

$$F(x_1)=\sum_n\int_0^1 d_{n-1}x\int_0^\infty\prod^n\frac{d^2k_\perp}{16\pi^3}\sum_s\left|\Psi_n(x_i,k_\perp^i,s)\right|^2, \tag{3.23}$$

$$\int_0^1 dx_1\,F(x_1)=1.$$

We want to point out here the following. We calculate in next chapters various two- and three-particle wave functions $\varphi_i(x_{1,2}), \varphi_i(x_{1,2,3})$. The wave functions which we use are defined through the matrix elements of gauge-invariant bi- and three-local operators and correspond to the "collinear basis".* The two-particle wave function of the leading twist, $\varphi(x_{1,2})$, is simply connected with $\Upsilon(x_{1,2}, k_\perp)$, see (3.19). But our two-particle wave functions of the nonleading twist, three-particle wave functions, etc., are connected with $\Upsilon_n(x_i, k_\perp^i, s)$ in a very complicated way.

3.4. THE "IMPROVED PARTON PICTURE"

The "parton picture" means here the approximation used above in ch.2 (i.e. logarithmic corrections are neglected). Namely, the target (pion) is characterized by the non-perturbative "soft" wave function $\varphi(\xi, \mu^2) \equiv \varphi(\xi)$ and the form factor has the form:

$$\Phi \to \int_{-1}^{1} d\xi_1 \int_{-1}^{1} d\xi_2 \, \overset{*}{\varphi}(\xi_2) \, \Phi_{Born}(\xi_1, \xi_2, \alpha_s) \varphi(\xi_1), \tag{3.24}$$

where Φ_{Born} is the form factor of the two "soft" quark system, calculated in the Born approximation (see (3.3),(3.4)).

As it has been shown above in sects. 3.2 and 3.3, when the leading logarithimic corrections are taken into account, the answer can be represented in the form /1.26,1.28,1.29/:

$$\Phi(Q^2) \to \int_{-1}^{1} d\xi_1 \int_{-1}^{1} d\xi_2 \, \overset{*}{\varphi}(\xi_2, Q^2) \, \Phi_{Born}(\xi_1, \xi_2, \alpha_s(Q^2)) \varphi(\xi_1, Q^2). \tag{3.25}$$

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* The use of the collinear basis for the matrix element $\langle 0 | O_k | P_{+} \to \infty \rangle$ means that all the derivatives $D_\perp$ and D_- present in the operator O_k, are expressed with a help of equations of the motion through D_+ and operators of additional particles.

The "improved wave function" $\varphi(\xi, Q^2)$ in (3.25) is (see (3.10), (3.11),(3.15)):

$$\varphi(\xi, \tau) = \varphi_{as}(\xi) \sum_{n=0} f_\pi^{(n)} C_n^{3/2}(\xi) e^{-\varepsilon_n \tau}, \quad f_\pi^{(n)} = \int_{-1}^{1} d\xi \, \varphi(\xi) C_n^{3/2}(\xi), \tag{3.26}$$

$$\varphi_{as}(\xi) = \frac{3}{4}(1-\xi^2), \quad \tau = \frac{1}{b_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(Q)} \geqslant 0, \quad \varphi(\xi) \equiv \varphi(\xi, \tau=0).$$

The use of the "improved wave function" $\varphi(\xi, \tau)$ can be inconvenient in some cases, however, because the contributions of the large ($\sim 1/\mu$) and small ($\sim 1/Q$) distance interactions are mixed therein. It seems natural to separate these contributions and to express the answer through the "soft" wave function directly /1.25,2.1/. With this purpose, let us introduce the Green function:

$$G(\xi, \xi', \tau) = \sum_{n=0} C_n^{3/2}(\xi) C_n^{3/2}(\xi') e^{-\varepsilon_n \tau}, \quad G(\xi, \xi', \tau \to \infty) \to 1, \tag{3.27}$$

$$G(\xi, \xi', \tau=0) = \frac{\delta(\xi - \xi')}{\sqrt{\varphi_{as}(\xi)\varphi_{as}(\xi')}}, \quad C_{n=0}^{3/2}(\xi) = 1, \quad \varepsilon_{n=0} = 0, \quad \varepsilon_{n \geq 1} > 0.$$

One can represent now (3.26) in the form:

$$\varphi(\xi, \tau) = \varphi_{as}(\xi) \int_{-1}^{1} d\xi' \, G(\xi, \xi', \tau) \varphi(\xi'), \tag{3.28}$$

$$\varphi(\xi, \tau \to \infty) \to \varphi_{as}(\xi), \quad \int_{-1} d\xi' \, \varphi(\xi') = 1.$$

Substituting (3.28) into (3.25), let us write the pion form factor as follows (compare with (3.24)):

$$\Phi(Q^2) \to \int_{-1}^{1} d\xi_1 \int_{-1}^{1} d\xi_2 \, \overset{*}{\varphi}(\xi_2) \, \Phi(\xi_1, \xi_2, \alpha_s(Q), \tau) \varphi(\xi_1), \tag{3.29}$$

$$\Phi(\xi_1, \xi_2, \alpha_s(Q), \tau) = \int_{-1}^{1} d\xi_1' \int_{-1}^{1} d\xi_2' \, \tilde{G}(\xi_1, \xi_1', \tau) \, \Phi_{Born}(\xi_1', \xi_2', \alpha_s(Q)) \tilde{G}(\xi_2, \xi_2', \tau),$$

$$\tilde{G}(\zeta,\zeta',\tau) = G(\zeta,\zeta',\tau)\,\varphi_{as}(\zeta'). \tag{3.30}$$

The function $\Phi(\zeta_1,\zeta_2,\alpha_s(Q),\tau)$ in (3.29),(3.30) is the form factor of the two soft quark system, calculated in the LLA.

The formula (3.29) has a simple meaning, fig.3.12. There arise two quark pairs in the small ($\sim 1/Q$) vicinity of the point "0" during a hard stage of the process, and this stage is described by the amplitude $\Phi_{Born}(\zeta_1,\zeta_2,\alpha_Q)$. The quarks have virtualities $\sim Q^2$ and the longitudinal momenta $(x_1'P_1, x_2'P_1)$ and $(y_1'P_2, y_2'P_2)$.

These two quark pairs evolve then independently of each other. For instance, the left quarks at fig.3.12 exchange gluons between themselves, diminish their virtualities from $\sim Q^2$ down to $\sim \mu^2$ and have after this the momenta (x_1P_1, x_2P_1). The Green function $\tilde{G}(\zeta,\zeta',\tau)$ describes just this evolution of the quark pair.

The formula (3.27) for the Green function is, in fact, the expansion over the partial waves in the $\bar{q}q$-channel. Let the initial (i,e, before an evolution) quark pair to have the "angular distribution" of the form $C_n^{3/2}(\zeta)$, $\zeta = \cos\theta$, which corresponds to the state with the "angular momentum" n*. Because the "angular momentum" n is conserved, each partial wave which describes the state with the "energy" ε_n, evolves then independently with the "time" τ.

Hence, we have after this evolution two quark pairs with the parameters (x_1,x_2,μ^2) and (y_1,y_2,μ^2). The function

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* The quantum number "n" is the value of the conformal angular momentum /3.9,1.28,3.10,3.11/ which is conserved in the LLA.

$\Phi(\zeta_1,\zeta_2,\alpha_Q,\tau)$ is just the form factor of such a quark pair, and it can be calculated completely within the QCD pert.th. Finally, the non-perturbative wave function $\varphi(\zeta)$ describes a transformation of the soft quark pair into the pion.

The advantage of the formulae (3.29),(3.30) is that all the information about the large ($\gtrsim 1/\mu$) distance interactions is concentrated in the non-perturbative wave functions $\varphi(\zeta)$, while the small ($< 1/\mu$) distance interactions are factorized out and can be computed explicitly within the pert.th.

In particular, it is convenient to represent Φ, (3.29), in the form:

$$\Phi(\zeta_1,\zeta_2,\alpha_Q,\tau) = \Psi(\zeta_1,\tau)\,\Phi_{Born}(\zeta_1,\zeta_2,\alpha_Q)\,\Psi(\zeta_2,\tau), \tag{3.31}$$

$$\Psi(\zeta,\tau) = \varphi_{as}(\zeta)\sum_{n=0,2,4\ldots}\left[\frac{8}{3}\,\frac{2n+3}{(n+1)(n+2)}\right]^{1/2} C_n^{3/2}(\zeta)\,e^{-\varepsilon_n\tau}, \quad \Psi(\zeta,\tau=0)=1.$$

The function $\Psi(\zeta,\tau)$ has the meaning of an effective two-quark wave function which arised due to an interaction. The formula (3.31) gives the possibility to trace the behaviour of the two-quark form factor Φ at $|\zeta_{1,2}| \to 1$. When one of two quarks carries a small momentum fraction, $|\zeta| \to 1$, then nearly the whole momentum is carried by the rest quark. The form factor of such two-quark system will then be proportional to the quark form factor and hence, it will be suppressed. Indeed, keeping in this limit only the leading logs in each order of the pert.th. (i.e. the terms $\sim \left(\alpha_s \ln\frac{Q^2}{\mu^2}\ln\frac{1}{1-\zeta^2}\right)^n$), one obtains from (3.31) (compare with (3.9)):

$$\Psi(\zeta,\tau) \sim \exp\left\{-\frac{\alpha_s}{2\pi}C_F \ln\frac{Q^2}{\mu^2}\ln\frac{1}{1-\zeta^2}\right\}, \quad \frac{\mu}{Q} \ll (1-\zeta^2) \ll 1, \tag{3.32}$$

and this resembles the quark form factor in the double logarithm approximation. The behaviour (3.32) is correct, in fact, at not too large Q^2 only. More precisely (see (3.31)):

$$\Upsilon(\zeta,\tau) \sim \begin{cases} (1-\zeta^2)^{2C_F\tau}, & 2C_F\tau < 1 \\ (1-\zeta^2), & 2C_F\tau > 1 \end{cases}, \quad \tau = \frac{1}{b_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(Q)}. \qquad (3.33)$$

Therefore, logarithmic corrections "improve" the behaviour of the form factor $\Phi(\zeta_{1,2},\tau)$ at $|\zeta_{1,2}|\to 1$, as compared with $\Phi_{Born}(\zeta_{1,2})$. That is, the effective wave function $\Upsilon(\zeta,\tau)$ suppresses contributions of those regions where one of two quarks becomes a "wee". As a result, writing the pion form factor in the form (see (3.29), (3.31), (3.33)):

$$\Phi(Q^2) = \alpha_s(Q)\left|I(\tau)\right|^2, \quad I(\tau) = \int_{-1}^{1}\frac{d\zeta}{1-\zeta^2}\Upsilon(\zeta,\tau)\varphi(\zeta), \qquad (3.34)$$

we see that the region $|\zeta\to 1|$ does not influences the asymptotic behaviour for any nonsingular wave function $\varphi(\zeta)$.

3.5. THE ROLE OF NON-LEADING LOGARITHMS /3.12/.

There are double logarithm contributions ($\sim\alpha_s \ln^2 Q^2$ per each loop) in the QCD pert.th. diagrams for the hadron form factor. All "superfluous" logs cancel in the sum of diagrams in each order of the pert.th. due to a neutrality of hadrons in colour. There remains, however, the "trace" of these double logs in the form of additional $\ln x$ or $\ln(1-x)$, i.e. there are terms from $\sim\left(\alpha_s \ln\frac{Q^2}{\mu^2}\ln\frac{1}{x}\right)^n$ (leading logs) down to $\sim\left(\alpha_s\ln^2\frac{1}{x}\right)^n$ (non-leading logs). All such logs are of potential importance in the region $x\to 0$. For this reason, let us consider the properties of the pert.th. contributions from this region in more details.

Roughly, the contribution of the region where one of two quarks is a "wee" (for instance, the lower quark line at fig.3.13 at $x_2\sim y_2 \lesssim M/Q$), is evident beforehand. Because the wee quark does not influences a dependence on Q, the meson form factor reduces to the form factor of the rest energetic quark (the upper line at fig.3.13). This is the Sudakov form factor: $S(Q^2,\mu^2)\simeq \exp\left\{-\frac{C_F}{b_0}\ln\frac{Q^2}{\mu^2}\ln\ln\frac{Q^2}{\mu^2}\right\}$. Hence, the contribution of this region into the pion form factor is:

$$\Delta F_\pi(Q^2) \simeq \frac{32\pi\alpha_s}{9Q^2} f_\pi^2\, S(Q^2,\mu^2)\int_{1-M/Q}^{1}\frac{dx\,\varphi_\pi^A(x,\mu)}{1-x}\int_{1-M/Q}^{1}\frac{dy\,\varphi_\pi^A(y,\mu)}{1-y}, \qquad (3.35)$$

(the wave function with the small virtuality enters here, because the gluon virtuality at fig.3.13 is $\sigma^2\sim\mu^2$ in this case). If $\varphi_\pi^A(x,\mu)\sim(1-x)$ at $x\to 1$ (see chs.1,4), then the contribution (3.35) is:

$$\Delta F_\pi(Q^2) \sim \frac{M^4}{Q^4} S(Q^2,\mu^2), \quad S(Q^2,\mu^2)\simeq\left(\frac{\mu^2}{Q^2}\right)^{C_F\tau}, \quad \tau = \frac{1}{b_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(Q)}, \qquad (3.36)$$

i.e. it is highly suppressed at large Q^2.

It is clear moreover, that the behaviour in the region $\frac{M}{Q}\lesssim x_2 \ll 1$ should go over smoothly into the above considered behaviour in the "wee-region" $0 < x_2 \lesssim M/Q$. Let us consider how this happens. We confine ourselves here by the double logarithm approximation, i.e. we keep only the terms $\sim\alpha_s\ln^2 z$ per each loop, whatever is the argument Z of the logarithm: Q^2/μ^2 or $x_2\to 0$. The results of the two-loop calculations are presented in /1.25/ (see also /3.13/ where a complete calculation of the one-loop correction is presented) and are:

$$\Phi(Q^2) = \Phi_{Born}\left\{1 - \frac{\alpha_s}{2\pi}C_F\Omega + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}C_F\Omega\right)^2\right\} \to \Phi_{Born}\exp\left\{-\frac{\alpha_s}{2\pi}C_F\Omega\right\}, \qquad (3.37)$$

$$\Omega = \left(\ln\frac{Q^2}{\mu^2} - \frac{1}{2}\ln\frac{1}{x_2 y_2}\right)\ln\frac{1}{x_2 y_2}\,, \qquad \frac{M}{Q} \lesssim x_2, y_2 \ll 1.$$

It is seen from (3.37) that the result coincides with the first three terms in the expansion of $\exp\left\{-\frac{\alpha_s}{2\pi}C_F\Omega\right\}$. Hence, we assume that the all orders answer is $\exp\left\{-\frac{\alpha_s}{2\pi}C_F\Omega\right\}$, and deal with the first order contribution only.

How can one interpret the correction $\left(1-\frac{\alpha_s}{2\pi}C_F\Omega\right)$ in (3.37)? It is clear from (3.31), (3.32) that the term

$$\left(1-\frac{\alpha_s}{2\pi}C_F\ln\frac{Q^2}{\mu^2}\ln\frac{1}{x_2 y_2}\right) \simeq \left(1-\frac{\alpha_s}{2\pi}C_F\ln\frac{Q^2}{\mu^2}\ln\frac{1}{x_2}\right)\left(1-\frac{\alpha_s}{2\pi}C_F\ln\frac{Q^2}{\mu^2}\ln\frac{1}{y_2}\right) \tag{3.38}$$

in (3.37) is the leading logarithm contribution into $I(\tau) = \int \frac{d\zeta\, \varphi(\zeta,\tau)}{1-\zeta^2}$, which is caused by the evolution of the wave function. What is the meaning of the non-leading logarithm in (3.37)? This becomes clear when the correction is written in the form:

$$\left(1-\frac{\alpha_s}{2\pi}C_F\Omega\right) \simeq \left(1-\frac{\alpha_s}{2\pi}C_F\ln\frac{\sigma^2}{\mu^2}\ln\frac{1}{x_2}\right)\left(1-\frac{\alpha_s}{4\pi}C_F\ln^2\frac{Q^2}{\sigma^2}\right)\cdot$$
$$\left(1-\frac{\alpha_s}{2\pi}C_F\ln\frac{\sigma^2}{\mu^2}\ln\frac{1}{y_2}\right), \quad \sigma^2 \simeq x_2 y_2 Q^2 + \mu^2. \tag{3.39}$$

The first and third factors in (3.39) describe (in the used approximation) the effects due to an evolution of the wave function from the initial scale σ^2, determined by the gluon virtuality at fig.3.13 diagram, down to μ^2. The second factor in (3.39) is the Sudakov form factor which arises because there is the leap of a virtuality at the photon vertex in the hard kernel: from the initial virtuality Q^2 down to the gluon virtuality σ^2, from which the wave function evolution starts. Therefore, on account of non-leading logs the form factor can be written in the form:

$$\Phi(Q^2) = \int_{-1}^{1} d\zeta_1 \int_{-1}^{1} d\zeta_2\, \varphi(\zeta_2,\sigma^2)\left\{\Phi_{Born}\, S(Q^2,\sigma^2)\right\}\varphi(\zeta_1,\sigma^2),$$

$$\Phi_{Born} = \frac{\alpha_s(\sigma^2)}{(1-\zeta_1)(1-\zeta_2)}\,, \quad \sigma^2 = \left(\frac{1-\zeta_1}{2}\right)\left(\frac{1-\zeta_2}{2}\right)Q^2 + \mu^2, \tag{3.40}$$
$$S(Q^2,\sigma^2) = \left(\frac{\sigma^2}{Q^2}\right)^{C_F\tau}, \qquad \tau = \frac{1}{b_0}\ln\frac{\alpha_s(\sigma^2)}{\alpha_s(Q^2)},$$

where $S(Q^2,\sigma^2)$ is the Sudakov form factor which accounts for the virtuality leap from "Q^2" to σ^2, and $\varphi(\zeta,\sigma^2)$ is the wave function with the constituent virtualities from $\sim\mu^2$ up to σ^2.

The expression (3.40) coincides with (3.35) at $x_2 = \left(\frac{1-\zeta_1}{2}\right) \sim y_2 = \left(\frac{1-\zeta_2}{2}\right) \sim M/Q$. In the LLA: $S(Q^2,\sigma^2) \simeq 1$, $\varphi(\zeta,\sigma^2) \simeq \varphi(\zeta,Q^2)$, and (3.40) coincides with (3.25), (3.29), (3.34)*.

The approximation used in (3.40) does not spoil a ladder-like structure of accounted diagrams in the planar gauge. But the evolution equation for the wave function becomes more complicated, because the limits of the integrals over $k_\perp$ depend now on the corresponding longitudinal momentum fractions.

In conclusion of this section, let us note the following.

a) The formula (3.40) does not correspond to the operator expansion and the renormalization group, because it sums non-leading logs in an another way.

b) The above given interpretation of (3.40) implies that the form (3.40) is sufficiently universal and is applicable not to the pion form factor only, but to other cases as well. The effects considered are especially important when the Born amplitude $\Phi_{Born}(\zeta_i)$ is highly singular at $|\zeta_i| \to 1$, and both the usual LLA and the renormalization group are inapplicable.

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* Of course, not all non-leading logs are accounted for in (3.40), only those which are most important at $|\zeta_{1,2}| \to 1$.

c) Although the Sudakov form factor will suppress at sufficiently high Q^2 any power singularity of $\Phi_{Born}(\xi_i)$ at $|\xi_i|\to 1$ and so these regions will not influence the asymptotic behaviour, this suppression can be insufficient at experimentally accessible values of Q^2. The power corrections $\sim \left(k_\perp^2+M^2/Q^2\right)$ which are always present in the denominators of quark and gluon propagators, can really be more important at such Q^2, and just these terms serve then as cut off parameters.

3.6. <u>SOME RESULTS IN THE FORMAL LIMIT $Q^2\to\infty$</u>.

We present in this section some results which can be obtained in the formal limit $Q^2\to\infty$, when not only power corrections $\sim M^2/Q^2$, but the logarithmic corrections $\sim e^{-\varepsilon_n\tau} = \left(\frac{\alpha_Q}{\alpha_\mu}\right)^{\varepsilon_n/b_0}$ can be neglected.

One can retain in this limit only the leading term in the expansion (3.26) over the partial waves, because the "energies" ε_n increase with increasing the "angular momentum" n, see (3.11). For the pion:

$$\varphi_\pi^A(\xi,\tau)\to\varphi_{as}(\xi)=\frac{3}{4}(1-\xi^2),\quad e^{-\varepsilon_0\tau}=1,\ \varepsilon_0=0 \qquad (3.41)$$

(ε_0 is the anomalous dimensionality of the axial-vector current $\bar d(0)\gamma_\mu\gamma_5 u(0)$). Substituting (3.41) into (3.3),(3.25), one obtains:

$$I_\pi(\tau)=\int_{-1}^{1}\frac{d\xi}{1-\xi}\varphi_\pi^A(\xi,\tau)\to\frac{3}{2},\quad \alpha_s(Q^2)\to\frac{4\pi}{b_0\ln Q^2},\quad b_0=11-\frac{2}{3}n_{f},$$

$$F_{\pi^+}(Q^2)\to\frac{32\pi}{9Q^2}\alpha_s(Q^2)|f_\pi|^2\left(\frac{3}{2}\right)^2=\frac{8\pi\alpha_s(Q^2)|f_\pi|^2}{Q^2}=\frac{32\pi^2|f_\pi|^2}{b_0Q^2\ln Q^2},$$

$$\langle 0|\bar d(0)\gamma_\mu\gamma_5 u(0)|\pi(p)\rangle=iP_\mu f_\pi,\quad |f_\pi|\simeq 133\text{ MeV},\quad 8\pi|f_\pi|^2\simeq 0.45\text{ GeV}^2. \qquad (3.42)$$

The pion form factor asymptopia (3.42) was obtained for the first time in 1977 independently by D.R.Jackson /1.26/ and in /1.25/ and reproduced later in many papers /1.28,1.29,1.30/.

Evidently, the same asymptotic form have the form factors of $\rho^+_{\lambda=0}$ and K^+ mesons (s is the meson spin):

$$F_{\rho^+_{\lambda=0},K^+}(Q^2)\to\frac{32\pi^2|f_{\rho,K}|^2}{b_0Q^2\ln Q^2}(-1)^s,\quad f_\rho\simeq 200\text{ MeV},\quad f_K\simeq 165\text{ MeV},$$

$$|F_{\rho^+}(Q^2)|:F_{K^+}(Q^2):F_{\pi^+}(Q^2)\to 2.3:1.5:1.0\,, \qquad (3.43)$$

$$\langle 0|\bar s(0)\gamma_\mu\gamma_5 u(0)|K(p)\rangle=iP_\mu f_K,\quad \langle 0|\bar d(0)\gamma_\mu u(0)|\rho_{\lambda=0}(p)\rangle=e_\mu^{\lambda=0}m_\rho f_\rho.$$

The wave function of the tensor $A^+_{2,\lambda=0}$ meson, $\varphi_{A_2}(\xi)$, is antisymmetric in ξ in the isotopic spin symmetry limit:

$$\langle 0|\bar d(z)\gamma_\nu u(-z)|A_{2,\lambda=0}(p)\rangle_\mu\to P_\nu\int_{-1}^{1}d\xi\, e^{i\xi(zp)}f_{A_2}\varphi_{A_2}(\xi,\mu^2),$$

$$\langle 0|\bar d(0)\gamma_\nu i\overleftrightarrow{D}_\rho u(0)|A_{2,\lambda=0}\rangle_\mu=-P_\nu P_\rho f_{A_2},\quad \int_{-1}^{1}d\xi\,\xi\,\varphi_{A_2}(\xi)=1. \qquad (3.44)$$

Therefore, the asymptotic behaviour of the $A_{2,\lambda=0}$ meson form factor is determined by the first nonzero matrix element f_{A_2} in (3.44). One has from (3.3),(3.11),(3.26),(3.44):

$$\varphi_{A_2}(\xi,\tau)\to\frac{15}{4}f_{A_2}(1-\xi^2)\xi e^{-\varepsilon_1\tau},\quad \varepsilon_1=\frac{32}{9},\quad \tau=\frac{1}{b_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(Q)}\to\infty,$$

$$I_{A_2}(\tau)=\int_{-1}^{1}\frac{d\xi}{1-\xi}\varphi_{A_2}(\xi,\tau)\to\frac{5}{2}f_{A_2}\left[\alpha_s(Q)/\alpha_s(\mu)\right]^{32/9b_0},$$

$$F_{A_2^+}(Q^2)\to\frac{200\pi}{9}\alpha_s(Q)\left(\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right)^{\frac{64}{9b_0}}\frac{|f_{A_2}(\mu)|^2}{Q^2}. \qquad (3.45)$$

The value of the matrix element f_{A_2} in (3.44) was found in /3.14/ using the QCD sum rules:

$$|f_{A_2}(\mu=1\text{ GeV})|\simeq 100\text{ MeV}.$$

The asymptotic behaviour of the $\gamma\gamma\pi^0$ form factor $F_{\gamma\pi}(q^2,0)$ (2.19),(2.24) has the form /1.29/:

$$F_{\gamma\pi}(q^2,0) \to \frac{\sqrt{2}\, f_\pi}{-q^2} = F_{\gamma\pi}(0,0)\,\frac{4\pi^2 f_\pi^2}{q^2} \simeq F_{\gamma\pi}(0,0)\,\frac{0.7\,\mathrm{GeV}^2}{q^2}. \qquad (3.46)$$

Our experience shows that two questions arise usually in connection with the above given formulae.

1) Because D.R.Jackson /1.26/ summed only the subset of all leading diagrams, why the answer he obtained for the asymptopia of $F_\pi(q^2)$ coincides with the exact answer (3.42)? The reason is that the ladder contributions, fig.3.3, which were summed by him (in the Feynman gauge) correspond to the operator expansion (3.2), but with the gauge non-invariant operators $\bar\psi\gamma_\mu\gamma_5(i\overleftrightarrow{\partial}_z)^n\psi$ with the anomalous dimensions: $\tilde\varepsilon_n = C_F\left[1-\frac{2}{(n+1)(n+2)}\right] \neq \varepsilon_n$ at $n\neq 0$ (compare with (3.11)). The $F_\pi(q^2)$ asymptopia is determined, however, by the $n=0$ operator $\bar\psi\gamma_\mu\gamma_5\psi$ and, therefore, the answers for the first leading terms coincide.

2) Having the experience with the deep-inelastic scattering, many physicists get accustomed that the deep-inelastic amplitude $\langle P|\bar\psi(z)\exp\{ig\int_{-z}^{z} d\sigma_\nu B_\nu\}\gamma_\mu\psi(-z)|P\rangle_Q$ should be expanded into a series of local operators ($\Gamma\equiv\gamma_\mu$):

$$\left[\bar\psi(z)\exp\{\dots\}\Gamma\psi(-z)\right]_Q = \left[\bar\psi(0)\Gamma\psi(0)\right]_\mu e^{-\varepsilon_0\tau} + \frac{1}{2!}\left[\bar\psi(0)\Gamma(z\overleftrightarrow{D})^2\psi(0)\right]e^{-\varepsilon_2\tau}+\dots,$$

and the leading contribution gives at $Q^2\to 0$ the first local term $\bar\psi(0)\Gamma\psi(0)$, because the anomalous dimensions ε_n increase with n.

Then by analogy, because $\langle 0|\bar d(0)\Gamma_5(\overleftrightarrow{D})^n u(0)|\pi(p)\rangle \to \int_{-1}^{1} dz\, z^n \varphi_\pi^A(z)$, see (1.13), it seems that the asymptopia of $F_\pi(Q^2)$ should be determined by the first local term (i.e. without the derivatives

$(\overleftrightarrow{D})^n$):

$$I_\pi(\tau)=\int_{-1}^{1}\frac{dz}{1-z}\varphi_\pi^A(z,\tau)=\sum_{n=0}\int_{-1}^{1}dz\,z^n\varphi_\pi^A(z,\tau)\xrightarrow{?}\int_{-1}^{1}dz\,\varphi_\pi^A(z,\tau)=1.$$

But this result differs by the factor (3/2) from the true result (3.42).

The reason is, of course, that the operators $\bar\psi\Gamma_5(z\overleftrightarrow{D})^n\psi$ do not renormalize multiplicatively. The correct expansion has the form:

$$\left[\bar\psi(z)\Gamma_5\exp\{\dots\}\psi(-z)\right]_Q = \left[\bar\psi(0)\Gamma_5\psi(0)\right]_Q + \frac{1}{2!}\left[\bar\psi(0)\Gamma_5(z\overleftrightarrow{D})^2\psi(0) - \frac{1}{5}(z\partial)^2\bar\psi(0)\Gamma_5\psi(0)\right]_Q + \frac{1}{5}\frac{(z\partial)^2}{2!}\left[\bar\psi(0)\Gamma_5\psi(0)\right]_Q+\dots = \left[\bar\psi(0)\Gamma_5\psi(0)\right]_\mu e^{-\varepsilon_0\tau} + \frac{1}{2!}\left[\bar\psi(0)\Gamma_5(z\overleftrightarrow{D})^2\psi(0) - \frac{1}{5}(z\partial)^2\bar\psi(0)\Gamma_5\psi(0)\right]_\mu e^{-\varepsilon_2\tau} + \frac{1}{5}\frac{(z\partial)^2}{2!}\left[\bar\psi(0)\Gamma_5\psi(0)\right]_\mu e^{-\varepsilon_0\tau}+\dots$$

Hence, the leading contribution is really

$$\left[\bar\psi(z)\Gamma_5\exp\{\dots\}\psi(-z)\right]_Q \to e^{-\varepsilon_0\tau}\left\{1+\frac{1}{5}\frac{(z\partial)^2}{2!}+\dots\right\}\left[\bar\psi(0)\Gamma_5\psi(0)\right]_\mu .$$

One can say in this sense, that the leading contributions give not only the local operator $\bar\psi(0)\Gamma_5\psi(0)$, but all the operators $\left[\frac{1}{2n!}(z\partial)^{2n}\bar\psi(0)\Gamma_5\psi(0)\right]$ as well, and the "superfluous" factor $3/2=\left(1+\frac{1}{5}+\dots+\frac{3}{(2n+1)(2n+3)}+\dots\right)$ is caused by just the contributions of these operators. All these operators give no contribution into the deep-inelastic scattering, because $\partial^n\langle P|\bar\psi(0)\Gamma\psi(0)|P\rangle=0$.

3.7 FLAVOUR-SINGLET MESONS AND QUARK-GLUON MIXING

Flavour-singlet mesons have two types of wave functions - quark and gluonic ones. For this reason, the transition form factor for such mesons includes the fig. 3.14 diagrams, in addition to the fig. 3.3 one.

Two-gluon wave functions of the leading twist 2 can be introduced in complete analogy with the two-quark ones. Because both gluons have the helicities $\lambda_{1,2}=\pm 1$, only the mesons with the helicities $\lambda=0$ and $\lambda=\pm 2$ have the leading twist two-gluon wave functions. For natural mesons with $\lambda=0$:

$$\langle 0| G_{\mu\nu}(z)\exp\left\{ig\int_{-z}^{z} d\sigma_\rho B_\rho(\sigma)\right\} G_{\nu\lambda}(-z)|q,\lambda=0\rangle_{\mu_0} = f_\$^L\, q_\mu q_\lambda \times$$

$$\int_{-1}^{1} d\zeta\, e^{i\zeta(zq)}\phi_\$^L(\zeta,\mu_0)\,,\quad \int_{-1}^{1} d\zeta\, \phi_\$^L(\zeta,\mu_0)=1,\quad q_z\to\infty. \qquad (3.47)$$

For natural mesons with $\lambda=\pm 2$:

$$\langle 0| G_{\mu\nu}(z)\exp\{\ldots\} G_{\alpha\beta}(-z)|q,|\lambda=2|\rangle_{\mu_0} = f_\$^\perp \int_{-1}^{1} d\zeta\, e^{i\zeta(zq)}\phi_\$^\perp(\zeta,\mu_0)\times$$

$$\left[\left(q_\mu e^\perp_{\nu\beta} - q_\nu e^\perp_{\mu\beta}\right)q_\alpha - \left(q_\mu e^\perp_{\nu\alpha} - q_\nu e^\perp_{\mu\alpha}\right)q_\beta\right], \qquad (3.48)$$

where $e^\perp_{\alpha\beta}=e^\perp_\alpha e^\perp_\beta$ is the polarization tensor. For unnatural mesons: $G_{\mu\nu}\to \tilde G_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}$ in (3.47), (3.48). The constants $f_\$^L$ and $f_\$^\perp$ have the dimensionality of the mass and are analogous to $f_\pi, f_\rho \ldots$ The asymptotic form of the above wave functions is /3.11, 3.15-3.19/: $\phi_\$^L(\zeta,\mu_0\to\infty)=\phi_\$^\perp(\zeta,\mu_0\to\infty)=\frac{15}{16}(1-\zeta^2)^2$ (see appendix B), and nothing more is known about these wave functions at present. The constants $f_\$^L$ and $f_\$^\perp$ are also unknown. The reason is that the QCD sum rules for the gluonic operators have peculiar properties which hamper their treatment /5.1/.

At the same time, various formal questions connected with properties of logarithmic corrections for these wave functions are investigated in details /3.11, 3.15-3.19/. Analogously to the deep in-elastic scattering /3.20, 3.21/, quark and gluon operators mix with each other due to higher order pert.th. effects. If this mixing is neglected for a time, then the multiplicatively renormalized operators have a form /3.11, 3.15-3.19/:

$$\$_n = G_{\mu\nu}\bar C^{5/2}_{n-2}\left(z\overleftrightarrow{D}/z\partial\right)G_{\nu\lambda}\,;\quad P_n=\tilde G_{\mu\nu}\bar C^{5/2}_{n-2}\left(z\overleftrightarrow{D}/z\partial\right)G_{\nu\lambda}\,;$$

$$\perp_n = G_{\mu\nu}\bar C^{5/2}_{n-2}\left(z\overleftrightarrow{D}/z\partial\right)G_{\alpha\beta}\,;\quad z^2=0,\ n\geq 2\,, \qquad (3.49)$$

where $C_n^{5/2}(\zeta)$ are the Gegenbauer polinomials normalized by

$$\int_{-1}^{1} d\zeta\,(1-\zeta^2)^{\nu-\frac{1}{2}}\left[\bar C^\nu_n(\zeta)\right]^2 = 2\pi\,\frac{\Gamma(n+2\nu)}{4^\nu (n+\nu)\Gamma(n+1)\Gamma^2(\nu)}\,.$$

Anomalous dimensions of the operators (3.49) can be found, for instance, by calculating in a standard way the divergencies in the Figs. 3.15, 3.16 diagrams and are $(n\geq 2)$:

a) for $\lambda=0$:

$$\$_n:\ (\varepsilon_n)^g_g = C_A\left[\frac{1}{3}+\frac{2n_{fe}}{3N}-\frac{4}{(n-1)n}-\frac{4}{(n+1)(n+2)}+4\sum_2^n\frac{1}{j}\right] \quad /3.15/,$$

$$P_n:\ (\tilde\varepsilon_n)^g_g = C_A\left[\frac{1}{3}+\frac{2n_{fe}}{3N}-\frac{8}{n(n+1)}+4\sum_2^n\frac{1}{j}\right] \quad /3.16-3.18/, \qquad (3.50)$$

b) for $|\lambda|=2$:

$$\perp_n:\ \varepsilon^\perp_n = C_A\left[\frac{1}{3}+\frac{2n_{fe}}{3N}-\frac{8}{(n-1)n}+4\sum_2^n\frac{1}{j}\right] \quad /3.19/.$$

Let us remind that for the leading twist two-quark operators:

$$V_n=\bar{\psi}\gamma_\mu C^{3/2}_{n-1}\psi:\ (\varepsilon_n)^q_q=C_F\left[1-\frac{2}{n(n+1)}+4\sum_2^n\frac{1}{j}\right],$$

$$A_n=\bar{\psi}\gamma_\mu\gamma_5 C^{3/2}_{n-1}\psi:\ (\tilde{\varepsilon}_n)^q_q=(\varepsilon_n)^q_q,\quad n\geq 1, \tag{3.51}$$

$$T_n=\bar{\psi}\sigma_{\alpha\beta}C^{3/2}_{n-1}\psi:\ \varepsilon^T_n=C_F\left[1+4\sum_2^n\frac{1}{j}\right].$$

Besides, the operators V_n and A_n correspond to helicity zero mesons and T_n -to helicity one mesons. Therefore, only the operators $(S_n\leftrightarrow V_n)$ and $(P_n\leftrightarrow A_n)$ mix, when the contributions like those shown at Fig. are taken into account:

$$\left(G_{\mu\nu}C^{5/2}_n G_{\nu\lambda}\right)\leftrightarrow\left(\bar{\psi}\gamma_\mu C^{3/2}_{n+1}\psi\right),$$

$$\left(\tilde{G}_{\mu\nu}C^{5/2}_n G_{\nu\lambda}\right)\leftrightarrow\left(\bar{\psi}\gamma_\mu\gamma_5 C^{3/2}_{n+1}\psi\right).$$

It can be shown /3.15-3.19 / that the operators $(S_n\leftrightarrow V_n)$ (or $P_n\leftrightarrow A_n$) mix "as a whole", i.e. the multiplicatively renormalized operators have a form: $(a_n S_n+b_n V_n)$, $(\tilde{a}_n P_n+\tilde{b}_n A_n)$, where a_n, b_n, $\tilde{a}_n$ and $\tilde{b}_n$ are numerical coefficients. As a result, the mixing matrix for (S_n, V_n) coincides with the corresponding matrix for the unpolarized deep in-elastic scattering /3.20 /, while the mixing matrix for (P_n, A_n) coincides with that for the polarized deep in-elastic scattering /3.21/:

$$\gamma_{S-V}=\begin{bmatrix}(\varepsilon_n)^g_g & (\varepsilon_n)^q_g\\ (\varepsilon_n)^g_q & (\varepsilon_n)^q_q\end{bmatrix};\quad \gamma_{P-A}=\begin{bmatrix}(\tilde{\varepsilon}_n)^g_g & (\tilde{\varepsilon}_n)^q_g\\ (\tilde{\varepsilon}_n)^g_q & (\tilde{\varepsilon}_n)^q_q\end{bmatrix},$$

$$(\varepsilon_n)^g_q\,(\varepsilon_n)^q_g=4C_F n_{fl}\frac{(n-1)(n+2)}{n^2(n+1)^2}, \tag{3.52}$$

$$(\tilde{\varepsilon}_n)^g_q\,(\tilde{\varepsilon}_n)^q_g=4C_F n_{fl}\frac{(n^2+n+2)^2}{n(n+1)^2(n+2)^2(n+3)}.$$

The eigenfunctions and eigenvectors of the mixing matrix (3.50)-(3.52) determine the evolution with Q^2 of the quark and gluon components of the flavour-singlet meson wave function. All anomalous dimensions $\varepsilon^{\pm}_n$, $\tilde{\varepsilon}^{\pm}_n$ grow, as usually, with n and so, only the operators with minimal anomalous dimensions survive at $Q^2\to\infty$.

Let us consider, for instance, the asymptotic behaviour of the process $e^+e^-\to\gamma\eta'$ /3.18/. The form factor $F_{\gamma\eta'}$ is determined analogously to $F_{\gamma\pi^0}$, see (2.19), (2.22):

$$iT_{\mu\nu}\to\frac{4}{q_1^2}e_{\mu\nu\lambda\sigma}q_1^\lambda\left[e_u^2\bar{u}\gamma_\sigma\gamma_5 u+(u\to d)+(u\to s)\right],$$

$$\langle\eta'(p)|\frac{1}{\sqrt{3}}\left(\bar{u}\gamma_\sigma\gamma_5 u+\bar{d}\gamma_\sigma\gamma_5 d+\bar{s}\gamma_\sigma\gamma_5 s\right)|0\rangle_{q_1^2}\to -if_{\eta'}p_\sigma\varphi^A_{\eta'}(\xi,q_1^2),$$

$$F_{\gamma\eta'}(q_1^2,q_2^2=0)\to-\frac{f_{\eta'}}{q_1^2}\,\frac{e_u^2+e_d^2+e_s^2}{\sqrt{3}}\int_{-1}^{1}\frac{d\xi}{1-\xi^2}\varphi^A_{\eta'}(\xi,q_1^2). \tag{3.53}$$

The quark-gluon mixing plays no role in the formal limit $|q_1^2|\to\infty$, the minimal anomalous dimensionality has the axial-vector current, and so:

$$F_{\gamma\eta'}(q_1^2,q_2^2=0)\to-f_{\eta'}/\sqrt{3}\,q_1^2. \tag{3.54}$$

The constant $f_{\eta'}$ is analogous to f_π: $f_{\eta'}\simeq 100$ MeV.

The process $e^+e^- \to \gamma h_c$ has been considered in /3.16/, where h_c is the charmonium ground state. The form factor of this process can analogously be expressed through the integral:

$$I(q^2)=\int_{-1}^{1}\frac{d\xi}{1-\xi^2}\varphi^A_{h_c}(\xi,q^2)\,,\quad \langle 0|\bar{c}\gamma_\mu\gamma_5 c|h_c(p)\rangle \xrightarrow[q^2]{} i f_{h_c}\varphi^A_{h_c}(\xi,q^2) \tag{3.55}$$

The wave function of h_c has the form: $\varphi^A_{h_c}(\xi,\mu^2\simeq 4M_c^2\simeq 9\,\text{GeV}^2)\simeq\delta(\xi)$ in the non-relativistic approximation, and $I(q^2\simeq 4M_c^2)=1$. At $|q^2|\to\infty$ $\varphi^c_h(\xi,q^2)\to\frac{3}{4}(1-\xi^2)$ and $I(q^2)\to\frac{3}{2}$. The evolution of $I(q^2)$ with q^2 is shown at Fig. 3.17 /3.16/, and all logarithmic effects are accounted for here, including the quark-gluon mixing. It is seen from the Fig. 3.17 that the evolution is extremely slow.

In conclusion of this section let us point the following. The experience with calculations of logarithmic effects due to anomalous dimensions, the quark-gluon mixing, etc., shows that all these effects are not large numerically and become really significant at very large Q^2 only. At the same time, exclusive cross sections have power fall off with Q^2, and can hardly be measured at very large Q^2. It seems therefore, that the logarithmic effects described above in this section are of academic interest mainly. Further details of the questions considered in this section can be found in the original works /3.11, 3.15 - 3.19/.

3.8 THRESHOLD BEHAVIOUR OF INCLUSIVE STRUCTURE FUNCTIONS

In what way the threshold behaviour of structure functions $F_i(x,Q^2)$ (see the sect.2.3.6) will change when logarithmic corrections are taken into account? This question has been considered in a number of papers /3.22, 2.1, 1.31, 1.32, 3.23/ The described below approach follows mainly to / 2.1 /.

We consider here the pion structure function $F^\pi_\perp(x,Q^2)$, because this is a good example to illustrate main characteristic features. Other structure functions can be considered analogously.

Let us consider the process $\gamma_\perp(q)+\pi^+(p)\to X$ in the region $Q^2=-q^2\to\infty$, $\mu^2\ll s=(p+q)^2\ll Q^2$. The typical Born diagram is shown at Fig. 3.18a. Two final quarks with momenta P_1 and P_2 can be considered as free ones at $s\gg\mu^2\sim 1\,\text{GeV}^2$. The cross section in the Born approximation can be calculated in a standard way and has the form:

$$Q^2\sigma_\perp(s,Q^2)=N_0(4\pi\bar{\alpha}_s)^2\int_{-1}^{1}d\xi_1 d\xi_2 dz_1 dz_2\,\frac{\varphi^A_\pi(\xi_1)}{D(\xi_1)}\,\frac{1}{1-z_1+\Delta}\,\delta(z_1-z_2)\times$$

$$\times\frac{1}{1-z_2+\Delta}\,\frac{\varphi^A_\pi(\xi_2)}{D(\xi_2)}+O\left(\frac{f_\pi^2 s}{Q^4}\right),\qquad N_0=\frac{4}{9\pi}(e_u^2+e_d^2)\frac{f_\pi^2 s}{Q^4}\,,$$

$$D(\xi)=(1-\xi)^2/(1-\tfrac{1}{3}\xi)\,,\quad \Delta=\frac{M_0^2}{s}\,,\quad s\equiv(1-x)Q^2/x. \tag{3.56}$$

Here: $\varphi^A_\pi(\xi)$ is the leading twist pion wave function, $z=\cos\theta$, where θ is the scattering angle in the photon-pion c.m.s., M_0 is the infrared cut off. The formula (3.56) gives (with the logarithmic accuracy): $Q^2\sigma_\perp(s,Q^2)\sim(f_\pi^2 s^2/Q^4M_0^2)\sim(1-x)^2$, in accordance with (2.27). It is seen from (3.56) that the cross section is highly sensitive to the infrared cut off M_0 (in spite that the gluon virtuality at Fig. 3.18a is large: $\sigma^2\simeq M_0^2Q^2/s\simeq(M_0^2/1-x)\gg M_0^2$), and this agrees with the general considerations presented in the sect.2.3.6.

Let us consider now the role of loop logarithmic corrections, Fig. 3.18b, and write the answer in the form:

$$Q^2\sigma_\perp(s,Q^2)=N_0\int_{-1}^{1}d\xi_1 d\xi_2 dz_1 dz_2\,\frac{\varphi^A_\pi(\xi_1,\sigma_1^2)\,\alpha_s(\sigma_1^2)\,s_1}{D(\xi_1)}\,(4\pi)^2\times$$

$$\times \frac{\bar{G}(z_{1,2};\sigma^2_{1,2};M_0^2)}{(1-z_1+\Delta)(1-z_2+\Delta)}\ \frac{\varphi_\pi^A(\zeta_2,\sigma_2^2)\,\alpha_s(\sigma_2^2)\,\$_2}{D(\zeta_2)} \equiv N_0 I. \quad (3.57)$$

There is a number of effects.

1. The pion wave function evolution:

$$\varphi_\pi^A(\zeta_1)\to\varphi_\pi^A(\zeta_1,\sigma_1^2),\quad \varphi_\pi^A(\zeta_2)\to\varphi_\pi^A(\zeta_2,\sigma_2^2),$$
$$|\sigma^2_{1,2}|\simeq\left(\frac{1-\zeta_{1,2}}{2}\right)\left(\frac{1-z_{1,2}+\Delta}{2}\right)Q^2. \quad (3.58)$$

The pion wave function $\varphi_\pi^A(\zeta,\sigma^2)$ describes the evolution of the quark pair from the virtuality $\simeq\sigma^2$ down to $\simeq M_0^2$ and a subsequent formation of the pion state (see sects.3.4,3.5).

2. The "Green function" of the final quark pair:

$$\bar{G}(z_{1,2};\sigma^2_{1,2};M_0^2)=\int_{-1}^{1}dz'\,G(z_1,z';\sigma_1^2,M_0^2)\,G(z',z_2;M_0^2,\sigma_2^2), \quad (3.59)$$

$$G(z_i,z';\sigma_i^2,M_0^2)\simeq\sum_n P_n(z_i)P_n(z')e^{-\varepsilon_n\tau_i},\quad \tau_i=\frac{1}{b_0}\ln\frac{\alpha_s(M_0^2)}{\alpha_s(\sigma_i^2)}\geq 0,$$

where $\{P_n(z)\}$ is the system of orthogonalized polinomials corresponding to multiplicatively renormalized operators, ε_n are the corresponding anomalous dimensions, $\varepsilon_n\simeq 4C_F\ln(n)$ at $n\gg1$, τ_1 and τ_2 determine the evolution intervals. The function $G(z,z';\sigma^2,M_0^2)$ describes the evolution of the final quark pair from the virtualities $\simeq\sigma^2$ down to M_0^2. The function $\bar{G}(z_{1,2};\sigma^2_{1,2};M_0^2)$ describes the evolution from the virtualities $\simeq\sigma_1^2$ down to M_0^2 and again up to $\simeq\sigma_2^2$. Roughly:

$$\bar{G}(z_{1,2};\sigma^2_{1,2};M_0^2)\simeq\sum_n P_n(z_1)P_n(z_2)e^{-\varepsilon_n(\tau_1+\tau_2)},$$
$$\bar{G}(z_i,\tau_i=0)=\delta(z_1-z_2),\quad \bar{G}(z_i,\tau_i\to\infty)\to\text{const}. \quad (3.60)$$

3. The functions $\$_{1,2}$ are the Sudakov form factors:

$$\$_i=\$(Q^2,\$+|\sigma_i^2|)\simeq\exp\left\{-\frac{C_F}{b_0}\ln\left(\frac{Q^2}{\$+|\sigma_i^2|}\right)\ln\frac{\alpha_s(\$+|\sigma_i^2|)}{\alpha_s(Q^2)}\right\}.$$

The total leap of virtuality of the quark is from Q^2 down to M_0^2. The radiation of gluons with momenta K_i (see Fig. 3.18 b) diminishes this interval down to $(Q^2\to\$)$. The evolution of the quark pair from σ^2 down to M_0^2, which is accounted for in the function $G(z,z';\sigma^2,M_0^2)$, diminishes this interval down to $(Q^2\to|\sigma^2|)$. On the whole, there remains the Sudakov suppression in the interval $(Q^2\to\$+|\sigma^2|)$.

The behaviour of the integral I in (3.57) is roughly as follows. At small "evolution time" $\tau_1\simeq\tau_2\simeq0$: $\bar{G}\simeq\delta(z_1-z_2)$, and the integral over z in (3.57) diverges linearly at $z\to1$: $I\sim\int^1 dz/(1-z+\Delta)^2=\frac{1}{\Delta}=(\$/M_0^2)\gg1$, and is highly sensitive to the infrared cut off M_0. The "evolution time" $\tau_{1,2}$ is large in the deep asymptotic region $Q^2\to\infty$ and/or $(1-x)\to0$, $\bar{G}\to O(1)$ in this formal limit, and the answer becomes insensitive to the infrared cut off M_0 and can be calculated unambiguously.

In more details, the contribution of the region $(1-z_1)\sim(1-z_2)\lesssim\Delta$ has the form: $\Delta=\frac{M_0^2}{\$}=\frac{M_0^2}{(1-x)Q^2}\ll1,\quad |\sigma_1^2|\sim|\sigma_2^2|\sim\Delta Q^2,$

$$\bar{G}\sim\left[\frac{1}{(1-z_1)+(1-z_2)}\right]^{1-\tau_0},\quad \tau_0=4\frac{C_F}{b_0}\ln\frac{\alpha_s(M_0^2)}{\alpha_s(\Delta Q^2)}>0,$$

$$\$_1=\$_2=\bar{\$}\simeq\exp\left\{-\frac{C_F}{b_0}\ln\left(\frac{Q^2}{\$+\Delta Q^2}\right)\ln\frac{\alpha_s(\$+\Delta Q^2)}{\alpha_s(Q^2)}\right\},$$

$$I_\Delta\sim\left(\frac{1}{\Delta}\right)^{1-\tau_0}\bar{\$}^2=\left[\frac{(1-x)Q^2}{M_0^2}\right]^{1-\tau_0}\bar{\$}^2. \quad (3.61)$$

Therefore, this contribution is small at $\tau_0 > 1$. The main contribution gives in this case the region $(1-z_1) \sim (1-z_2) \sim O(1)$, and one can neglect Δ in comparison with $(1-z_{1,2})$ in (3.57).

On the whole, the structure function has the form:

$$Q^2 \sigma_L(\xi, Q^2) \to c_0 \frac{|f_\pi|^2 \xi}{Q^4} \left\{ 1 + c_1 \left(\frac{\xi}{M_0^2} \right)^{1-\tau_0} \bar{\xi}^2 \right\}, \qquad (3.62)$$

where c_0 and c_1 depend on the asymptotic variables Q^2, $(1-x)$ and on M_0^2 only weakly (logarithmically). The first term in (3.62) dominates at $\tau_0 > 1$ and therefore, the structure function can be calculated unambiguously in this region.

Analogously, for the nucleon:

$$F_L^N(\xi, Q^2) \to c_2 \frac{|f_N|^2 \xi}{Q^6} \left\{ 1 + c_3 \left(\frac{\xi}{M_0^2} \right)^{2-t_0} \bar{\xi}^2 \right\}, \qquad (3.63)$$

and the answer has no strong dependence on M_0 at $t_0 > 2$.

We do not pursue these questions further here, because, it seems, they are of academic interest only (the "evolution time" $\tau_0 \simeq 1$ at $(1-x) \lesssim 10^{-2}$ only).

3.9 CONCLUSIONS

Two methods are widely used at present for the calculation of loop logarithmic corrections: the operator expansions and the renormalization group on the one hand, and the Bethe-Salpeter equation (the evolution equation) on the other hand. Use of "physical" gauges for the exclusive (as well as inclusive) processes leads to a simple parton-like picture.

The properties of loop corrections are investigated at present in great details and, it seems, the main properties of both

leading and non-leading logs are well understooded now.

One of the main results has been obtained yet in the pioneer papers and reads as follows: the summary effect of loop corrections is mild and does not change the power behaviour, determined by the Born diagrams for three-point functions (form factors, two-particle decays, ...)*. For instance, the selection rules described in ch.2 remain true when loop corrections are accounted for.

All double logs cancel in colourless channels, and rest logs describe here a slow evolution with increasing maximal virtuality. At the same time, one of the most characteristic features inherent to theories with vector gluons, is the presence of "internal Sudakov effects" (see sects.3.5, 3.8). - If the regions with large leaps of virtualities are encountered in coloured channels when integrating over "internal variables" (longitudinal momentum fractions x_i, say), there arise Sudakov form factors which suppress contributions from such regions.

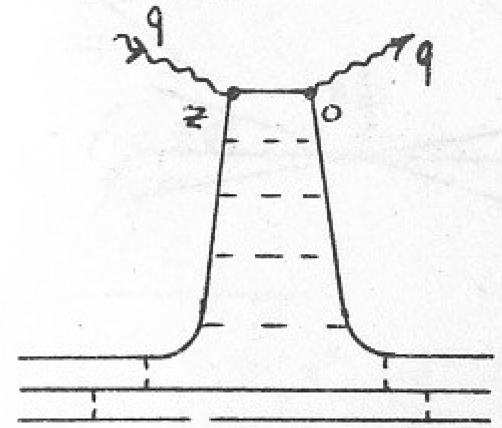
In practice, howewer, loop logarithmic corrections play a mild role at experimentally accessible values of Q^2, and are really significant at enormously large Q^2 only. The exclusive cross sections fall off quickly with Q^2 howewer (unlike to inclusive ones), and it seems, their measurement at $Q^2 \gg 100\ \text{GeV}^2$ ($Q^2 \simeq 100\ \text{GeV}^2$ is the Υ region) will be hardly possible. As a result, the knowledge of non-perturbative hadronic wave functions is much

---

* This is unlike, for instance, to the deep in-elastic scattering, where the longitudinal structure function $F_L(x, Q^2) \sim \frac{1}{Q^2} \varphi_L(x)$ at the Born level, and $F_L(x, Q^2) \sim \frac{\alpha_s}{\pi} \tilde{\varphi}_L(x)$ in the next order in α_s.

more important at experimentally accessible values of Q^2.

The investigation of properties of various hadronic wave functions is described in the next chapters.



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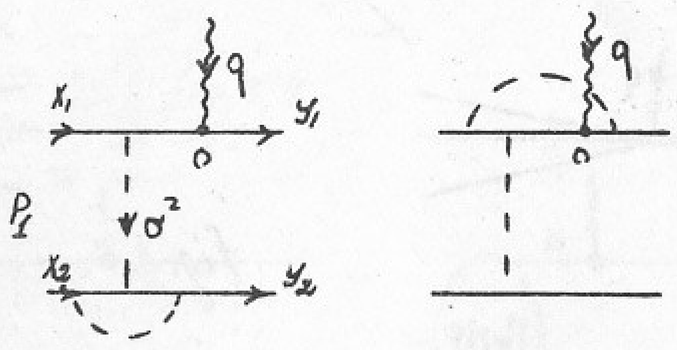

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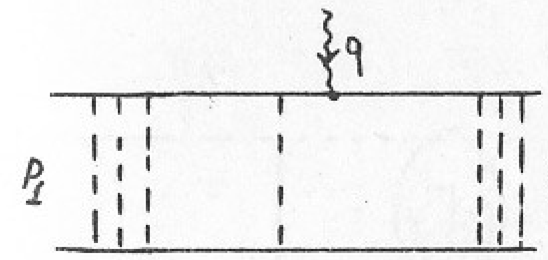

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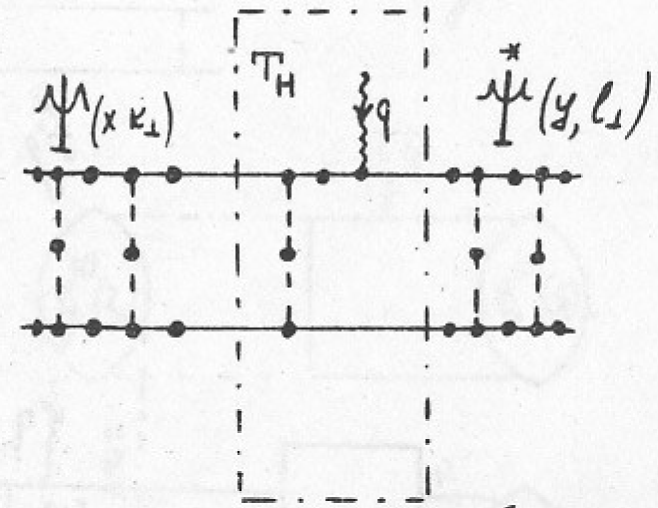

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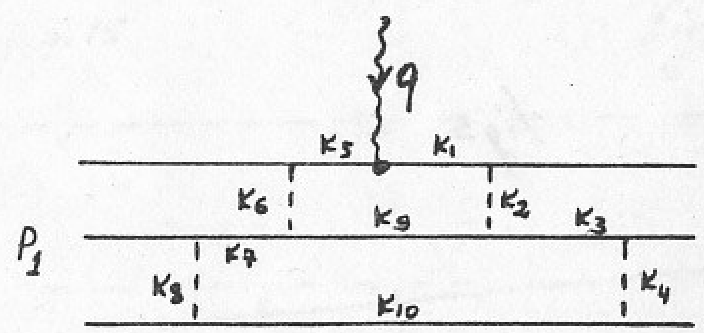
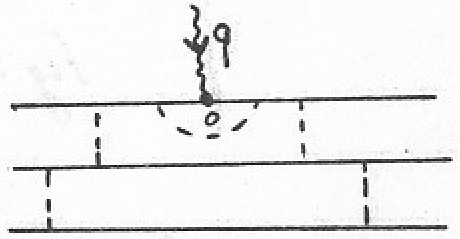


fig. 3.5

fig. 3.6



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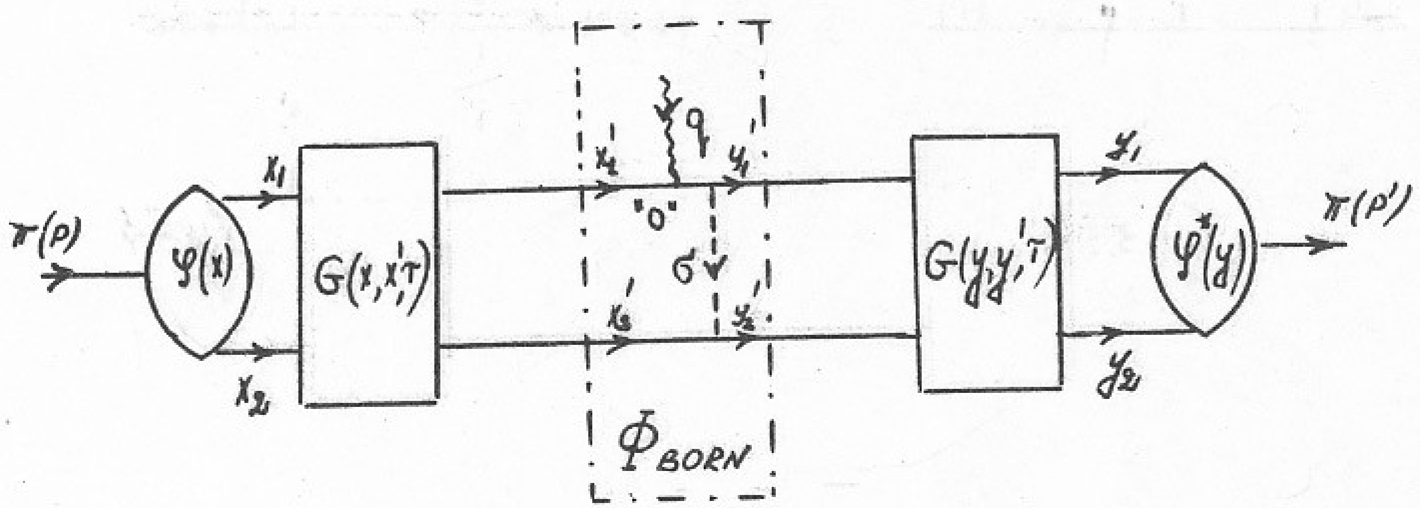


fig 3. 12

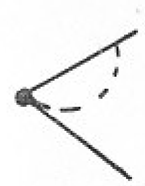
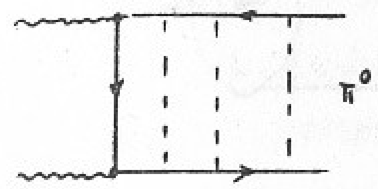


fig. 3.8

fig. 3.9



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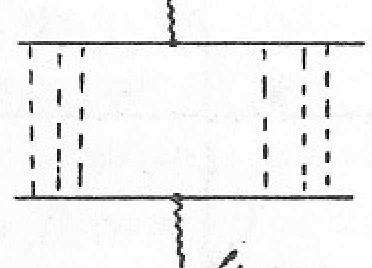

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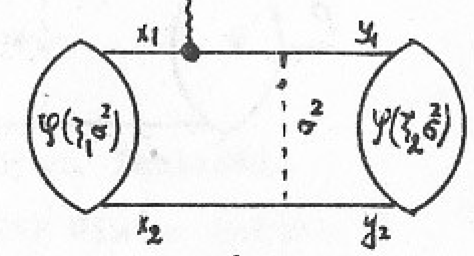

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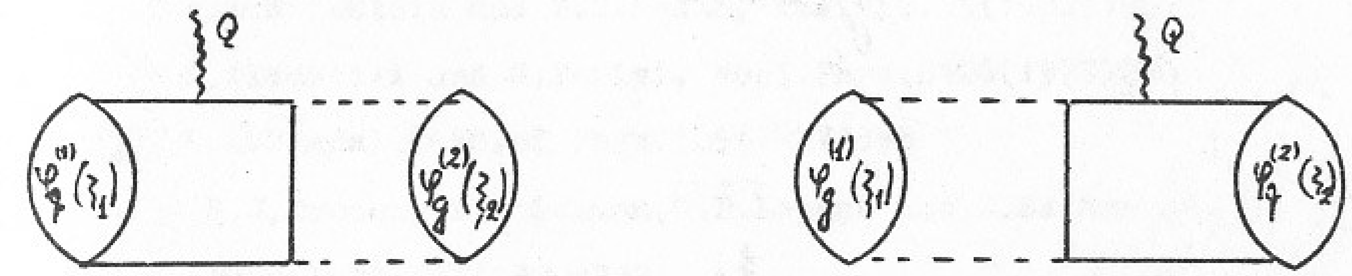
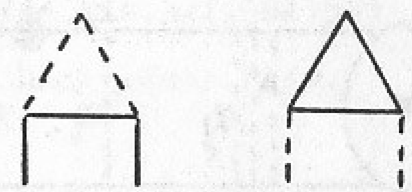


fig. 3.14

fig. 3.15



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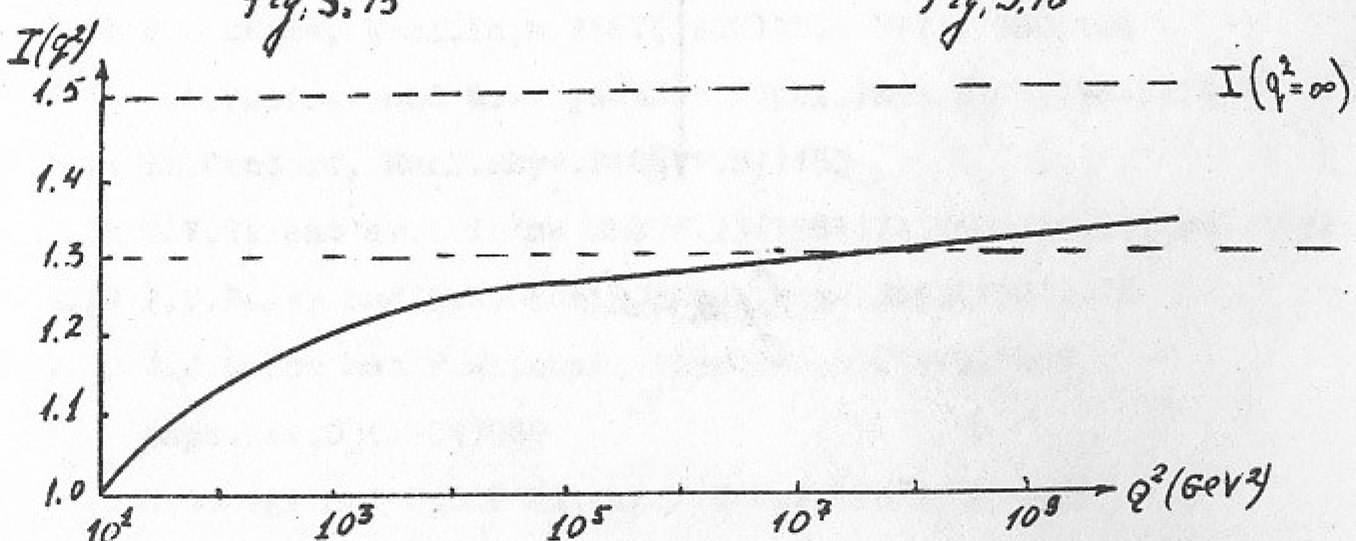

fig. 3.17

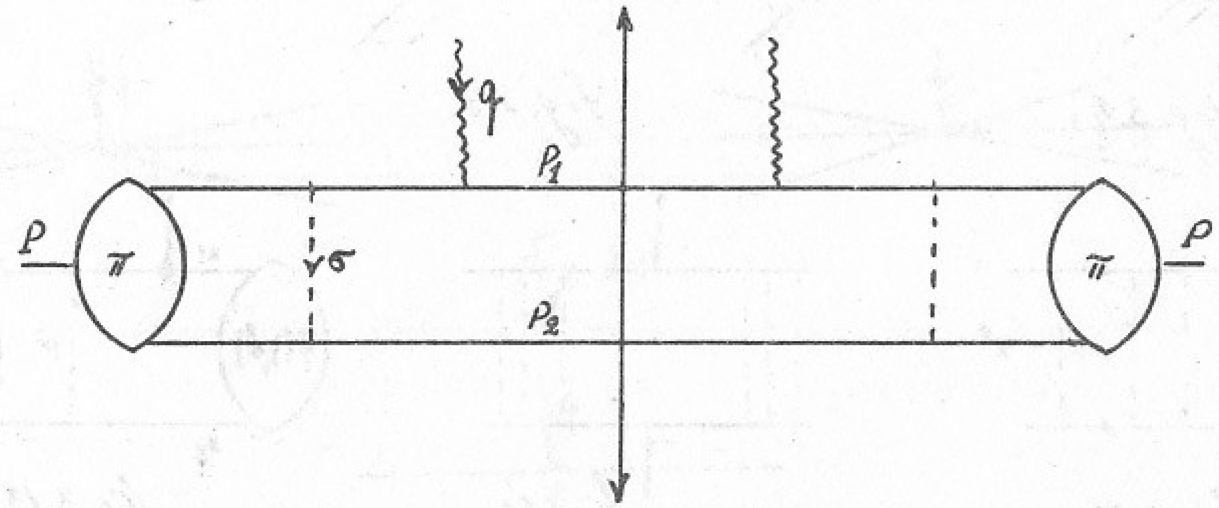

fig. 3.18 a

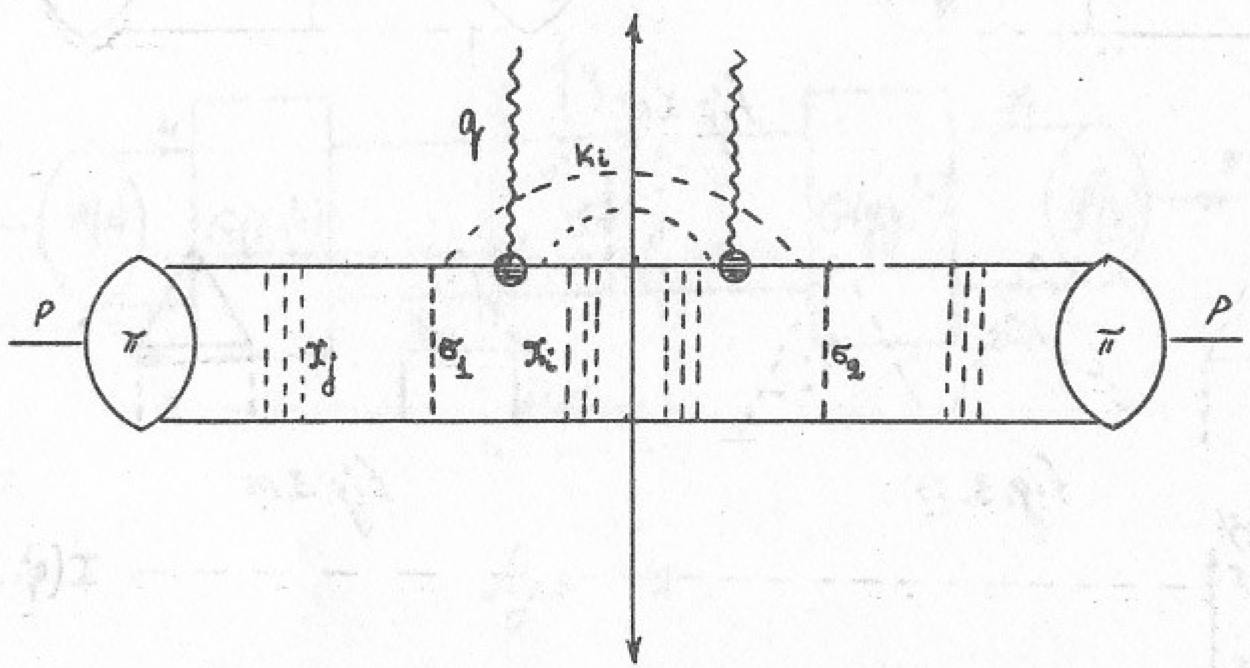

fig. 3.18 b

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АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ ЭКСКЛЮЗИВНЫХ ПРОЦЕССОВ В КХД

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