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IN A LONG-WAVE VACUUM FIELD IN QCD

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ABSTRACT

The analitical properties of the photon polarization operator $\Pi_{\alpha\beta}(q)$ is considered using the Lorentz-invariant averaging over the covariant constant field $G_{\mu\nu}$ orientation in QCD. It is shown that the usual operator expansion is valid below the threshold. It is turned out that $\Pi_{\alpha\beta}(q)$ has no cut in the complex plane q^2. This means the quark confinement in this field.

1. As one believes now, a vacuum state in QCD has a complex non-perturbative structure. The possible type of the vacuum fluctuations (VF) is instantons (for the detailed consideration of instanton effects see Ref./1/). The presence of the quark condensate ($\langle\bar\psi\psi\rangle \neq 0$) /2/ and the gluon condensate ($\left\langle\frac{\alpha_s}{\pi} G^2\right\rangle \neq 0$) /3/ is an important feature of the QCD vacuum. Instantons of different sizes ρ contribute to these condensates. The interaction of an instantons with a long-wave VF and the interaction between instantons increase very rapidly with increasing ρ /1,4/. These interactions become important starting from comparatively small ρ /4,5/. Therefore, the form of a long-wave VF differs from an instanton one considerably.

One can suppose that a variety of the non-perturbative phenomena in QCD come from different field configurations. Very likely, VF of the sizes $\rho\sim 1/\mu$ (μ is the characteristic hadron scale) is significant for the forming of the low-lying resonances. However, taking into account the VF with arbitrarily large sizes ρ is, probably, required for the consideration of the confinement mechanism. The reason for this is an increase of the distances, on which the quarks and gluons can propagate, with increasing the energy.

In the present paper we consider the effects of the VF whose amplitude is substantially larger than their inverse size $1/\rho$. Such VF in the limit of large ρ correspond to a homogeneous field in the space-time. This limit means a neglect of the operators with covariant derivatives $\mathcal{D}_\mu$ in the operator expansion terms. One can formulate the gauge-invariant and covariant condition for the field homogeneity /6/ as follows:

$$[\mathcal{D}_\lambda, G_{\mu\nu}] = 0. \qquad (1)$$

This condition defines the so-called covariant constant field configurations /6/. In our paper we shall restrict ourselves to the colour group $SU_c(2)$. For this group the field potential has the form /6/ :

$$B_\mu = -\frac{\varkappa}{2} F_{\mu\nu} x_\nu ; \qquad (2)$$

here $\varkappa = n^a \cdot \frac{\tau^a}{2}$, τ^a are Pauli matrices, vector n^a determines the field orientation in the colour space. Then $G_{\mu\nu} = \varkappa F_{\mu\nu}$. The expression (2) corresponds to the gauge condition $x_\mu \cdot B_\mu = 0$ /7/ if we neglect the terms containing covariant derivatives: $[\mathcal{D}_{\mu_1}, ..[\mathcal{D}_{\mu_n}, G_{\mu\nu}]]$.

It has been shown in the papers /6/ from the consideration of the Yang-Mills effective action for the covariant constant field in the one-loop approximation that the "vacuum state" containing the chromomagnetic field has the energy less than the "perturbative vacuum". In the papers /8/ the instability of a homogeneous chromomagnetic field was found. It has been argumented in Refs./8/ that this instability results in the appearance of flux tubes in the QCD vacuum. However, in the analysis of Refs./8/ both the Lorentz invariance and the rotational invariance in the colour space were absent.

In the present paper the analytical properties of the vector polarization operator $\Pi_{\alpha\beta}(q)$ in the one-loop approximation are studied in the covariant constant field. In our consideration the Lorentz invariance and rotational invariance in the colour space are restored by averaging over the field $G_{\mu\nu}$ orientation. We show, in particular, the existence of the quark

confinement in the field under consideration.

2. Let us consider the photon polarization operator in the field (2) in the one-loop approximation. We shall perform first the calculations in Euclidean space and pass to Minkowski space-time by means of Wick rotation, $q_4 \to -i q_0$ (q_μ is photon momentum).

The Lorentz-invariant averaging over the field $F_{\mu\nu}$ is the principal point of our investigation. Due to this averaging the polarization operator (we set quark electric charge $Q_f e = e$)

$$\Pi_{\mu\nu}(q) = i e^2 \, Tr \left\langle\!\left\langle 0 \left| \frac{1}{\hat{\mathcal{P}} - m + i0} \gamma_\mu \frac{1}{\hat{\mathcal{P}} - \hat{q} - m + i0} \gamma_\nu \right| 0 \right\rangle\!\right\rangle \qquad (3)$$

has the form

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2) \cdot \Pi(q^2). \qquad (4)$$

In eq.(3) $\mathcal{P}_\mu = i\mathcal{D}_\mu = i\partial_\mu + gB_\mu$, $\hat{\mathcal{P}} = \mathcal{P}_\mu \cdot \gamma_\mu$, m is the quark mass and the trace is taken over the Lorentz and colour indices. The second averaging in eq.(3) is carried out over $F_{\mu\nu}$. Let us introduce the invariant measure for the integration over $F_{\mu\nu}$:

$$d\mu = \frac{1}{a} d\vec{E}\, d\vec{H} \cdot \delta(\mathcal{F} - \mathcal{F}_0) \cdot \delta(\mathcal{G}^2 - \mathcal{G}_0^2), \qquad (5)$$

where $\vec{E}$ and $\vec{H}$ are chromoelectric and chromomagnetic Abelian fields, and

$$\begin{aligned} & E_i = F_{4i}, \quad H_i = -\tfrac{1}{2}\epsilon_{ijk} F_{jk}, \\ & \mathcal{F} = \tfrac{1}{4} F_{\alpha\beta} F_{\alpha\beta} = \tfrac{1}{2}(\vec{H}^2 + \vec{E}^2), \\ & \mathcal{G} = \tfrac{1}{4} F^*_{\alpha\beta} F_{\alpha\beta} = \vec{E} \cdot \vec{H}, \\ & F^*_{\alpha\beta} = \tfrac{1}{2} \epsilon_{\alpha\beta\gamma\delta} F_{\gamma\delta}, \quad \epsilon_{1234} = 1, \end{aligned} \qquad (6)$$

The constant a is chosen so that $\int d\mu = 1$:

$$a = \frac{8\pi^2}{\mathcal{G}_0}\left(\mathcal{F}_0^2 - \mathcal{G}_0^2\right)^{1/2}. \tag{7}$$

One can verify the invariance of the measure (5) by direct calculation of the Jacobian which corresponds to rotations in Euclidean space. There are two independent invariants $\mathcal{F}_0$ and $\mathcal{G}_0$ characterizing the covariant constant "condensate" of the gluon fields. After the integration over $d\mu$ the arbitrary amplitude in the field (2) can be multiplied by some function $f(\mathcal{F}_0, \mathcal{G}_0)$ and integrated over invariants $\mathcal{F}_0$ and $\mathcal{G}_0$. The latter integration has no influence on Lorentz invariance and corresponds to taking into account the dynamical degrees of freedom in the functional integral. Note that the field (2) should not be considered as giving the main contribution to the functional integral, for any amplitudes and distances. One can suppose that the configuration (2) is the result of averaging over the small scale vacuum fluctuations and it may be used as an effective background field for an approximate description of the dynamics on large distances r. Thus we are interested in the Fourier components of the gluon vacuum field $G_{\mu\nu}(k_\lambda)$ with small momenta $k^2 \ll 1/r^2$, in Euclidean formulation. The particular form of the function $f(\mathcal{F}_0, \mathcal{G}_0)$ is not very important for the results and conclusions obtained.

In the model under consideration the vacuum expectation values $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$ and $\left\langle (G^*\cdot G)^2 \right\rangle$ are equal to

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right\rangle = g^2 \frac{\langle \mathcal{F}_0 \rangle}{\pi^2},$$
$$\left\langle \left(G^{*a}_{\mu\nu} G^a_{\mu\nu}\right)^2 \right\rangle = 16 \cdot \left\langle \mathcal{G}_0^2 \right\rangle, \tag{8}$$

where the averaging in the right-hand sides of eqs.(8) corresponds to the integration over $\mathcal{F}_0$ and $\mathcal{G}_0$ with the factor

$f(\mathcal{F}_0, \mathcal{G}_0)$. Invariant $\mathcal{F}_0$ characterizes the intensity of the gluon vacuum field and $\mathcal{G}_0$ characterizes the topological charge density fluctuations. Emphasize that $\langle G^*\cdot G \rangle = 0$. It should be noted that the polarization operator $\Pi_{\mu\nu}(q)$ has a complex tensor form before the integration over $d\mu$ and the scalar functions attached to different tensor structures depend not only on q^2, but on $q\cdot G^2\cdot q$ too.

We use here the photon polarization operator in a homogeneous external field, which was obtained by means of an operator diagram technique in QED /9/. Using the explicit representation for the electron Green function in this field the polarization operator in QED was obtained in Ref./10/ in another form.

The calculation of various physical amplitudes in the field (2), in particular $\Pi_{\mu\nu}(q)$, has to be performed using the measure (5). In our case it is convenient to use the following simplification. Let us multiply the expression for $\Pi_{\mu\nu}(Q)$ from Ref./9/ by $\delta(q^2 - Q^2)$ and take the integral over Q_μ, with due regard for the fact that the scalar function Π in (4) depends on q^2 only. After that the integration over $d\mu$ becomes trivial. Performing these calculations, for the renormalized polarization operator we have

$$\Pi^R(q^2; \mathcal{F}_0, \mathcal{G}_0) = \frac{\alpha}{6\pi}\,\mathrm{tr}\int_0^1 d\tau \int_0^\infty \frac{ds}{s} e^{-sm^2} \int_{-1}^{1} du \left(e^{\Phi} - 1\right)\cdot \frac{xy}{\mathrm{sh}x\cdot\mathrm{sh}y}\cdot$$
$$\left[\mathrm{ch}x\tau\cdot\mathrm{ch}y\tau - \mathrm{cth}x\cdot\mathrm{cth}y\cdot\mathrm{sh}x\tau\cdot\mathrm{sh}y\tau + \left(\frac{1+u}{2}\frac{\mathrm{ch}y}{\mathrm{sh}x}\zeta(x) + \binom{x\leftrightarrow y}{u\to -u}\right)\right], \tag{9}$$

where

$$\Phi = \frac{q^2}{4} s\left[\frac{1+u}{x}\zeta(x) + \frac{1-u}{y}\zeta(y)\right],$$
$$\zeta(z) = \frac{\mathrm{ch}z - \mathrm{ch}z\tau}{\mathrm{sh}z}, \quad x = \varkappa s \mathcal{E}, \quad y = \varkappa s \mathcal{H}. \tag{10}$$

Here $\mathcal{E} = g\left(\mathcal{F}_0 - \sqrt{\mathcal{F}_0^2 - \mathcal{G}_0^2}\right)^{1/2}$, $\mathcal{H} = g\left(\mathcal{F}_0 + \sqrt{\mathcal{F}_0^2 - \mathcal{G}_0^2}\right)^{1/2}$.

In the calculations we deformed the contour of integration over s so that it coincided with the imaginary half-axis: $s \in [0, -i\infty)$. This deformation is possible in Euclidean formulation due to the positive definiteness of the operator $m^2 - \hat{\mathcal{P}}^2$. Equations (9) and (10 have been derived in Euclidean space. However, the analysis of eq.(9) shows that Wick rotation may be performed in the integral representation (9) because of integral convergence at arbitrary q^2. Thus, eqs.(9) and (10) determine also the polarization operator in Minkowski space with $q^2 = q_0^2 - \vec{q}^{\,2}$. The integral over u is trivial, but for the sake of simplicity and convenience of the asymptotics calculations we use here the expression (9). We make the renormalization of the polarization operator so that $\Pi^R(0; \mathcal{F}_0, \mathcal{G}_0) = 0$.

The trace in eq.(9) is taken over the colour indices. The averaging over $G_{\mu\nu}$ colour orientation should be carried out by integration over all matrices $U \in SU_c(2)$ and in the averaging amplitude we have $\varkappa \to \varkappa_U = U \varkappa U^{-1}$. In our case this integration is trivial due to Π^R-colourlessness. The polarization operator Π^R is an even function on $\varkappa$. Using this fact and the relation $(n^a \tau^a)^2 = 1$ we obtain that the trace taking reduces to the substitutions $\varkappa \to \frac{1}{2}$, $tr \to 2$. It is worth noting that going over to the colour group $SU_c(3)$ has no principal difficulties.

The interesting limit of eq. (9) is the polarization operator in the non-relativistic approximation ($|q^2 - 4m^2| \ll m^2$; $\mathcal{E}, \mathcal{H} \ll m^2$):

$$\Pi^R(E; \mathcal{F}_0, \mathcal{G}_0) = \frac{\alpha}{4(\pi m)^{1/2}} \int_{-1}^{1} du \int_0^\infty \frac{d\tau}{\tau^{3/2}} \left(e^{E\tau - \chi\tau^3} - 1\right) + \frac{16\alpha}{9\pi}, \qquad (13)$$

where $\chi = \frac{\mathcal{E}^2(1+u) + \mathcal{H}^2(1-u)}{96m}$, $E = \frac{q^2}{4m} - m$. The polarization operator $\Pi^R(E; \mathcal{F}_0, \mathcal{G}_0)$ (13) satisfies the relation (the constant $16\alpha/9\pi$ is omitted)

$$\Pi^R(E; \mathcal{F}_0, \mathcal{G}_0) = -\frac{4\pi\alpha}{m^2} G_s^R(\vec{0}, \vec{0}; E),$$

here

$$G_s^R(\vec{0}, \vec{0}; E) = \lim_{\vec{\tau}', \vec{\tau} \to 0} \left[G_s(\vec{\tau}', \vec{\tau}; E) - G_s(\vec{\tau}', \vec{\tau}; 0)\big|_{F_{\mu\nu}=0} \right],$$

where the Green function of the quark and antiquark in the colour singlet state

$$G_s(\vec{\tau}', \vec{\tau}; E) = -\frac{1}{2}\left(\frac{m}{4\pi}\right)^{3/2} \int_{-1}^{+1} du \int_0^\infty \frac{d\tau}{\tau^{3/2}} \exp\left[E\tau - \frac{m(\vec{\tau}' - \vec{\tau})^2}{4\tau} - \chi\tau^3\right] \cdot \frac{sh\lambda\tau}{\lambda\tau},$$

$$\lambda^2 = 3m\chi(\vec{\tau}' + \vec{\tau})^2, \qquad (14)$$

may be got from the non-relativistic Green function of the charge particle in electric field (see,e.g./7/,/11/) by averaging over $F_{\mu\nu}$ with the measure (5)and the replacement of m by $m/2$.

3. Let us consider now the properties of the polarization operators Π^R (9),(13).

Our model is the non-trivial theoretical-field example, which confirms the existence of the non-perturbative operator expansion in the under-threshold domain ($q^2 < 4m^2$). If $4m^2 - q^2 \gg (g^2\mathcal{F}_0)^{1/2}, (g^2\mathcal{G}_0)^{1/2}$, then the characteristic distance , on which the quark and antiquark can propagate, is $\tau \sim (4m^2 - q^2)^{-1/2} \ll (g^2\mathcal{F}_0)^{-1/4}, (g^2\mathcal{G}_0)^{-1/4}$. In this case one can prove that the expression (9) can be expanded into the series of field invariants $\mathcal{F}_0$ and $\mathcal{G}_0$. Then, the integrals for the coefficient functions converge exponentially over s and the characteristic proper time is $s \sim \frac{d/2}{4m^2 - q^2}$ (d is the dimension of the corresponding operator). The leading, field-dependent term in Π^R-expansion (proportional to $\mathcal{F}_0$, see (8)) coincides with the results of Refs./3,12/. The following terms of expansion, containing the field in the combinations $\mathcal{F}_0^2$ and $\mathcal{G}_0^2$, agree*) with the contribution of the appropriate gluon operators (linear combination of the operators O_4^{1+2} and O_4^{3+4} in the terms of Ref./13/) to the vector polarization operator from the papers /13/. In Ref./13/ the contributions to the current correlators with the quantum numbers $J^P = 0^{\pm}, 1^{\pm}$ of the operators of dimensions 6 and 8 were obtained using the analytical computer calculations.

As it follows from the analysis of eq. (9), the operator expansion in the under-threshold domain is of the asymptotic character. Therefore, the fact of the large coefficients for the operators of dimension 8 in Ref./13/ is not so surprising.

For $-q^2 \gg m^2, (g^2\mathcal{F}_0)^{1/2}, (g^2\mathcal{G}_0)^{1/2}$ the polarization operator Π^R (9) can be represented in the form $\Pi^R = \Pi_0 + \Pi_1 + \Pi_2$, where Π_0 is the polarization operator without field, Π_2 doesn't depend on q^2 and equals zero, if the field is zero. Then Π_1 equals to:

$$\Pi_1 = \frac{\alpha}{\pi q^4}\left[\frac{\mathcal{E}^2+\mathcal{H}^2}{6} - m^2\int_0^\infty ds\, e^{-sm^2}\left(\mathcal{E}\mathcal{H}\,\mathrm{cth}\frac{\mathcal{E}s}{2}\cdot\mathrm{cth}\frac{\mathcal{H}s}{2} - \frac{4}{s^2}\right)\right]. \tag{15}$$

The vacuum expectation value of the second term in (15) is equal to $8\pi^2 m\langle\bar\psi\psi\rangle$. This fact may be verified directly from the definition of the quark condensate, generated by the field:

$$\langle\bar\psi\psi\rangle = -i\lim_{z'\to z}\mathrm{Tr}\left[S(z',z) - S(z',z)\big|_{F_{\mu\nu}=0}\right] = -\frac{m}{2\pi^2}\int_0^\infty ds\, e^{-sm^2}\left(\frac{\mathcal{E}}{2}\frac{\mathcal{H}}{2}\mathrm{cth}\frac{\mathcal{E}s}{2}\,\mathrm{cth}\frac{\mathcal{H}s}{2} - \frac{1}{s^2}\right), \tag{16}$$

here $S(z',z)$ is the quark Green function in the field. The vacuum expectation value of the right-hand side of eq.(15),

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*) As we were informed by A.V.Radyushkin, the preprint in Ref. /13/ contains the misprint: in eq.(2.16) in the contribution of the operator O_4^{3+4} the number 249 should be replaced by 294.

expressed via $\left\langle\frac{\alpha_s}{\pi^2}G^2\right\rangle$ and $m\langle\bar\psi\psi\rangle$, agrees with the appropriate expression in Ref./3/. If $m^2 \gg \mathcal{E},\mathcal{H}$ then $\langle\bar\psi\psi\rangle = -\frac{1}{12m}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle$. This formula is in agreement with Ref./3/.

It is interesting that eq.(16) give us the possibility to estimate the value of $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle$ in this model, using the value of $\langle\bar\psi\psi\rangle$. It follows from the definition of the invariants $\mathcal{E}$ and $\mathcal{H}$ that $\mathcal{E} \leqslant \mathcal{H}$. However, there are some arguments and estimates to suppose that $\mathcal{E} \ll \mathcal{H}$ in the QCD vacuum. Fig.1 demonstrates the dependence of $\langle\bar\psi\psi\rangle$ on m (for the colour group $SU_c(2)$). Note that $\langle\bar\psi\psi\rangle \sim N_c$ in $1/N_c$-expansion. Taking, for example, the value of $\langle\bar\psi\psi\rangle_{SU_c(3)}$ for light quarks, $\langle\bar\psi\psi\rangle_{SU_c(3)} \approx 1{,}5\cdot 10^{-2}\,GeV^3$, known from the current algebra, and using the relation $\langle\bar\psi\psi\rangle_{SU_c(2)} \approx (2/3)\langle\bar\psi\psi\rangle_{SU_c(3)}$ (about the Monte-Carlo calculations of $\langle\bar\psi\psi\rangle_{SU_c(2)}$ on the lattice see Ref. /14/) we make an estimation of

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{SU_c(2)} = (0.07 \pm 0.02)\,GeV^4 \approx \frac{2}{3}(0.1 \pm 0.03)\,GeV^4.$$

This estimation is based on the use of the average value of $\langle\bar\psi\psi\rangle_{SU_c(2)}$ in the mass interval $0.1 < m\cdot\mathcal{H}^{-1/2} < 0.5$, where $\langle\bar\psi\psi\rangle_{SU_c(2)}$ varies sligtly (the indicated accuracy is due to this variation). The obtained value may be compared with ($SU_c(3)$) $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.012\ GeV^4$ /3/ and $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \approx 0.1\ GeV^4$ /15/.

The properties of the polarization operator Π^R (9) in the under-threshold domain are quite ordinary, but in the physical domain these are very remarkable. For real $q^2 > 4m^2$ the imaginary part of Π^R is strictly equal to zero:

$$\mathrm{Im}\,\Pi^R(q^2;\mathcal{F}_0,\mathcal{G}_0)\big|_{\mathrm{Im}\,q^2=0} = 0. \tag{17}$$

The absence of the cut of Π^R in the complex plane q^2 means the impossibility for quark to exist in a free state. Non-triviality of the asymptotics of Π^R in the physical domain is connected with this property of Π^R. For $q^2 \gg m^2, \mathcal{E}, \mathcal{H}$, we obtain from eq.(9) that

$$\Pi^R(q^2;\mathcal{F}_0,\mathcal{G}_0)=\frac{2\alpha}{3\pi}\cdot\frac{\mathcal{E}^2\mathcal{H}^2}{\mathcal{H}-\mathcal{E}}\cdot\frac{1}{q^6}\cdot$$

$$\left[e^{\frac{q^2}{\mathcal{E}}}\left(\frac{\mathcal{E}}{q^2}\right)^{\frac{2m^2}{\mathcal{E}}}\cdot\Gamma\left(\frac{m^2}{\mathcal{E}}+1\right)\cdot\left(\Gamma\left(\frac{m^2}{\mathcal{E}}+1\right)+\left(\frac{\mathcal{E}}{q^2}\right)^{\frac{\mathcal{H}}{\mathcal{E}}-1}\cdot\Gamma\left(\frac{m^2+\mathcal{H}}{\mathcal{E}}\right)\right)-(\mathcal{E}\leftrightarrow\mathcal{H})\right] \quad (18)$$

is exponentially large: $\Pi^R \sim e^{\frac{q^2}{\mathcal{E}}}$ ($\mathcal{E}<\mathcal{H}$). It follows from eq.(18) the non-analytical behaviour of Π^R for $\mathcal{E},\mathcal{H}\to 0$. In the model under consideration the quark confinement has the completely non-perturbative nature. Quark confinement takes place for arbitrary small values of the invariants $\mathcal{E}$ and $\mathcal{H}$. However, if $\mathcal{E}\equiv 0$, then Π^R acquires the cut and confinement is absent. Therefore, for the mechanism of confinement in the field (2) the presence of the topological charge density fluctuations ($\mathcal{G}_0 \neq 0$) is of the principal importance.

One can convince oneself that, for $E>0;\ \epsilon\equiv\left(\frac{\mathcal{E}^2}{mE^3}\right)^{1/2}\ll 1$, $h\equiv\left(\frac{\mathcal{H}^2}{mE^3}\right)^{1/2}\ll 1$, Π^R(13) has the similar behaviour as (18):

$$\Pi^R(E;\mathcal{F}_0,\mathcal{G}_0)=\alpha\left[\frac{3}{32}\left(\frac{E}{m}\right)^{1/2}\frac{\epsilon^4 e^{\frac{8}{3\epsilon}}-h^4 e^{\frac{8}{3h}}}{h^2-\epsilon^2}+\frac{16}{9\pi}\right]. \quad (19)$$

Quark confinement for any energies is reflected in an increase of Π^R with increasing energy.

It follows from eqs.(9) and (13) that the polarization operator Π^R has no poles also, a bound states are absent.

We have considered the polarization operator in the one-loop approximation only and the finite size VF have not been taken into account. In this case, the created quark and antiquark cannot be "blanched".

Due to the mentioned analytical properties of the polarization operator (9) the usual dispersion relations is violated.

One can hopes th... ...g into consideration the finite size VF and going out of the framework of the one-loop approximation ensures the fulfilment of the dispersion relations.

The physical reason for quark confinement in this model is the presence of the field (2) with infinite correlation radius and invariant $\mathcal{E}\neq 0$ in QCD vacuum. The naive analytical continuation of the coefficient functions of the operator expansion from the under-threshold domain to the physical is not fulfilled for any energies in this case.

The non-relativistic Green function (14) enables one to estimate the characteristic distances τ_0 on which the quark and antiquark can propagate. For $E>0;\ \epsilon, h\ll 1$ the proper times, giving the leading contribution to the integral (14) are $\tau_0\sim(mE)^{1/2}/\mathcal{E}$, and the corresponding distance τ_0 is

$$\tau_0\sim\frac{E}{\mathcal{E}}. \quad (20)$$

This relation has the analogy with the well-known formula $E(\tau)=\tau/2\pi\alpha'$ ($\alpha'\approx 0.9\ \mathrm{GeV}^{-2}$ is the Regge slope).

It is interesting to consider in our model the vacuum expectation value of the Wilson loop

$$W(C)=\frac{1}{N_c}\left\langle tr\, P\exp\left(ig\oint_C dx_\mu B_\mu(x)\right)\right\rangle. \quad (21)$$

Averaging over field $G_{\mu\nu}$ with the use of (5), we obtain

$$W(C)\Big|_{SU_c(2)}=\int_0^\infty d\mathcal{H}\int_0^{\mathcal{H}} d\mathcal{E}\cdot f(\mathcal{F}_0,\mathcal{G}_0)\cdot 2\frac{\cos\left(\frac{\mathcal{E}}{2}S\right)-\cos\left(\frac{\mathcal{H}}{2}S\right)}{\left(\frac{S}{2}\right)^2\cdot(\mathcal{H}^2-\mathcal{E}^2)}, \quad (22)$$

where S is the plane contour C area. Another way of the Wilson loop averaging, which is based on the use of the factorization hypothesis /3/ for gluon operators, has been considered in Ref. /16/. In the wide class of functions f the asymptotics of expression (22) for $S \gg \frac{1}{\mathcal{E}^*}, \frac{1}{\mathcal{H}^*}$ has the "area law" form

$$W(C)\big|_{SU_c(2)} \sim e^{-\frac{\mathcal{E}^*}{2} S}, \qquad (23)$$

where $\mathcal{E}^*$ is the characteristic value $\mathcal{E}$ for the function $f(\mathcal{F}_0, \mathcal{G}_0)$.

It should be emphasized that the main non-Abelian effect in this quark confinement mechanism is the existence of the Lorentz-invariant gluon condensate with the topological charge density fluctuations ($\mathcal{G}_0 \neq 0$) in the QCD vacuum (another mechanisms of confinement have been discussed recently in Refs. /17/,/18/).

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FIGURE CAPTION

Fig.1 The dependence of the quark condensate $\langle\bar{\psi}\psi\rangle_{SU_c(2)}$, generated by the covariant constant field $G_{\mu\nu}$, on quark mass m ($\delta \equiv \frac{\varepsilon}{\varkappa}$).

Curve 1 : δ = 0.001,

curve 2 : δ = 0.01,

curve 3 : δ = 0.025,

curve 4 : δ = 0.05.

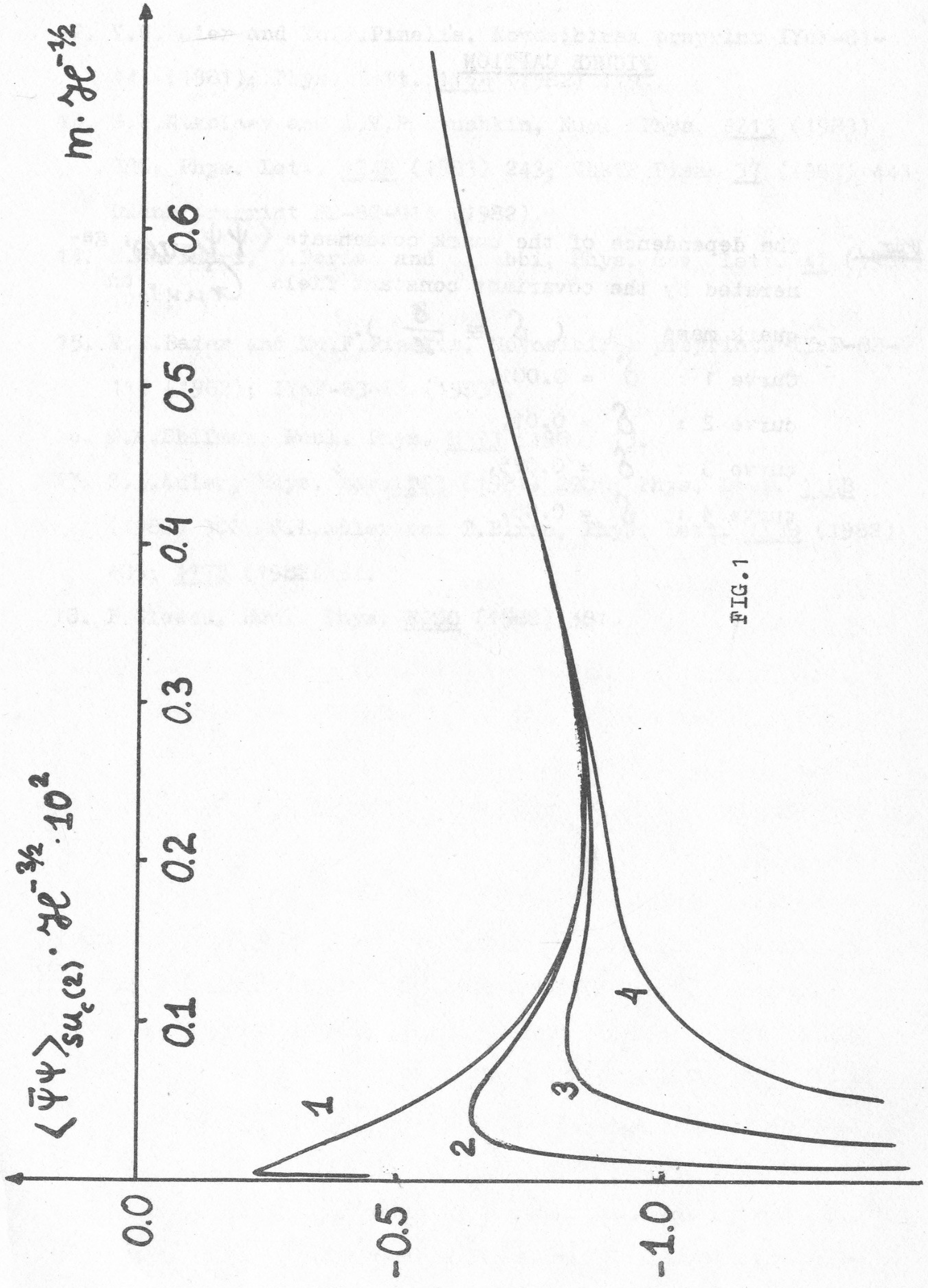

FIG.1

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