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DETERMINATION OF FIXED POINTS AND SOME CHARACTERISTICS OF NONLINEAR SYSTEMS, II

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DETERMINATION OF FIXED POINTS AND SOME CHARACTERISTICS OF NONLINEAR SYSTEMS, II

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A b s t r a c t

A many-dimensional version of the method of finding the periodic orbits in dynamical oscillator systems, which has been suggested in Ref. [1], is described. The techniques under discussion are used for a study of the sequences of period-doubling bifurcations of the particular four-dimensional Hamiltonian mappings. The process of storing such bifurcations, which goes on as one of the parameters is varied, is shown to be not subjected to the regularities revealed not long ago in the two-dimensional mappings [2,3].

1. Introduction

A lot of information about the nonlinear oscillator Hamiltonian systems has been gained from the study of the motion in the vicinity of the resonance states [4]. However, only the systems with a small number of degrees of freedom have been analysed in considerable detail. This is due, to some extent, to the difficulties encountered in a search for the many-dimensional periodic orbits. At the same time, there are important physical phenomena which can be studied and cleared up only in a many-dimensional phase space since they have no analogues at a small dimensionality. One of the examples of such a kind is the Arnold diffusion - a very fine mechanism of the universal instability of the perturbed Hamiltonian systems with the number of degrees of freedom more than two [4]. The study of this diffusion is connected with the necessity for locating the initial states inside the stochastic layer and the simplest way to do this is to search for an unstable periodic orbit belonging to this layer [5].

In recent years, the investigation of the sequences of period-doubling bifurcations has acquired especial significance (at this bifurcation a periodic orbit loses its stability and a stable orbit of twice of the period appears simultaneously). It is worth emphasizing that the studies dealing with dissipative systems have been carries out most thoroughly, since for this systems there are the effective algorithms of searching for many-dimensional periodic orbits (see, e.g.[6]). The processes of storing the bifurcations mentioned above, which proceed in the physical, chemical, biological and others systems as one parameter is varied exhibit a remarkable universality and a scale-invariant description can be made [7,8]. As to the Hamiltonian systems, similar features for them are found only in the two-dimensional mappings [2.3]. Moreover, the many-dimensional dissipative systems turned out to behave as the one-dimensional ones in such processes (if the dissipation in not too small [8]). This important circumstance enables one to study 'with legal reason' a simple motion rather than the complex one. As has been pointed out in [2], it seems expedient to find out anything similar for Hamiltonian systems.

The solution of the problems indicated above and of the others is strongly hampered because of the absence of a practical algorithm to search for periodic orbits of the many-dimensional Hamiltonian systems of general form. Indeed, the only approach to this problem, which is known to the author [9,10], makes the success only if the dynamical variables appear as simple powers in the Hamiltonian (the method is presented and discussed in the second chapter of the book by A.J.Lichtenberg and M.A.Lieberman [11]).

In what follows a many-dimensional version of the method, suggested in Ref. [1] and based on the representations on partially periodic manifolds and their intersections, is described. As an illustration (see Section 3), we consider the four-dimensional Hamiltonian mapping, for which the initial parts of the chains of period-doubling bifurcations are determined. It turns out that upon variation of one of the parameters such bifurcations are not subjected to the regularities discovered comparatively recently in the two-dimensional mappings [2,3].

2. A search for fixed points in a many-dimensional phase space by the crossing method

Let us consider an oscillator dynamical system with m degrees of freedom, which undergoes the external periodic action with period T_S. Location of this system in a phase space is determined by the vector

$$y=(y_1,y_2,\dots y_{2m})=(q_1,p_1,q_2,p_2,\dots q_m,p_m), \quad (1)$$

where $(q_1,\dots q_m)=q$ and $(p_1,\dots p_m)=p$ are the generalized coordinates and momenta.

The problem of finding a periodic orbit with period T_0 proves to be solved if one succeeds, at initial moment of time t_0, in indicating such a state of the system for which the equalities

$$y_j(t_0)=y_j(t_0+T_0) \quad (2)$$

are satisfied over all the variables $j=1,2,\dots 2m$ simultaneously. Let us refer to the trajectory passing through the point $y(t_0)$ as a partially periodic one with respect to some dynamical variable y_k if the equality (2) holds for $j=k$ with the behavior of the remaining variables being not taken into consideration. For a very broad class of dynamical systems, the set of all trajectories, partially periodic over n variables, forms the $(2m-n+1)$ -dimensional surface S_n in an extended phase space. This circumstance can be useful for the construction of a very simple, in its idea, method of finding periodic orbits. It is relatively simple to "arrive at" the most extended surface S_1. It is the thing that we do, providing the partial periodicity with respect to the variable y_1. Without loss in this quality, we go over to the surface $S_2\subset S_1$ and add the periodicity with respect to y_2, etc. Such a process looks as the motion along the chain $S_1\supset S_2\supset \dots\dots \supset S_{2m-1}\supset S_{2m}$ of the subsets of decreasing dimensionality, imbedded into each other. The periodic trajectory S_{2m} is found as a results of this motion. The 'crossing method' is described in detail in [1] for the case of a phase plane; its generalization to the 2m-dimensional phase space is given below.

Further treatment is more convenient to perform in terms of the point mappings of the phase space onto itself. Such mappings naturally appear when studying the continuous systems by means of the Poincaré cross section surface method [11]; furthermore, some problems in the nonlinear dynamics are stated, from the very beginning, as the mappings (see, e.g. [5]).

Let us assume that the time interval between the successive intersections with the surface of section and the period of orbits T_0 are both divisible by the period of driving force T_S. In this case, one or several fixed points will serve as an image of the periodic orbit and instead of the period T_0 it is more convenient to speak about the minimum number of iterations N_0, which provides the mapping of fixed point onto itself.

Let us define the operating region in the phase space as the $2m$ -dimensional hypercube $\mathcal{D}$, the one-dimensional faces (edges) of which are oriented along the coordinate axes. It is necessary to find the fixed belonging to $\mathcal{D}$ points of a certain mapping. The set of all phase points, partially periodic with period N_0 with respect to the first n variables $y_1, y_2, \dots y_n$, forms the $(2m-n)$-dimensional surface $\mathcal{P}_n$ in the phase space. It follows from this that: 1) the intersection (if it is not empty) of $\mathcal{P}_n$ with the $(n+1)$ -dimensional face of $\mathcal{D}$ is a line and we refer to it as the n -line; 2) the intersection (if it is not empty) of $\mathcal{P}_n$ with the n-dimensional face of $\mathcal{D}$ is a point and we refer to it as the n -point. With these definitions taken into account, one can say that the problem consists in searching for the $2m$-points belonging to $\mathcal{D}$. Below one of the versions of such a search is presented.

Beginning from the vertex of $\mathcal{D}$, we go along its first face $\{y_1\}$, $y_i = const$, $i = 2, 3, \dots 2m$ and search for the 1-point lying on it. Beginning from this point, we go along the 1-line (which contains this point) in the plane $\{y_1, y_2\}$, $y_i = const$, $i = 3, 4, \dots 2m$ and search for the 2-point on it. This offers the possibility of going over to the 2 -line located in the three-dimensional face $\{y_1, y_2, y_3\}$, $y_i = const$, $i = 4, 5, \dots 2m$, etc. The last stage is the motion along the $(2m-1)$-line in $\mathcal{D}$ and the finding of the unknown fixed $2m$-point on it. The major element of the search is the motion along the n -line in the $(n+1)$ -dimensional face of $\mathcal{D}$ and some details need to be explained.

Let us define the vector $f(y, N_0) = \{f_i\}$ and the matrix $G(y, N_0) = \{g_{ij}\}$:

$$f_i(y, N_0) = y_i^* - y_i , \quad (3)$$

$$g_{ij}(y, N_0) = \frac{\partial f_i}{\partial y_j}, \quad i, j = 1, 2, \dots 2m. \quad (4)$$

where y^* is the image of the point y after making N_0 iterations of the mapping. It is easy to test the equality

$$G(y, N_0) = M(y, N_0) - I , \quad (5)$$

where I is the unit matrix and $M(y, N_0)$ stands for the matrix of small divergences at the point y (see formula (12)). The accuracy of determining of the n -point is convenient to estimate by the quantity:

$$Q_n(y, N_0) = \sum_{i=1}^{n} abs[f_i(y, N_0)]. \quad (6)$$

Let $y^{(1)}$ belong to the n -line along which it is necessary to shift by one step of d long and to occupy a new position $y^{(2)}$. The accuracy of arriving at the n -line, found according to formula (6), should be not worse then the given value Q^*. One should search for the first $n+1$ components of the shift vector $\delta y^{(1)} = y^{(2)} - y^{(1)}$ (the remaining components of $\delta y^{(1)}$ are zero since the motion occurs in the $(n+1)$ -dimensional face of $\mathcal{D}$). For this purpose, we will solve the set of linear equations (here and below the summation over the indices writing twice is assumed to be made):

$$g_{i\alpha}(y^{(1)}, N_0) \cdot \delta y_\alpha^{(1)} = 0, \quad i = 1, 2, \dots n; \ \alpha = 1, 2, \dots n+1; \quad (7)$$

with further normalization $|\delta y^{(1)}| = d$. The meaning of the system (7) is simple: the motion must occur in a tangent to the n -line, i.e. orthogonally to the gradients of the first n components of f. Generally speaking the point $y^{(3)} = y^{(1)} + \delta y^{(1)}$, found in this fashion, proves aside from the n -line and the required accuracy Q^* cannot be achieved. For compensation of such a deviation the corrections $\delta y^{(3)}$ to the value of $y^{(3)}$ need to be found from the solution of the second set of equations:

$$\left. \begin{aligned} f_i(y^{(3)}, N_0) + g_{i\alpha}(y^{(3)}, N_0) \cdot \delta y_\alpha^{(3)} = 0, \\ (y_\alpha^{(3)} - y_\alpha^{(1)}) \cdot \delta y_\alpha^{(3)} = 0, \\ i = 1, 2, \dots n, \quad \alpha = 1, 2, \dots n+1. \end{aligned} \right\} \quad (8)$$

The first n equations in (8) guarantee, in linear approximation, a partial periodicity with respect to the variables $y_1, y_2, \dots y_n$, whereas the last equation makes the point to approach the n-line perpendicularly to the direction of the tangent. That remains to be done is to find out whether the fixed $(n+1)$-point lies on the section $y^{(1)} < y < y^{(2)} = y^{(3)} + \delta y^{(3)}$. To do this, it is sufficient to compare the quantities $f_{n+1}(y^{(1)}, N_0)$ and $f_{n+1}(y^{(2)}, N_0)$: if they have different signs, then the $(n+1)$-point is, as a matter of fact, on this section and can be localized with the required accuracy by means of the decreasing of the step d.

Any stage of the search can fail: when we go along the n-line within the operating region, we cannot find the fixed $(n+1)$-point. In this case, the order of introducing the dynamical variables q_i, p_i into the phase vector y (see formula (1)) has to be changed. If all possible versions of this ordering fail, there is nothing to do but to conclude that there are no fixed points of period N_0 in the operating region.

Some of the practical methods, used on the plane and described in [1] (dividing of the operation region into several adjacent subregions, remembering of the "tails" of the already passed parts of the n-lines, etc.) prove to be suited for the many-dimensional case as well. Upon motion along the n--line, observation for the behaviour of the functions f_j, $j > n$ can favor substantial acceleration of the process.

In analysing the nonlinear dynamical problems it is sometimes important to elucidate what is going on with the fixed points and with their stability as the parameters $a = (a_1, a_2, \dots)$ entering into the system, are varied. It could be permissible, after the variation of the set of parameters, to address again to a global search in the hypercube, but this problem can be settled in a simpler way.

Let us assume that, at $a = a^{(0)}$, the fixed $2m$-point is already known and its new position needs to be found at $a = a^{(1)}$ fairly close to $a^{(0)}$. Recall that the $2m$-point has been found in going along the $(2m-1)$-line and that this line itself has been uniquely determined by indicating

the set of parameters $a^{(0)}$. The new set $a^{(1)}$ also determines unequally the 'own' $(2m-1)$-line being in phase space somewhere nearly with the already found one. This enables one to consider the fixed $2m$-point (which connected with $a^{(0)}$) as the 'incorrect hit' on the $(2m-1)$-line (which connected with $a^{(1)}$) and to use the correcting algorithm (8). The transition to the new $(2m-1)$-line and going along it in the direction of decreasing the quantity $abs(f_{2m})$ enable the new $2m$-point to be found.

Further application of the above representations is more convenient to illustrate by means of a specific example.

3. Period-doubling bifurcations of the discrete Hamiltonian system of coupling oscillators

As an illustration, let us consider a time-dependent system with two degrees of freedom and with the Hamiltonian

$$H = \frac{1}{2}(p_1^2 + p_2^2) + \delta_1 \cdot \left[\frac{\alpha}{4}(x_1^4 + x_2^4) - \mu x_1 x_2 + \varepsilon x_1 \Phi(t)\right], \quad (9)$$

where δ_1 is the periodic delta-function with a period of unity. This system is a discrete analogue of two nonlinear oscillators with a linear coupling and a driving force acting upon one of them [4,5]. Let us introduce the phase vector $y = (x_1, p_1, x_2, p_2)$ and write down the mapping $y \to \bar{y}$ explicitly induced by the Hamiltonian (9):

$$\left.\begin{aligned} \bar{p}_1 &= p_1 - \alpha x_1^3 + \mu x_2 + \varepsilon \Phi(t), \\ \bar{x}_1 &= x_1 + \bar{p}_1, \\ \bar{p}_2 &= p_2 - \alpha x_2^3 + \mu x_1, \\ \bar{x}_2 &= x_2 + \bar{p}_2. \end{aligned}\right\} \quad (10)$$

It may be observed, by the way, that it is the mapping (10) which was studied in considerable detail with respect to the Arnold diffusion [5].

The matrix of small deviations per one iteration at the point y is equal to

$$M_1(y) = \begin{pmatrix} 1-3\alpha x_1^2 & 1 & \mu & 0 \\ -3\alpha x_1^2 & 1 & \mu & 0 \\ \mu & 0 & 1-3\alpha x_2^2 & 1 \\ \mu & 0 & -3\alpha x_2^2 & 1 \end{pmatrix} \qquad (11)$$

The matrix of small deviations at the same point for N_0 iterations is defined by the product of the form

$$M(y,N_0) = M_1(y(N_0-1)) \times \ldots \times M_1(y(1)) \times M_1(y(0)), \qquad (12)$$

where $y(0) \equiv y \to y(1) \to \ldots \to y(N_0-1) \to y(N_0) \equiv y^*$ is the sequence of phase points, which is formed by the mapping. We will be interested in the fixed points $y(0) = y(N_0)$ of period N_0, the stability of small oscillations around of which is determined by the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and λ_4 of the matrix $M(y,N_0)$. Recall that by virtue of the conservation of the phase space volume and the symplecticity, the eigenvalues are pairwise complex conjugated and pairwise reciprocal. Therefore, only five, qualitatively different locations of these values on the complex plane are possible among which only one location (on a unit circle) corresponds to the stable small oscillations [12]. Thus, after a stable fixed point has lost its stability, various situations can occur, but the "birth" of the twice-of-period fixed points is observed if only two eigenvalues arrive (through the eigenvalue $\lambda_1 = \lambda_2 = -1$) at the real axis, whereas the two others remain on the unit circle.

The mapping (10) contains two internal parameters α and μ. At $\mu = 0$ this mapping is, in essence, degenerated in the system with one degree of freedom $\{x_1, p_1\}$ which demonstrates the chain of period-doubling bifurcations (see Appendix A). This process confirms properly the regularities comparatively recently discovered for two-dimensional Hamiltonian mappings (see, e.g., Refs. [2,3]). In particular, the bifurcative values of α_k have the cluster point in coming nearer to which the Feigenbaum variable [7]

$$\delta_k = (\alpha_{k-2} - \alpha_{k-1})/(\alpha_{k-1} - \alpha_k), \quad k = \lg_2 N_0. \qquad (13)$$

tends to the limit $\delta_\infty = 8.721097\ldots$ [2, 3].

With $\mu = const \neq 0$ and with variation of the parameter α in the four-dimensional phase space of the system (10), the sequences of period-doubling bifurcations are also observed; the initial stages of such processes are considered by means of two examples in Appendix A for the values of $\mu = 0.001$ and $\mu = 0.002$. The results show that in varying one of the parameters entering into four-dimensional mapping (10), the latter behaves in a significantly different way than in the case of a two-dimensional mapping. It is not safe, even, to say that the chains of bifurcations, described in Appendix A for $\mu \neq 0$, 'reach the end'. For $\mu = 0.005$, for example, such a chain breaks already at $N_0 = 16$, since, instead of period-doubling bifurcation, the other bifurcation arises: all the four eigenvalues, remaining the complex quantities, leave simultaneously the unit circle (in this case the simultaneity is also the consequence of the conservation of the phase space volume and the symplecticity).

In order to obtain the numerical results containing in Appendix A, use is made of not only the above methods, but some specific features inherent in the period-doubling bifurcations. In Appendix B, a simple example of the first bifurcation of the two-dimensional mapping, corresponding to the system (10) at $\mu = 0$, is considered. It is shown that at this bifurcation a) the twice-of-period points scatter from the location of their 'birth' along the eigenvector direction; b) immediately after their 'birth' these 'daughters' and their 'mother' (the unstable point of ordinary period) are linked by the lines of partially periodicity. Both these facts are likely to be of fairly general character since they are observed for all the bifurcations given in the Appendix A in two - ($\mu = 0$) and four-dimensional ($\mu \neq 0$) phase space. The first fact sharply decreases the uncertainly in the choice of the hypercube inside of which the global search is arranged. The second enables the twice-of-period points to be found with the help of a simple motion over (any of four) the 3-line passing through an unstable point of ordinary period. True, in this case, the points which have been just born are searched for and one has to shift them over the parameter for a long time (see text of the end of section 2) until they themselves "give birth to" the following bifurcation. At large values of the period N_0, this seems, however, to be more convenient than a time-consuming global search in the hypercube.

4. Conclusion

The presented method of searching for periodic orbits (fixed points) has, in our opinion, some attractive features: 1) absence of principal limitations on the dimensionality of a space 2) independence from the character of stability of the orbit to be searched for; 3) applicability to the discrete and continuous, Hamiltonian and dissipative systems of general form. As mentioned earlier, there are some proper computational methods for the latter, but they use most often the Newton's iterative procedure (see, e.g. Ref. [6]) and their success is dependent on the availability of a "good" initial approximation. If there is no such an approximation, one should make use of the crossing method which is capable of starting the search 'from a distance'.

This method, however, needs to be improved, especially in the case of a large number of degrees of freedom m and of large magnitudes of the period N_0. Some of the partially periodic points, found of the beginning of the search, are then excluded because their connection with the periodic orbit of the required period N_0 is not confirmed. The number of such points strongly increases with increasing m and N_0 and it is desirable to have a criterion for their rapid recognition. In addition, it may be well to be able to quickly find out whether there is the object to be searched for in the operating region.

As has been indicated in the Introduction, the process of storing the period-doubling bifurcations, as one of the parameters in many-dimensional and in one-dimensional dissipative systems is varied, proceeds, in essence, in the same way [8]. The results of section 3 permit one to assume that

this is not, apparently, the case for Hamiltonian systems. It is likely that with a simultaneous change of both internal parameters (α and μ) the Hamiltonian mapping (10) can demonstrate a sequence of bifurcations similar to that in two-dimensional mappings [2,3], but this problem is need in a special analysis.

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APPENDIX A

Bifurcative values of the parameter α of the system (10) at $\varepsilon\Phi(t)\equiv 1$ and $\mu = 0.$; 0.001 and 0.002.

These values are listed in the Table. In considering the latter, it is necessary to bear in mind the following:

1. The Table presents the values of α at which the fixed point of period N_0 loses its stability and the fixed points of period $2N_0$ are born, i.e. the period-doubling bifurcation occurs.

2. At such a bifurcation on the complex plane, two eigenvalues of the matrix of small oscillations shift, through the point $\lambda_1 = \lambda_2 = -1$, to the real axis, while the two others remain on the unit circle. The value of the parameter α is regarded as a bifurcative one if the smallest of two real eigenvalues is within the interval $-1.001 < \lambda_{min} < -1.000$.

3. The fixed point is assumed to be a localized one if the accuracy of mapping it onto itself, calculated according to formula (6), is not worse than 10^{-15}.

4. The Feigenbaum variable δ has been found by means of formula (13).

Table

$\mu = 0$

$N_0 = 2^k$	α_k	δ_k
1	2.370370370370370	-
2	3.372727748379688	-
4	3.509168261718750	7.346479
8	3.525084649658203	8.572329
16	3.526914030761719	8.700422
32	3.527123843383789	8.719118
64	3.527147902197266	8.720821
128	3.527150660902405	8.721051
256	3.527150977227295	8.721113
512	3.527151013498535	8.721094

$\mu = 0.001$

$N_0 = 2^k$	α_k	δ_k
1	2.370047607421875	-
2	3.372343750000000	-
4	3.508776214599609	7.346463
8	3.524691786193848	8.572263
16	3.526520958709717	8.700969
32	3.526730719604492	8.720274
64	3.526754785897827	8.715962
128	3.526757544002897	8.725662
256	3.526757859548889	8.740738
512	3.526757895440722	8.791582

Table

$\mu = 0.002$

$N_0 = 2^k$	α_k	δ_k
1	2.369556250000000	-
2	3.371760351562500	-
4	3.508180048007812	7.346477
8	3.524094434925781	8.572099
16	3.525923831467041	8.699254
32	3.526133731735596	8.715551
64	3.526157729800781	8.746550
128	3.526160459800781	8.790500
256	3.526160759374258	9.112956

APPENDIX B

The first period-doubling bifurcation of the mapping (10) at $\Phi(t) \equiv 1$ and $\mu = 0$

At $\mu = 0$ it suffices to study a two-dimensional mapping (in what follows we use simple notations x and p instead of x_1 and p_1):

$$\left.\begin{aligned} \bar{p} &= p - \alpha x^3 + \varepsilon , \\ \bar{x} &= x + \bar{p} . \end{aligned}\right\} \qquad (B.1)$$

The coordinates of the fixed point of period $N_0 = 1$ on the $\{x, p\}$ plane and the eigenvalues $\lambda_1 = \lambda, \lambda_2 = \lambda^{-1}$ of the matrix of small deviations from this point are equal to

$$x_{01} = \left(\frac{\varepsilon}{\alpha}\right)^{1/3}, \quad p_{01} = 0 . \qquad (B.2)$$

$$\lambda = 1 + \frac{3}{2}\alpha x_{01}^2 \cdot \left(\sqrt{1 - \frac{4}{3\alpha x_{01}^2}} - 1\right). \qquad \text{(B.3)}$$

Bifurcation occurs at the critical value of $\lambda = \lambda_{CR} = -1$ which the following values of the parameter α and of the coordinates correspond to:

$$\alpha_{CR} = \left(\frac{4}{3}\right)^3 \cdot \varepsilon^{-2}, \qquad \text{(B.4)}$$

$$x_{CR} = \frac{3}{4}\varepsilon, \quad p_{CR} = 0. \qquad \text{(B.5)}$$

At $\lambda = \lambda_{CR}$ the eigenvector is directed along a line given by the equation

$$p - p_{CR} = \frac{3}{2}\alpha_{CR} x_{CR}^2 \cdot (x - x_{CR}) = 2 \cdot (x - x_{CR}). \qquad \text{(B.6)}$$

The coordinates of the twice-of-period $N_n = 2$ fixed points can be found, for any $\alpha > \alpha_{CR}$, from the relation

$$x_{02} = p_{02} + \left(\frac{\varepsilon - 2p_{02}}{\alpha}\right)^{1/3} = \left(\frac{2p_{02} + \varepsilon}{\alpha}\right)^{1/3}. \qquad \text{(B.7)}$$

If we restrict our consideration to the condition $\delta\alpha = \alpha - \alpha_{CR} \ll 1$, we have

$$\delta p_{02} = p_{02} - p_{CR} = \pm \frac{9}{4\sqrt{5}} \varepsilon^{5/6} \cdot \sqrt{\alpha^{1/3} - \alpha_{CR}^{1/3}} + \mathcal{O}(\delta p_{02}^5) = \qquad \text{(B.8)}$$

$$= \pm \frac{3}{4}\sqrt{\frac{3}{5}} \cdot \varepsilon^{5/6} \cdot \alpha_{CR}^{-1/6} \cdot \left(\frac{\delta\alpha}{\alpha_{CR}}\right)^{1/3} + \mathcal{O}\left[\left(\frac{\delta\alpha}{\alpha_{CR}}\right)^{3/2}\right].$$

$$\delta x_{02} = x_{02} - x_{CR} = \frac{2}{3} \cdot \frac{1}{\alpha_{CR} x_{CR}^2} \cdot \delta p_{02} = \frac{1}{2}\delta p_{02}. \qquad \text{(B.9)}$$

It is worth emphasizing two facts:

a) Comparing the equalities (B.9) and (B.6), we see that just after their 'birth', the twice-of-period points (x_{02}, $\pm p_{02}$) are scattered from the point, which has given born to them (x_{CR}, p_{CR}) along the line containing the critical eigenvector.

b) Let us fix any, close to α_{CR}, value of the parameter $\alpha > \alpha_{CR}$, find the coordinates (x_{01}, p_{01}) of the ordinary-period unstable point by means of (B,1) and construct the x- (periodicity over x only) and p-lines (periodicity over p only) of period $N_0 = 2$, which pass through this point. It turns out that these lines also pass through both 'daughter' points (x_{02}, p_{02}) of twice of the period.

The facts mentioned above are observed in all the successive bifurcations of the mapping (B.1) (see Appendix A and the discussion in section 3).

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ОПРЕДЕЛЕНИЕ НЕПОДВИЖНЫХ ТОЧЕК И СВЯЗАННЫХ С НИМИ ХАРАКТЕРИСТИК НЕЛИНЕЙНЫХ СИСТЕМ. II.

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