16

И Н С Т И Т У Т
ЯДЕРНОЙ ФИЗИКИ СОАН СССР

ПРЕПРИНТ И Я Ф 77 - 21

D.D. Ryutov, Toshio Tange

STATISTICAL ACCELERATION OF IONS BY SUCCESSIVE CHARGE-EXCHANGE PROCESSES

Новосибирск
1977

STATISTICAL ACCELERATION OF IONS BY SUCCESSIVE CHARGE-EXCHANGE PROCESSES

D.D. Ryutov*), Toshio Tange**)

Abstract

It is shown that in an incompletely ionized plasma a longitudinel electric field perpendicular to the magnetic field can produce high-energy ions via successive acts of neutralization and ionization through resonant charge-transfer processes.

*) Nuclear Physics Institute, Novosibirsk, USSR
**) Hiroshima University, Hiroshima, Japan

In this paper, we consider the behavior of ions in a system where ionization is incomplete, and neutral and ionized particles coexist. We will show that if in such a system there exists an electric field perpendicular to the magnetic field, then a quite general mechanism of accelerating heavy particles works connected with successive acts of ionization and neutralization of heavy particles.

Besides general physical interests, this mechanism could to some extent be responsible for the formation of ion high-energy tails in the experiments where strong radial electric field exists in a plasma column. One such example is the discharge across a magnetic field between two coaxial electrodes (e.g. of homopolar type [1]). Other examples are some turbulent heating experiments where the electron temperature T_e is relatively high and a radial potential difference of the order of T_e / e is formed due to the ambipolar electric field.

Let us first consider a simple model where a constant homogeneous magnetic field B is directed along the z-axis and a constant electric field E, which is in the x-direction, exists only in the half-space x 0, the other half-space (region II) being free of the electric field (as we shall show below, at $\vec{B}(\vec{r})$ = const., our heating mechanism works only in the case when the electric field is inhomogeneous, this being the reason for the assumption made). We then consider the following cycle of particle motion (see, Fig.1); namely, an ionized particle initially gyrating in region II is neutralized and moves to region I following a straight orbit; then gets ionized and starts a gyrating drift motion in the y-direction; gets neutralized again and moves back to region II and finally becomes reionized. For ionization and neutralization, we consider resonant charge-transfer processes whose cross sections are much greater than those of the ordinary pair collisions. We can therefore neglect the latter throughout the entire cycle. Also, the momentum exchange associated with the electron transfer between two heavy particles can be ignored. Then in the cycle, the magnitude of the particle velocity can change only via gyrating drift motion in region I. The change in the speed between the initial and the final states of the cycle depends on the phase difference between the starting and the terminating points of gyration in region I, and takes a value from -2cE/B to 2cE/B. An example of gaining the speed is shown in Fig. 1. Those particles which repeatedly experience these cycles can then be statistically accelerated to velocities well above the drift velocity cE/B. It is these particles which constitute the high-energy tail in the present model. We note that this statistical acceleration mechanism works only when the particle makes repeated excursions to both regions. If the particle stays in region I, say, it feels a constant electric field which can be transformed away by going to the drifting frame, whence the particle energy is conserved in this frame. Alternatively stated, the particle staying in region I keeps its phase memory even after many neutralization and ionization events and hence can never be accelerated by an amount greater than 2cE/B. Excursion to region II makes the particle lose its phase memory, and therefore the process becomes stochastic.

For a quantitative analysis, we consider a more realistic situation in which the electric field (directed in the x-direction) is a function of x with the characteristic gradient length much greater than the Larmour radius. The velocity representing the gyrating drift motion can then be written as

$$v_x(t) = a\sin(\theta - \Omega t), \tag{1}$$
$$v_y(t) = -a\cos(\theta - \Omega t) - cE(x)/B,$$

where a and θ are constants determined by the initial condition and Ω is the cyclotron frequency. The time averaged velocity of this motion over one gyrating period can be written as

$$\bar{v} = \int_0^{2\pi} \frac{d\theta}{2\pi}\left[a^2 + \left(\frac{cE}{B}\right)^2 + 2\,\frac{cE}{B}\,a\cos\theta\right]^{1/2}. \tag{2}$$

We shall be interested in the particles in the high-energy tail. For these particles, a is much greater than cE/B, so that we have

$$\bar{v} \simeq a + O(a[cE/Ba]^2)$$

Consider a particle which is neutralized at time t_1 and at position x_1 and then reionized at time t_2 and at position x_2. Let $\vec{v}(t)$ and $\vec{v}'(t)$ denote the velocities of gyrating drift motions before t_1 and after t_2, respectively. Since the neutral particle moves with a constant velocity, we must have $\vec{v}(t_1) = \vec{v}'(t_2)$, from which we can calculate $\Delta v \equiv \bar{v}' - \bar{v}$ as

$$\Delta v = \frac{c\Delta E}{B}\left[-\cos(\theta - \Omega t_1) + \frac{\varepsilon}{2} + \frac{\varepsilon^2}{4}\cos^2(\theta - \Omega t_1)\right] + a\left(\frac{c\Delta E}{aB}\right)^2\left\{\frac{1}{4} + \sin^2(\theta - \Omega t_1) + \frac{\varepsilon}{2}\cos(\theta - \Omega t_1) + \frac{\varepsilon^2}{4}\left[1 + \frac{3}{2}\varepsilon^2\cos^2(\theta - \Omega t_1)\right]\right\} + o\left[\left(\frac{c\Delta E}{aB}\right)^2\right], \tag{3}$$

$$\Delta E \equiv E(x_2) - E(x_1), \quad \varepsilon \equiv cE/aB.$$

It is small as compared with $\bar{v}$, so we can use the Fokker-Planck equation for the fast-ion distribution function, $f(v)$ (the particle number between v and $v+dv$ is proportional to $f(v)dv$), i.e.

$$\frac{\partial}{\partial t} f(v) = -\frac{\partial}{\partial v}\left[f(v)\frac{\langle \Delta v\rangle}{\Delta t}\right] + \frac{1}{2}\frac{\partial^2}{\partial v^2}\left[f(v)\frac{\langle (\Delta v)^2\rangle}{\Delta t}\right] \tag{4}$$

where the angular bracket denotes the average over the initial phase θ and Δt is the time step for each successive event of neutralization and ionization. One can choose Δt as the sum of the mean life times, τ_i and τ_n for the ion and the neutral particle, respectively, against the charge-transfer process, i.e. $\Delta t = \tau_i + \tau_n$. Using relation (3), one can then calculate the coefficients of the Fokker-Planck equation.

For simplicity, we consider the case when dE/dx is almost constant over the distance of the order of $\tau_n \bar{v}$. Then we can approximate ΔE by $(x_2 - x_1)\,dE/dx \simeq \bar{v}\sin(\theta - \Omega t_1)\tau_n\,dE/dx$.

We then find

$$\frac{\langle \Delta v \rangle}{\Delta t} \simeq \frac{\bar{v}}{4(\tau_i + \tau_n)} \left[\frac{c\tau_n}{B} \frac{dE}{dx} \right]^2$$

$$\frac{\langle \Delta v^2 \rangle}{\Delta t} \simeq \frac{1}{8(\tau_i + \tau_n)} \left[\frac{c\bar{v}\tau_n}{B} \frac{dE}{dx} \right]^2$$

For $\bar{V} \gg cE/B$, the second moment gives a contribution much greater than the first moment. If one retains only the second moment, eq. (4) gives a result which describes an indefinite acceleration of the particles. The acceleration will, however, be terminated by particle losses, say, by the escape from the plasma. Let L be the effective escape length of the particle; then including also the source $S(v)$ due to ionization, we can write eq. (4) as

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{FP} - \frac{v}{L} f(v) + S(v)$$

where $(\partial f / \partial t)_{FP}$ stands for the right-hand side of eq.(4). At stationary state, the high-energy tail of the distribution is determined by the balance of the last term of eq.(4) with the loss term in eq. (5) (since $S(v)$ is nonzero only at small energies). Noting that the mean free paths, λ_i and λ_n, for neutralization and ionization by charge exchange are almost independent of the energy in the high-energy region [2], one finally obtains

$$f \propto v^{-1} \exp[-v/v_0]$$

where $v_0 = [L/(\lambda_i + \lambda_n)]^{1/2} (c\lambda_n/4B) |dE/dx|$. This quantity substantially exceeds the drift velocity cE/B when L is much greater than $(\lambda_i + \lambda_n)$.

The authors wish to thank Prof. Kyoji Nishikawa for helpful discussions and critical reading of the manuscript.

This work was partially supported by the Scientific Research Fund of the Ministry of Education of Japan.

References

1 D.A. Baker, J.E. Hammel and F.L. Ribe, Phys. of Fluids 4 (1961) 1534.

2 E.W. McDaniel, Collision phenomena in ionized gases, (Wiley, New York, 1964) Chap. 6.

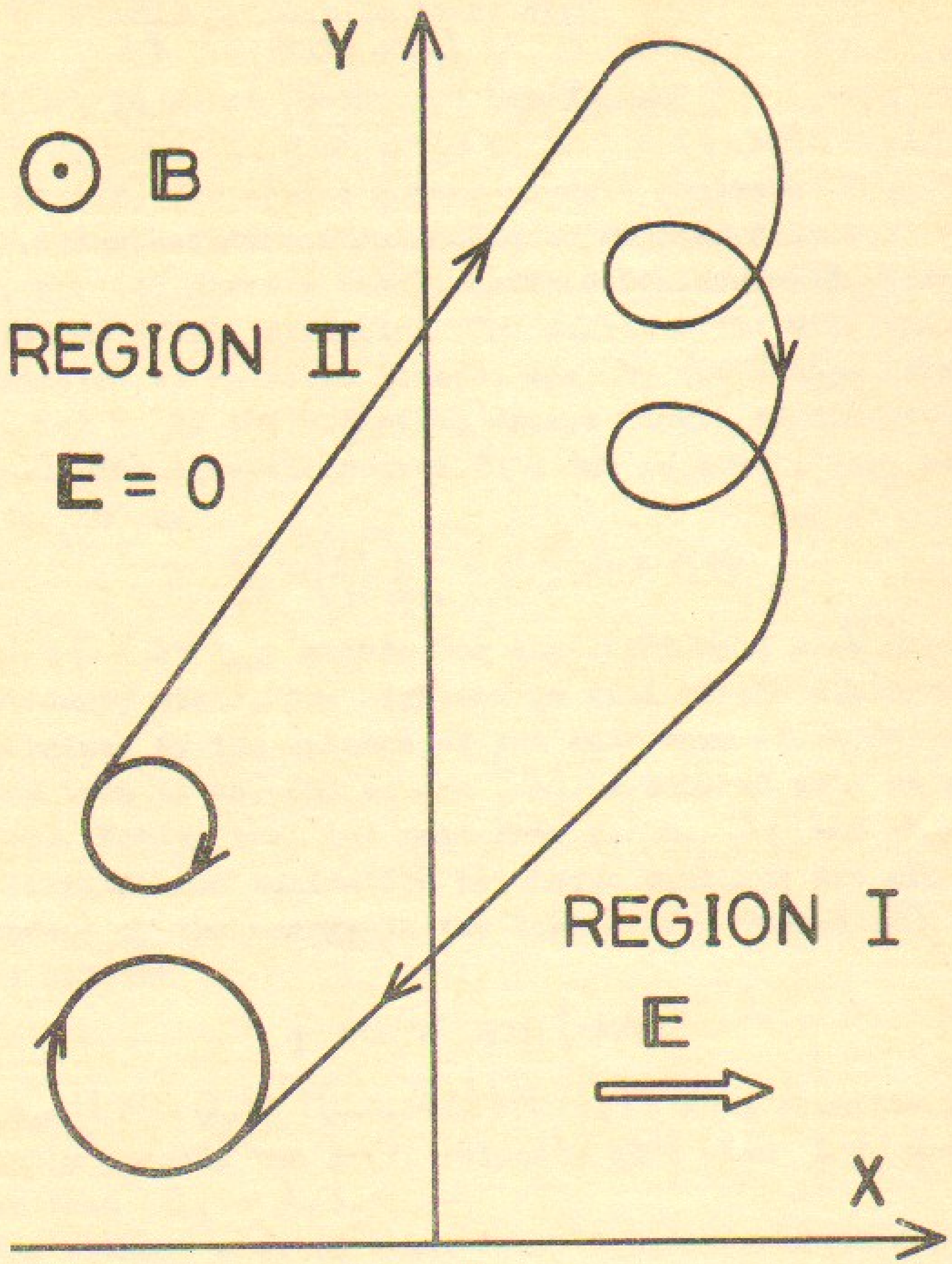

FIG. I

Работа поступила - 28 декабря 1976 г.

Ответственный за выпуск - С.Г.ПОПОВ
Подписано к печати I7.II-1977 г. МН 0266I
Усл. 0,5 печ.л., 0,4 учетно-изд.л.
Тираж 200 экз. Бесплатно
Заказ № 2I.

Отпечатано на ротапринте ИЯФ СО АН СССР