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Abstract

The contribution to the kernel of the non-forward BFKL equation from the two-gluon production is calculated for the case of the antisymmetric colour octet state of the two Reggeized gluons in the t -channel. The one-gluon contribution to the kernel in the one-loop approximationb is also obtained using the one-loop expression for the effective vertex of the one-gluon production in the Reggen-Reggeon collisions. The explicit form of the total kernel is presented.

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1 Introduction

The most common basis for the description of processes at small values of $x = Q^2/s$ (Q^2 is a typical virtuality and $\sqrt{s}$ is the c.m.s. energy) in the framework of the perturbative QCD is the BFKL equation [1], originally derived in the leading logarithmic approximation (LLA), which means resummation of all terms of the type $[\alpha_s \ln s]^n$ ($\alpha_s = g^2/(4\pi)$ is the QCD coupling constant). The calculation of the radiative corrections to the kernel of this equation has taken many years of a hard work [2]-[7]. Recently, the kernel was obtained in the next-to-leading approximation (NLA) [8] for the case of the forward scattering, i.e. for the momentum transfer $t = 0$ and the vacuum quantum numbers in the $t-$channel. In the $\overline{MS}$ renormalization scheme with a reasonable scale setting the corrections appear to be large. Now this problem is widely discussed in literature (see, for instance [9]). In this situation it is very important to be sure in correctness of both the basic hypothesys used and the calculations performed in the derivation of the equation.

Remind, that the derivation of the BFKL equation (in the NLA as well as in the LLA) is based on one of the remarkable properties of QCD - the gluon Reggeization [10], which was proved in the LLA [1],[11]. In the NLA this property was only checked in the first three orders of the perturbation theory [6]. Since the gluon Reggeization forms a basis of the derivation of the BFKL equation, it is clear, that more powerful tests are necessary.

As for the calculations of the radiative corrections to the kernel, they are very complicated and up to now only a part of them was independently performed [7] or checked [12]. Therefore, the calculations must be carefully verified.

The both goals - the stringent test of the gluon Reggeization and the examination of the calculations - can be solved simultaneously by check of the "bootstrap" equations [13],[14] appearing as the requirement of the compatibility of the gluon Reggeization with the s-channel unitarity. In fact, the BFKL equation is the equation for the Green's function of two Reggeized gluons. In the colour singlet state these Reggeized gluons create the Pomeron. The self-consistency requires that in the antisymmetric colour octet state the two Reggeized gluons must reproduce the Reggeized gluon itself ("bootstrap" condition). The above statements are valid in the NLA as well as in the LLA. Along with the stringent test of the gluon Reggeization, the check of the bootstrap equations provides a global test of the calculations of the NLA kernel, because these equations contain almost all the values appearing in the calculations.

In the BFKL approach amplitudes of high energy processes are expressed in terms of the above mentioned Green's function and impact factors of scattered particles, which are defined by Reggeon-particle scattering amplitudes. The non-forward impact factors for gluon [15] and quark [16] scattering were recently calculated in the NLA and the fulfillment of the bootstrap conditions for them was demonstrated [15], [16],[17] for both helicity conserving and non-concerving parts, in an arbitrary space-time dimension $D = 4 + 2\epsilon$.

The quark contribution to the non-forward BFKL kernel was also calculated [18] and the fulfillment of the bootstrap conditions for the kernel in the part concerning this contribution was explicitly demonstrated in the NLA [18],[19], also in an arbitrary space-time dimension. The only one (but most complicated) bootstrap condition remains unchecked - for the gluon part of the kernel. In this paper we calculate the gluon contribution to the non-forward colour octet kernel of the BFKL equation, having in mind subsequent examination of the bootstrap condition.

A significance of the the non-forward octet kernel is not limited by the check of the bootstrap condition. The kernel of the non-forward BFKL equation for an arbitrary colour state in the t -channel is expressed in terms of the gluon Regge trajectory and the part related to the real particle production in the Reggeon-Reggeon scattering ("real" part, for brevity). The trajectory is known [4] and enters into the kernel in the universal (not depending on a colour state) way [13]. The "real" part includes contributions from the one-gluon, two-gluon and quark-antiquark pair production. The first contribution for any colour state in the t -channel can differ from the contribution for the octet state only by a group coefficient. The last two contributions can be separated (for an arbitrary colour state in the t- channel) in two pieces, one of which is determined by the colour octet state. Therefore, the colour octet piece enters (with some group coefficient) in kernels for other colour states, in particular, for the colour singlet state (Pomeron channel). In the Pomeron channel the non-forward BFKL equation can be applied directly for the description of experimental data. Evidently, a region of applicability of this equation is mich wider than the forward-case one.

In the next Section we present the general form of the gluon contribution to the kernel and the explicit form of the gluon piece of the gluon trajectory. In Section 3 we derive the gluon part of the contribution to the kernel from the one-gluon production. In Section 4 we consider the two-gluon production in collisions of the two Reggeized gluons. The contribution of this process to the kernel and the result for the total gluon contribution to the kernel are presented in Sections 5 and 6 respectively.

2 Definitions and basic equations

In the BFKL aproach the high energy scattering amplitudes are expressed in terms of the impact factors Φ of the scattering particles and of the Green's function G for the scattering of Reggeized gluons [13]. Considering the Green's function we can take, without any loss of generality, masses of the colliding particles with momenta p_A and p_B equal zero: $p_A^2 = p_B^2 = 0, \quad (p_A + p_B)^2 = 2(p_A p_B) = s$. As usual in an analysis of high energy processes, we apply the Sudakov decomposition for particle momenta. The Mellin transform of the Green's function with the initial momenta of the Reggeized gluons in the $s-$channel $q_1 \simeq \beta p_A + q_{1\perp}$ and $-q_2 \simeq \alpha p_B - q_{2\perp}$ and the momentum transfer $q \simeq q_\perp$ obeys the equation [13]:

$$\omega G_\omega^{(\mathcal{R})} \left(\vec{q}_1, \vec{q}_2, \vec{q}\right) =$$

$$\vec{q}_1^{\,2} \vec{q}_1^{\,\prime\, 2} \delta^{(D-2)} \left(\vec{q}_1 - \vec{q}_2\right) + \int \frac{d^{D-2}r}{\vec{r}^{\,2} \vec{r}^{\,\prime\, 2}} \mathcal{K}^{(\mathcal{R})} \left(\vec{q}_1, \vec{r}; \vec{q}\right) G_\omega^{(\mathcal{R})} \left(\vec{r}, \vec{q}_2; \vec{q}\right) \ , \qquad (1)$$

where $\mathcal{R}$ denotes the representation of the colour group in the $t-$channel. The transverse momenta are spacelike and we use the vector sign for them. Here and below we use for brevity $v' \equiv v - q$ for any v. The space-time dimension $D = 4 + 2\epsilon$ is taken different from 4 to regularize the infrared divergences. We use the normalization adopted in [13].

The non-forward kernel, as well as the forward one, is given by the sum of the "virtual" part, defined by the gluon trajectory $j(t) = 1 + \omega\,(t)$, and the "real" part $\mathcal{K}_r^{(\mathcal{R})}$, related to the real particle production in the Reggeon-Reggeon collisions

$$\mathcal{K}^{(\mathcal{R})} \left(\vec{q}_1, \vec{q}_2; \vec{q}\right) =$$

$$\left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-\vec{q}_1^{\,\prime\, 2}\right)\right] \vec{q}_1^{\,2} \vec{q}_1^{\,\prime\, 2} \delta^{(D-2)} \left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r^{(\mathcal{R})} \left(\vec{q}_1, \vec{q}_2; \vec{q}\right) \ . \qquad (2)$$

As it is seen from (2), the gluon trajectory enters the equation in the universal (independent from $\mathcal{R}$) way. In the one-loop approximation (LLA) the trajectory is purely gluonic:

$$\omega^{(1)}(t) = \frac{g^2 N t}{2\,(2\pi)^{D-1}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2} \vec{q}_1^{\,\prime\,2}}\,, \tag{3}$$

where $t = q^2 = -\vec{q}^{\,2}$, N is the number of colours ($N = 3$ in QCD). In the NLA the trajectory was calculated in [4]. Since the quark contribution to the non-forward kernel was already considered [18], we present here the two-loop gluon contribution:

$$\omega_G^{(2)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \int \frac{d^{(D-2)} q_1}{\vec{q}_1^{\,2} \vec{q}_1^{\,\prime\,2}} \left[F_G(\vec{q}_1, \vec{q}) - 2 F_G(\vec{q}_1, \vec{q}_1) \right]\;, \tag{4}$$

where

$$F_G(\vec{q}_1, \vec{q}) = -\frac{g^2 N^2 \vec{q}^{\,2}}{4(2\pi)^{D-1}} \int \frac{d^{(D-2)} q_2}{\vec{q}_2^{\,2} (\vec{q}_2 - \vec{q})^2}$$
$$\times \left[\ln\left(\frac{\vec{q}^{\,2}}{(\vec{q}_1 - \vec{q}_2)^2} \right) - 2\psi(D-3) - \psi\left(3 - \frac{D}{2}\right) + 2\psi\left(\frac{D}{2} - 2\right) + \psi(1) \right.$$
$$\left. + \frac{2}{(D-3)(D-4)} + \frac{D-2}{4(D-1)(D-3)} \right]\;;\;\; \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}\;, \tag{5}$$

$\Gamma(x)$ is the Euler gamma-function. In Eqs. (3)-(5) and below g is the bare coupling constant related to the renormalized coupling g_μ in the $\overline{MS}$ scheme by the relation

$$g = g_\mu \mu^{-\epsilon} \left[1 + \left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N} \right) \frac{\bar{g}_\mu^2}{2\epsilon} \right]\;, \tag{6}$$

where n_f is the number of the quark flavours,

$$\bar{g}_\mu^2 = \frac{g_\mu^2 N \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\;. \tag{7}$$

Let us stress that in this paper we will systematically use the perturbative expansion in terms of the bare coupling g.

The "real" part for the non-forward case in the LLA is [1], [21]:

$$\mathcal{K}_r^{(\mathcal{R})B}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{g^2 c_R}{(2\pi)^{D-1}} \left(\frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\,2} + \vec{q}_2^{\,2} \vec{q}_1^{\,\prime\,2}}{(\vec{q}_1 - \vec{q}_2)^2} - \vec{q}^{\,2} \right)\;, \tag{8}$$

where the superscript B means the LLA (Born) approximation and the colour group coefficients c_R for the singlet ($R = 1$) and octet ($R = 8$) cases are

$$c_1 = N,\;\; c_8 = \frac{N}{2}\;. \tag{9}$$

In the NLA the "real" part of the kernel can be presented as [13]:

$$\mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\,\right)=\int_0^\infty\frac{ds_{_{RR}}}{(2\pi)^D}\mathcal{I}\,m\mathcal{A}_{RR}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\,\right)\theta\left(s_{_\Lambda}-s_{_{RR}}\right)$$
$$-\frac{1}{2}\int\frac{d^{D-2}r}{\vec{r}^{\,2}\vec{r}^{\,\prime\,2}}\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1,\vec{r};\vec{q}\,\right)\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{r},\,\vec{q}_2;\,\vec{q}\,\right)\ln\left(\frac{s_{_\Lambda}^2}{(\vec{r}-\vec{q}_1)^2(\vec{r}-\vec{q}_2)^2}\right)\ . \tag{10}$$

Here $\mathcal{A}_{RR}^{(\mathcal{R})}\left(q_1,q_2;q\right)$ is the scattering amplitude of the Reggeons with the initial momenta $q_1\ =\ \beta p_A+q_{1\perp}$ and $-q_2\ =\ \alpha p_B\ -\ q_{2\perp}$ at the momentum transfer $q\ =\ q_\perp$ and the representation $\mathcal{R}$ of the colour group in the $t-$channel, $s_{_{RR}}=\left(q_1-q_2\right)^2=s\alpha\beta-(\vec{q}_1-\vec{q}_2)^2$ is the squared invariant mass of the Reggeons. The $s_{_{RR}}-$channel imaginary part $\mathcal{I}m\mathcal{A}_{RR}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)$ is expressed in terms of the effective vertices for the production of particles in the Reggeon-Reggeon collisions [13]. The second term in the r.h.s. of Eq. (10) serves for the subtraction of the contribution of the large $s_{_{RR}}$ region in the first term, in order to avoid a double counting of this region in the BFKL equation. The intermediate parameter $s_{_\Lambda}$ in Eq. (10) must be taken tending to infinity. At large $s_{_{RR}}$ only the contribution of the two-gluon production does survive in the first integral, so that the dependence on $s_{_\Lambda}$ disappears in Eq. (10) due to the factorization property of the two-gluon production vertex [13].

The remarkable properties of the kernel are

$$\mathcal{K}_r^{(\mathcal{R})}(0,\vec{q}_2;\vec{q}\,)=\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,0;\vec{q}\,)=\mathcal{K}_r^{(\mathcal{R})}(\vec{q},\vec{q}_2;\vec{q}\,)=\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q};\vec{q}\,)=0\ , \tag{11}$$

and

$$\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2;\vec{q}\,)=\mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1^{\,\prime},\vec{q}_2^{\,\prime};-\vec{q}\,)=\mathcal{K}_r^{(\mathcal{R})}(-\vec{q}_2,-\vec{q}_1;-\vec{q}\,)\ . \tag{12}$$

The properties (11) mean that the kernel turns into zero at zero transverse momenta of the Reggeons and follow from the gauge invariance; Eqs.(11) are the consequences of the symmetry of the imaginary part of the Reggeon-Reggeon scattering amplitude (13).

Using the operator $\hat{\mathcal{P}}_\mathcal{R}$ for the projection of the two-gluon colour states in the $t-$channel on the irreducible representation $\mathcal{R}$ of the colour group we can present the imaginary part of the Reggeon-Reggeon scattering amplitude entering Eq. (10) in the form

$$\mathcal{I}m\mathcal{A}_{RR}^{(\mathcal{R})}\left(q_1,q_2;\vec{q}\right)=\frac{\langle c_1c_1^{\prime}|\hat{\mathcal{P}}_\mathcal{R}|c_2c_2^{\prime}\rangle}{2n_\mathcal{R}}\sum_{\{f\}}\int\gamma_{c_1c_2}^{\{f\}}\left(q_1,q_2\right)\left(\gamma_{c_1^{\prime}c_2^{\prime}}^{\{f\}}\left(q_1^{\prime},q_2^{\prime}\right)\right)^*d\rho_f\ \ . \tag{13}$$

Here $n_{\mathcal{R}}$ is the number of independent states in the representation $\mathcal{R}$, the sum $\{f\}$ is performed over all states f which are produced in the Reggeon-Reggeon collisions and over all discreet quantum numbers of these states, $\gamma_{c_1c_2}^{\{f\}}(q_1, q_2)$ is the effective vertex for the production of the state f and $d\rho_f$ is the phase space element for this state,

$$d\rho_f = \frac{1}{n!}\,(2\pi)^D\,\delta^{(D)}(q_1 - q_2 - \sum_{i\in f} k_i)\prod_{i\in f}\frac{d^{D-1}k_i}{(2\pi)^{D-1}\,2\epsilon_i} \quad , \tag{14}$$

where n is a number of identical particles in the state f . In the LLA only the one-gluon production does contribute in (13) and Eq. (10) gives for the kernel its LLA value (8); in the NLA the contributing states include also the two-gluon and the quark-antiquark states. The normalization of the corresponding vertices is defined in Ref. [13].

The most interesting representations $\mathcal{R}$ are the colour singlet (Pomeron channel) and the antisymmetric colour octet (gluon channel). We have for the singlet case

$$\langle c_1c_1'|\hat{\mathcal{P}}_1|c_2c_2'\rangle = \frac{\delta_{c_1c_1'}\delta_{c_2c_2'}}{N^2-1} \ , \quad n_1 = 1 \ \ , \tag{15}$$

and for the octet case

$$\langle c_1c_1'|\hat{\mathcal{P}}_8|c_2c_2'\rangle = \frac{f_{c_1c_1'c}f_{c_2c_2'c}}{N} \ , \quad n_8 = N^2-1 \ , \tag{16}$$

where f_{abc} are the structure constants of the colour group.

3 The one-gluon contribution to the kernel

The gluon contribution to the Reggeon-Reggeon-gluon (RRG) vertex was calculated in [3]. Remind that the complicated analitical structure [20, 3] of the vertex is irrelevant in the NLA where only the real parts of the production amplitudes do contribute (only these parts interfere with the LLA amplitudes, which are real). Remind also that in the NLA the vertex depends on the energy scale s_R used in the Regge factors. In Eqs. (10),(13) it was assumed that $s_R = \vec{k}^{\,2}$, where k is the produced gluon momentum. Neglecting the imaginary part, we have for the gluon contribution to the RRG vertex with this choice of s_R [21]

$$\gamma_{c_1c_2}^{G}(q_1, q_2) = gT^d_{c_1c_2}e^*_\mu(k)\left\{C^\mu(q_2, q_1)\left[1 + \frac{2g^2N\Gamma\,(1-\epsilon)}{(4\pi)^{2+\epsilon}}\left(f_1^{(G)} + f_2^{(G)}\right)\right]\right.$$

$$+\left(\frac{p_A}{(kp_A)}-\frac{p_B}{(kp_B)}\right)_\mu \frac{g^2 N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}[f_3^{(G)}-(2\vec{k}^2-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})f_2^{(G)}]\Bigg\} \ . \quad (17)$$

Here d is the colour index of the produced gluon, $e^\mu(k)$ is its polarization vector, $T^d_{c_1c_2}=-if_{dc_1c_2}$ are the matrix elements of the colour group generator in the adjoint representation, $k=q_1-q_2$ is the gluon momentum,

$$C(q_2,q_1)=-q_{1\perp}-q_{2\perp}+(q_1-q_{1\perp})\left(1-\frac{2\vec{q}_1^{\,2}}{\vec{k}^2}\right)+(q_2-q_{2\perp})\left(1-\frac{2\vec{q}_2^{\,2}}{\vec{k}^2}\right) \ , \quad (18)$$

and

$$2f_1^{(G)}=\frac{11}{6}\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\left(\vec{k}^{\,2}\right)^\epsilon\left(\frac{1}{\epsilon^2}-3\zeta(2)+2\epsilon\zeta(3)\right) \ ;$$

$$2f_2^{(G)}=\frac{\vec{k}^2}{3(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\left[\vec{q}_1^{\,2}+\vec{q}_2^{\,2}-2\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right] \ ;$$

$$f_3^{(G)}=\frac{11}{3}\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+\frac{\vec{k}^{\,2}}{6} \ , \quad (19)$$

where $\zeta(n)$ is the Riemann zeta-function. Note that the one-loop contribution to the RRG vertex is not known for arbitrary ϵ. Therefore Eq. (19), contrary to all the preceeding formulas, is valid only in the limit $\epsilon\to 0$. The only term of this equation which remains unexpanded in ϵ is $\left(\vec{k}^{\,2}\right)^\epsilon$. For this term the expansion is not performed because the RRG vertex is singular at $\vec{k}^{\,2}=0$ and in subsequent integrations of its contribution to the kernel the region $\epsilon|\ln(\vec{k}^{\,2})|\sim 1$ does contribute. In Eq. (19) all terms giving nonvanishing in the limit $\epsilon\to 0$ contributions after these integrations are kept.

The vertex (17) is explicitly invariant under the gauge transformation

$$e^\mu(k)\to e^\mu(k)+k^\mu\chi, \quad (20)$$

so that we can use the relation

$$\sum_\lambda e^{*\,(\lambda)}_\mu(k)e^{(\lambda)}_\nu(k)=-g_{\mu\nu}. \quad (21)$$

Substituting (17) in (13), using (21) for the sum over polarizations and

$$\delta_{c_1c_1'}\delta_{c_2c_2'}T^d_{c_1c_2}\left(T^d_{c_1'c_2'}\right)^*=N(N^2-1), \ \ f_{c_1c_1'c}f_{c_2c_2'c}T^d_{c_1c_2}\left(T^d_{c_1'c_2'}\right)^*=\frac{N^2(N^2-1)}{2} \quad (22)$$

for the sum over colour indices, with the help of Eqs. (18)-(19) and (14)-(16) we obtain from Eq. (10) the gluon part of the contribution to the kernel from the one-gluon production in the Reggeon-Reggeon collisions:

$$\mathcal{K}^{G\,(\mathcal{R})}_{RRG}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{g^2 c_{\mathcal{R}}}{(2\pi)^{D-1}} \left\{ \left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2} \right) \right.$$
$$\times \left(\frac{1}{2} + \frac{g^2 N\Gamma(1-\epsilon)}{2(4\pi)^{2+\epsilon}} \left[-(\vec{k}^{\,2})^{\epsilon} \left(\frac{2}{\epsilon^2} - \pi^2 + 4\,\epsilon\,\zeta(3) \right) - \ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \right] \right)$$
$$+ \frac{g^2 N\Gamma(1-\epsilon)}{6(4\pi)^{2+\epsilon}} \left(\left[\frac{(\vec{q}_1^{\,\prime\,2}-\vec{q}_2^{\,\prime\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})} - \frac{\vec{k}^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2} \left(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+4\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime}-2\vec{q}^{\,2} \right) \right] \right.$$
$$\times \left[\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}} \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) - \vec{q}_1^{\,2} - \vec{q}_2^{\,2} \right]$$
$$+11 \left[\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}} + \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}\,\vec{q}^{\,2} \right] \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right)$$
$$\left. \left. -2\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime} \right) \right\} + \frac{g^2 c_{\mathcal{R}}}{(2\pi)^{D-1}} \left\{ \vec{q}_i \longleftrightarrow \vec{q}_i^{\,\prime} \right\}. \tag{23}$$

The symmetry properties (12) of $\mathcal{K}^{G\,(\mathcal{R})}_{RRG}(\vec{q}_1,\vec{q}_2;\vec{q})$ are evident from (23). The properties (11) are not so evident, but can be easily checked.

4 The two-gluon production

Let us consider the production of two gluons with momenta k_1 and k_2 in collisions of two Reggeons with momenta q_1 and $-q_2$. We will use the Sudakov parametrization for the produced gluon momenta :

$$k_i = \beta_i p_A + \alpha_i p_B + k_{i\perp}\ , \qquad s\alpha_i\beta_i = -k_{i\perp}^2 \ = \vec{k}_i^{\,2}, \qquad i=1,2\ ,$$
$$\beta_1+\beta_2=\beta\ , \qquad \alpha_1+\alpha_2=\alpha\ \ , \qquad k_{1\perp}+k_{2\perp}=q_{1\perp}-q_{2\perp}\ , \tag{24}$$

and the denotation

$$k = k_1 + k_2 = q_1 - q_2\ , \tag{25}$$

so that $s_{_{RR}} = k^2$. For the effective vertex of the two-gluon production in the Reggeon-Reggeon collision we have:

$$\gamma^{GG}_{c_1 c_2}(q_1,q_2) \tag{26}$$

$$= g^2\, e^*_{\alpha_1}(k_1)\, e^*_{\alpha_2}(k_2)\, \left[T^{d_1}_{c_1 j}\, T^{d_2}_{j c_2}\, A^{\alpha_1\alpha_2}(k_1,k_2) + T^{d_2}_{c_1 j}\, T^{d_1}_{j c_2}\, A^{\alpha_2\alpha_1}(k_2,k_1) \right],$$

where d_i are the colour indices of the produced gluons, $e^\mu(k_i)$ are their polarization vectors. The tensor $A^{\alpha_1\alpha_2}(k_1,k_2)$ obtained in [2] satisfies the transversality conditions:

$$k_1^{\alpha_1} A_{\alpha_1\alpha_2}(k_1,k_2) \,=\, k_2^{\alpha_2} A_{\alpha_1\alpha_2}(k_1,k_2) \,=0. \tag{27}$$

Due to these conditions the two terms in (26) are separately invariant with respect to independent gauge transformations of the gluon polarization vectors

$$e^\alpha(k_i) \to e^\alpha(k_i) + k_i^\alpha \chi_i, \tag{28}$$

so that we can use different gauges for each of the produced gluons and for each of the terms. Choosing

$$e_\alpha(k_1)k_1^\alpha = e_\alpha(k_1)p_1^\alpha = 0\,, \quad e_\alpha(k_2)k_2^\alpha = e_\alpha(k_2)p_2^\alpha = 0\,, \tag{29}$$

we can present the polarization vectors as

$$e(k_1) = e_\perp(k_1) - \frac{(k_1 e_\perp(k_1))}{k_1 p_1} p_1,\; e(k_2) = e_\perp(k_2) - \frac{(k_2 e_\perp(k_2))}{k_2 p_2} p_2, \tag{30}$$

and their convolution with the tensor $A^{\alpha_1\alpha_2}(k_1,k_2)$ as

$$e^*_{\alpha_1}(k_1)\, e^*_{\alpha_2}(k_2)\, A^{\alpha_1\alpha_2}(k_1,k_2) \equiv 4 e^*_{\perp\alpha_1}(k_1)\, e^*_{\perp\alpha_2}(k_2)\, c^{\alpha_1\alpha_2}(k_1,k_2). \tag{31}$$

The tensor $c^{\mu\nu}(k_1,k_2)$ in the transverse space was defined in [6]. It can be presented in the form

$$\begin{aligned} c^{\mu\nu}(k_1,k_2) = {}& \frac{(q_1-k_1)_\perp^\mu (q_1-k_1)_\perp^\nu}{\tilde t_1} - \frac{(q_1-k_1)_\perp^\mu}{k^2}\left(k_1 - \frac{\beta_1}{\beta_2}k_2\right)_\perp^\nu + \left(k_2 - \frac{\alpha_2}{\alpha_1}k_1\right)_\perp^\mu \\ & \times \frac{(q_1-k_1)_\perp^\nu}{k^2} + \frac{k_{1\perp}^{\ \mu} k_{1\perp}^{\ \nu}}{k^2}\frac{t_2}{s\alpha_1\beta} + \frac{k_{2\perp}^{\ \mu} k_{2\perp}^{\ \nu}}{k^2}\frac{t_1}{s\alpha\beta_2} - \frac{k_{1\perp}^{\ \mu} k_{2\perp}^{\ \nu}}{k^2}\left(1 + \frac{\tilde t_1}{s\alpha_1\beta_2}\right) + \frac{k_{2\perp}^{\ \mu} k_{1\perp}^{\ \nu}}{k^2} \\ & -\frac{1}{2} g_\perp^{\mu\nu}\left(1 + \frac{\tilde t_1}{k^2} + \frac{s\alpha_1\beta_2}{\tilde t_1} - \frac{s\alpha_2\beta_1}{k^2} + \frac{s\alpha_1\beta_2}{k^2} - \frac{\alpha_2 t_1}{\alpha k^2} - \frac{\beta_1 t_2}{\beta k^2}\right), \end{aligned} \tag{32}$$

where the denotations

$$t_i = q_{i\perp}^2,\; i=1,2;\; \tilde t_1 = (q_1-k_1)^2 \tag{33}$$

are used, $g_\perp^{\mu\nu}$ is the metric tensor in the transverse plane:

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{p_A^\mu p_B^\nu + p_B^\mu p_A^\nu}{(p_A p_B)}. \tag{34}$$

Since the different gauges are used for each of the produced gluons, to obtain the contribution of the second term in (26) one has not only to change $k_1 \leftrightarrow k_2$ in the contribution of the first term, but to change also the gauges $(e(k)p_1 = 0 \;\leftrightarrow\; e(k)p_2 = 0)$, so that

$$e^*_{\alpha_1}(k_1)\, e^*_{\alpha_2}(k_2)\, A^{\alpha_2\alpha_1}(k_2,k_1) \tag{35}$$

$$= 4e^*_{\perp\alpha_1}(k_1)\, e^*_{\perp\alpha_2}(k_2)\, \Omega^{\alpha_1\beta_1}(k_1)\Omega^{\alpha_2\beta_2}(k_2)c_{\beta_2\beta_1}(k_2,k_1)\,,$$

where

$$\Omega^{\alpha\beta}(k) = g_\perp^{\alpha\beta} - 2\frac{k_\perp^\alpha k_\perp^\beta}{k_\perp^2}. \tag{36}$$

In order to calculate the two-gluon contribution to the imaginary part of the Reggeon-Reggeon scattering amplitude (13) entering Eq. (10) for the the kernel we need to sum over gluon polarizations and colour indices. The first sum can be obtained using the relation

$$\sum_\lambda \left(e_\perp^{\alpha(\lambda)}(k)\right)^* e_\perp^{\beta(\lambda)}(k) = -g_\perp^{\alpha\beta}, \tag{37}$$

and the second, for the most interesting singlet and octet representations, with the help of

$$\delta_{c_1c_1'}\delta_{c_2c_2'}T^{d_1}_{c_1i}T^{d_2}_{ic_2}T^{d_1}_{c_1'j}T^{d_2}_{jc_2'}$$

$$= N^2(N^2-1),\; \delta_{c_1c_1'}\delta_{c_2c_2'}T^{d_1}_{c_1i}T^{d_2}_{ic_2}T^{d_2}_{c_1'j}T^{d_1}_{jc_2'} = \frac{N^2(N^2-1)}{2},$$

$$f_{c_1c_1'c}f_{c_2c_2'c}T^{d_1}_{c_1i}T^{d_2}_{ic_2}T^{d_1}_{c_1'j}T^{d_2}_{jc_2'} = \frac{N^3(N^2-1)}{4},\; f_{c_1c_1'c}f_{c_2c_2'c}T^{d_1}_{c_1i}T^{d_2}_{ic_2}T^{d_2}_{c_1'j}T^{d_1}_{jc_2'} = 0\;. \tag{38}$$

Using these formulas, we obtain

$$\frac{\langle c_1c_1'|\hat{\mathcal{P}}_{\mathcal{R}}|c_2c_2'\rangle}{2n_{\mathcal{R}}} \sum_{GG} \gamma^{GG}_{c_1c_2}(q_1,q_2)\left(\gamma^{GG}_{c_1'c_2'}(q_1',q_2')\right)^*$$

$$= 8g^4N^2\left[\left(a_R c^{\alpha_1\alpha_2}(k_1,k_2)c'_{\alpha_1\alpha_2}(k_1,k_2)\right.\right.$$

$$+b_R c_{\alpha_1\alpha_2}(k_1,k_2)c'_{\beta_2\beta_1}(k_2,k_1)\Omega^{\alpha_1\beta_1}(k_1)\Omega^{\alpha_2\beta_2}(k_2)) + \Big(k_1 \leftrightarrow k_2\Big)\Big] \,, \tag{39}$$

where $c'_{\alpha_1\alpha_2}(k_1,k_2)$ is obtained from $c_{\alpha_1\alpha_2}(k_1,k_2)$ by the substitition $q_i \to q'_i$ and the coefficients a_R and b_R for the singlet ($R=1$) and octet ($R=8$) representations are

$$a_0 = 1, \quad b_0 = \frac{1}{2}; \quad a_8 = \frac{1}{4}, \quad b_8 = 0\,. \tag{40}$$

Evidently the term $(k_1 \leftrightarrow k_2)$ in (39) gives the same contribution to the kernel as the preceeding terms, so that in the following only these terms will be considered and their contribution to the kernel will be doubled.

To perform the integration in Eqs. (10), (13) over longitudinal components of the produced gluon momenta we will use the variable $x \equiv \beta_1/\beta$, so that

$$x_1 = x,\;\; x_2 = 1-x,\;\; x_i = \frac{\beta_i}{\beta},\;\; i=1,2. \tag{41}$$

The alternative choice is y, with

$$y_2 = y,\;\; y_1 = 1-y,\;\; y_i = \frac{\alpha_i}{\alpha},\;\; i=1,2. \tag{42}$$

The variables x and y are connected by the relations

$$y = \frac{x\vec{k}_2^{\,2}}{x\vec{k}_2^{\,2} + (1-x)\vec{k}_1^{\,2}},\;\; x = \frac{y\vec{k}_1^{\,2}}{y\vec{k}_1^{\,2} + (1-y)\vec{k}_2^{\,2}}, \tag{43}$$

which are inverse each to other. Remind that the vector sign is used for the transverse components. The integration measure in Eqs. (10), (13) with account of (14) has the same form for both choices :

$$\frac{dk^2}{(2\pi)}d\rho_{GG} = \frac{dx}{4x(1-x)}\frac{d^{D-2}k_1}{(2\pi)^{(D-1)}} = \frac{dy}{4y(1-y)}\frac{d^{D-2}k_2}{(2\pi)^{(D-1)}},$$

$$1 \ge x \ge 0,\;\; 1 \ge y \ge 0. \tag{44}$$

Note that

$$\vec{k}_1 + \vec{k}_2 = \vec{k} = \vec{q}_1 - \vec{q}_2 = \vec{q}_1^{\,\prime} - \vec{q}_2^{\,\prime} \tag{45}$$

is fixed.

The important symmetry properties of the convolutions

$$f_a(k_1,k_2) = c^{\alpha_1\alpha_2}(k_1,k_2)c'_{\alpha_1\alpha_2}(k_1,k_2) \tag{46}$$

and

$$f_b(k_1,k_2) = c_{\alpha_1\alpha_2}(k_1,k_2)c'_{\beta_2\beta_1}(k_2,k_1)\Omega^{\alpha_1\beta_1}(k_1)\Omega^{\alpha_2\beta_2}(k_2) \tag{47}$$

entering (39) are their invariance with respect to the "left-right" transformation

$$\vec{k}_1 \leftrightarrow \vec{k}_2, \quad \alpha_1 \leftrightarrow \beta_2, \quad \alpha_2 \leftrightarrow \beta_1, \quad \vec{q}_1 \leftrightarrow -\vec{q}_2, \quad \vec{q}_1^{\,\prime} \leftrightarrow -\vec{q}_2^{\,\prime}\,. \tag{48}$$

This invariance follows from the transformation law for the tensor $c^{\mu\nu}(k_1,k_2)$. It is easily seen from (32) that this tensor turns into $c^{\nu\mu}(k_1,k_2)$ under the transformation (48). In terms of variables $\vec{k}_i, x, y$ this transformation reads as

$$\vec{k}_1 \leftrightarrow \vec{k}_2,\, x \leftrightarrow y, \quad \vec{q}_1 \leftrightarrow -\vec{q}_2, \quad \vec{q}_1^{\,\prime} \leftrightarrow -\vec{q}_2^{\,\prime}\,. \tag{49}$$

One can check by a direct inspection that the tensor $c^{\mu\nu}(k_1,k_2)$ (32) is equal zero at zero transverse momentum of one of the Reggeons, i.e. at $\vec{q}_1 = 0$ or $\vec{q}_2 = 0$. It guarantees another important properties of the functions $f_a(k_1,k_2)$ and $f_b(k_1,k_2)$ - their turn into zero at zero transverse momenta of the Reggeons (cf.(11)).

Let us adopt the first choice $(x,\vec{k}_1)$ of variables for the integration in Eqs.(10), (13). Then the two-gluon contribution to the kernel is presented in the form

$$\mathcal{K}^{(\mathcal{R})}_{RRGG}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) \tag{50}$$

$$= \frac{4g^4N^2}{(2\pi)^{D-1}}\int\frac{d^{D-2}k_1}{(2\pi)^{(D-1)}}\int_0^1\frac{dx\theta\left(s_{\Lambda}-k^2\right)}{x(1-x)}\left[a_Rf_a(k_1,k_2)+b_Rf_b(k_1,k_2)\right]$$

$$-\frac{1}{2}\int\frac{d^{D-2}r}{\vec{r}^{\,2}\vec{r}^{\,\prime\,2}}\mathcal{K}^{(\mathcal{R})B}_r\left(\vec{q}_1,\vec{r};\vec{q}\right)\mathcal{K}^{(\mathcal{R})B}_r\left(\vec{r},\,\vec{q}_2;\,\vec{q}\right)\ln\left(\frac{s^2_{\Lambda}}{(\vec{r}-\vec{q}_1)^2(\vec{r}-\vec{q}_2)^2}\right)\,.$$

where the group coefficients a_R and b_R are defined in (40) and the functions $f_a(k_1,k_2)$ and $f_b(k_1,k_2)$ in (46) and (47) respectively. The functions must be expressed in terms of x and $\vec{k}_1$. It can be done using Eq.(41) and the relations

$$k^2 = \frac{\left((1-x)\vec{k}_1 - x\vec{k}_2\right)^2}{x(1-x)},\ \tilde{t}_1 = -\frac{1}{x}\left((1-x)\vec{k}_1^{\,2} + x(\vec{k}_1-\vec{q}_1)^2\right),$$

$$\vec{k}_1+\vec{k}_2 = \vec{k},\ \ t_i = \vec{q}_i^{\,2},\ \ \alpha_i = \frac{\vec{k}_i^{\,2}}{s\beta_i},\ \ i=1,2. \tag{51}$$

To analise the behaviour of the functions $f_a(k_1,k_2)$ and $f_b(k_1,k_2)$ in the integration region of (50) it is convenient to express the tensor $c^{\mu\nu}(k_1,k_2)$ in

(32) in terms of x and k_1. After this it is not difficult to show that for any x in the interval $[0,1]$ the tensor falls down as $1/\vec{k}_1^{\,2}$, so that the integration over $\vec{k}_1$ is well convergent in the ultraviolet region. As for the x- behaviour at fixed $\vec{k}_1$, it is easy to see that in the limit $x \to 0$ the tensor $c^{\alpha_1\alpha_2}(k_1,k_2)$ tends to zero, whereas at $x \to 1$ the tensor has a finite value. It means that the function $f_b(k_1,k_2)$ (see (47)) turns into zero both at $x=0$ and $x=1$, so that performing the integration of the term with $f_b(k_1,k_2)$ in (50) we can ignore the restrictions on the integration region imposed by $\theta\left(s_{_\Lambda} - k^2\right)$. Remind that the parameter $s_{_\Lambda}$ must be taken tending to infinity, therefore, due to the convergency of the integral over $\vec{k}_1$ in the ultraviolet region, the restrictions have the form:

$$1 - \frac{\vec{k}_2^{\,2}}{s_{_\Lambda}} \geq x \geq \frac{\vec{k}_1^{\,2}}{s_{_\Lambda}}\ . \tag{52}$$

From the discussion above it is clear that the restriction from below does not play any role, but the upper limit is important for the integration of $f_a(k_1,k_2)$ in (50).

The limit $x \to 1$ corresponds to the multi-Regge limit of large relative rapidities of the produced gluons, so that the value of $f_a(k_1,k_2)$ at this limit is related to the LLA kernel $\mathcal{K}_r^{(\mathcal{R})B}$. Indeed, using (41) and (51), we obtain from (32):

$$c^{\mu\nu}(k_1,k_2)|_{x=1} \tag{53}$$

$$= \frac{-1}{(\vec{q}_1-\vec{k}_1)^2}\left[q_1 - k_1 + \frac{(\vec{q}_1-\vec{k}_1)^2}{\vec{k}_1^{\,2}}k_1\right]^\mu_\perp \left[q_1 - k_1 - \frac{(\vec{q}_1-\vec{k}_1)^2}{\vec{k}_2^{\,2}}k_2\right]^\nu_\perp .$$

This result and Eqs.(30), (31) and (18) give us the relation

$$e^*_\mu(k_1)\, e^*_\nu(k_2)\, A^{\mu\nu}(k_1,k_2)|_{x=1} = -\frac{e^*_\mu(k_1)C^\mu(q_1-k_1,q_1)e^*_\nu(k_2)C^\nu(q_2,q_1-k_1)}{(\vec{q}_1-\vec{k}_1)^2}, \tag{54}$$

which means that in the multi-Regge limit the vertex for the two-gluon production is expressed in terms of the one-gluon production vertices. For the function $f_a(k_1,k_2)$ (46) we obtain from (53), using (8):

$$\left(\frac{2g^2c_R}{(2\pi)^{D-1}}\right)^2 f_a(k_1,k_2)|_{x=1}$$

$$= \frac{1}{(\vec{q}_1-\vec{k}_1)^2(\vec{q}_1^{\,\prime}-\vec{k}_1)^2}\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1, \vec{q}_1-\vec{k}_1; \vec{q}\right)\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1-\vec{k}_1,\, \vec{q}_2;\, \vec{q}\right).$$

Therefore the substraction term in (50) can be written as

$$-\frac{1}{2}\int \frac{d^{D-2}r}{\vec{r}^{\,2}\vec{r}^{\,\prime\,2}}\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1,\vec{r};\vec{q}\right)\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{r},\,\vec{q}_2;\,\vec{q}\right)\ln\left(\frac{s_\Lambda^2}{(\vec{r}-\vec{q}_1)^2(\vec{r}-\vec{q}_2)^2}\right)$$

$$= -\frac{1}{2}\left(\frac{2g^2c_R}{(2\pi)^{D-1}}\right)^2\int d^{D-2}k_1\int_{\frac{\vec{k}_1^{\,2}}{s_\Lambda}}^{1-\frac{\vec{k}_2^{\,2}}{s_\Lambda}}\frac{dx}{x(1-x)}f_a(k_1,k_2)|_{x=1}\,. \qquad (55)$$

Taking into account that $c_R^2 = N^2a_R$ (compare (9) and (40)), we obtain that the two-gluon contribution to the kernel can be presented as

$$\mathcal{K}_{RRGG}^{(R)}(\vec{q}_1,\vec{q}_2;\vec{q})$$

$$= \frac{4g^4N^2}{(2\pi)^{D-1}}\int_0^1\frac{dx}{x(1-x)}\int\frac{d^{2+2\epsilon}k_1}{(2\pi)^{D-1}}\left\{a_R\left[f_a(k_1,k_2)-x(f_a(k_1,k_2)|_{x=1})\right]\right.$$

$$\left.+b_Rf_b(k_1,k_2)\right\}+\frac{2g^4N^2}{(2\pi)^{D-1}}\int\frac{d^{2+2\epsilon}k_1}{(2\pi)^{D-1}}a_Rf_a(k_1,k_2)|_{x=1}\ln\left(\frac{k_1^2}{k_2^2}\right)\,. \qquad (56)$$

Remind that from general arguments the kernel must be symmetric (see (12)) with respect to the substitutions $\vec{q}_i \leftrightarrow \vec{q}_i^{\,\prime},\ i=1,2$ and $\vec{q}_1 \leftrightarrow -\vec{q}_2,\ \vec{q}_1^{\,\prime} \leftrightarrow -\vec{q}_2^{\,\prime}$ (note that at both of them $\vec{q}$ changes its sign). The symmetries of the two-gluon contribution to the kernel can be explicitly demonstrated using the representation (50). First of all, it is easy to show with the help of the expression (8) for the Born kernel that the substraction term is symmetric under these substitutions. After this the symmetry of the total contribution under the first transformation follows from the evident invariance of the convolutions (46) and (47) under this transformation. The symmetry under the second follows from the invariance of these convolutions as well as the integration measure (see (44)) with respect to the "left-right" transformation (48), (49). Turning to (56) we see that the contribution of $f_b(k_1,k_2)$ is symmetric with respect to both transformations. As for the terms with $f_a(k_1,k_2)$, the last of them gives the contribution antisymmetric under the substitution $\vec{q}_1 \leftrightarrow -\vec{q}_2,\ \vec{q}_1^{\,\prime} \leftrightarrow -\vec{q}_2^{\,\prime}$. Therefore, this term can be omitted together with antisymmetric contributions from remaining terms. So, the total contribution of all three terms with $f_a(k_1,k_2)$ can be obtained by the integration of the first term over x from zero to $1-\delta$ at arbitrary small δ with subsequent omission of terms proportional $\ln\delta$ and terms antisymmetric under the substitution $\vec{q}_1 \leftrightarrow -\vec{q}_2,\ \vec{q}_1^{\,\prime} \leftrightarrow -\vec{q}_2^{\,\prime}$.

5 The two-gluon contribution to the octet kernel

Up to now our results could be used for any colour representation $\mathcal{R}$. Starting from this point we will consider only the case of the gluon channel (antisymmetric octet representation). In this case only the function $f_a(k_1,k_2)$ does contribute to the kernel. From the discussion at the end of the preceeding Section it follows that the the two-gluon contribution to the octet kernel can be presented as

$$\mathcal{K}^{(8)}_{RRGG}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{g^4N^2}{(2\pi)^{D-1}}\hat{\mathcal{S}}\int_0^1 \frac{dx}{(1-x)_+}\int \frac{d^{2+2\epsilon}k_1}{(2\pi)^{D-1}}\frac{f_a(k_1,k_2)}{x}\ , \qquad (57)$$

where $\hat{\mathcal{S}}$ denotes the operator of symmetrization with respect to the substitution $\vec{q}_1 \leftrightarrow -\vec{q}_2,\ \vec{q}_1^{\,\prime} \leftrightarrow -\vec{q}_2^{\,\prime}$ and $(1-x)_+$ means the substraction:

$$\int_0^1 \frac{dx}{(1-x)_+} f(x) \equiv \int_0^1 \frac{dx}{(1-x)}[f(x)-f(1)]. \qquad (58)$$

According to (46) the function $f_a(k_1,k_2)$ is determined by the convolution of the tensor $c^{\mu\nu}(k_1,k_2)$ given by (32) with the tensor $c'_{\mu\nu}(k_1,k_2)$ given by the same formula with the substitution $q_i \to q'_i \equiv q_i - q$. We obtain (details of the calculation are given in Appendix A):

$$\begin{aligned} f_a(k_1,k_2) = &\Bigg\{\frac{1+\epsilon}{k^2}\Bigg[\frac{x^2(1-x)}{\Sigma}\vec{q}_1^{\,\prime\,2}\left((\vec{q}_1\vec{\Lambda}) + (1-2x)(1-x)\frac{\vec{q}_1^{\,2}(\vec{\Lambda}\vec{k})}{\Sigma}\right) \\ &+\frac{(\vec{q}_1^{\,\prime}\vec{\Lambda})(\vec{q}_1\vec{\Lambda})}{k^2}\Bigg] + \frac{x(1-x)\vec{q}_1^{\,\prime\,2}}{k^{\,2}\Sigma}\left(\frac{x(1-x)\vec{q}_1^{\,2}}{2k^{\,2}\Sigma}k_\perp^\mu k_\perp^\nu - \frac{k_\perp^\mu q_{1\perp}^\nu}{k^2}\right) \\ &\times\left(2(1+\epsilon)\Lambda_\mu\Lambda_\nu + \vec{\Lambda}^{\,2}g_{\mu\nu}^\perp\right) + \frac{x^2\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{4\vec{k}_1^{\,2}\Sigma} + \frac{x\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{4(1-x)\vec{k}_1^{\,2}k^2} \\ &+\left(\frac{x(1-x)}{\Sigma}\right)^2\frac{\vec{q}_1^{\,\prime\,2}\vec{q}_1^{\,2}}{2}\left(\frac{1+\epsilon}{2} - (3+2\epsilon)x(1-x)\right) \\ &-\frac{x(1-x)\vec{q}_1^{\,\prime\,2}}{2k^2\Sigma}\Big[(1-x)\left((1+\epsilon)(2(\vec{k}_1\vec{q}_1) - x\vec{q}_1^{\,2}) - \epsilon x(\vec{k}^{\,2} - \vec{q}_2^{\,2})\right) \\ &-(1+\epsilon)\vec{k}_2^{\,2} + 2\vec{q}_2^{\,2}\Big] + \frac{x\vec{q}_1^{\,2}\vec{q}^{\,2}}{4\tilde{t}_1\vec{k}_1^{\,2}} - (\vec{q}_1\vec{q})(1-x)\frac{(\vec{q}_1^{\,\prime\,2} - 2(\vec{k}_1\vec{q}_1^{\,\prime}))}{\tilde{t}_1\tilde{t}_1^{\,\prime}} \end{aligned}$$

$$
+\frac{(1+\epsilon)(1-x)^2}{4}\frac{(\vec{q}_1^{\,\prime\,2}-2(\vec{k}_1\vec{q}_1^{\,\prime}))(\vec{q}_1^{\,2}-2(\vec{k}_1\vec{q}_1))}{\tilde{t}_1\tilde{t}_1^{\prime}}
$$

$$
+\frac{1}{k^2\tilde{t}_1}\Big[2((\vec{q}_1\vec{q}_2)(\vec{k}_1\vec{q}_1^{\,\prime})-(\vec{q}_1^{\,\prime}\vec{q}_2)(\vec{k}_1\vec{q}_1))+x\vec{q}_2^{\,\prime\,2}\vec{q}_1^{\,2}-x\vec{k}^{\,2}(\vec{q}_1\vec{q})+\vec{q}_1^{\,2}(\vec{q}_1^{\,\prime}\vec{q})
$$

$$
-\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}\frac{(1-x)\vec{k}_1^{\,2}}{\Sigma}+\frac{1+\epsilon}{2}(1-x)\Big\{(\vec{q}_1^{\,2}-2(\vec{k}_1\vec{q}_1))(\vec{q}_1^{\,\prime\,2}(1-x)-2(\vec{q}_1^{\,\prime}\vec{\Lambda}))
$$

$$
+\frac{\vec{k}_1^{\,2}\vec{k}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\Sigma}\Big\}\Big]+\frac{(\vec{q}^{\,2})^2}{8\tilde{t}_1\tilde{t}_1^{\prime}}+\frac{x\vec{q}_1^{\,2}\vec{q}^{\,2}\vec{k}^{\,2}}{4k^2\tilde{t}_1\vec{k}_1^{\,2}}-\frac{x\vec{q}_1^{\,\prime\,2}(\vec{q}_2^{\,2})^2}{4k^2\Sigma\tilde{t}_1}-\frac{x\vec{q}_2^{\,\prime\,2}(\vec{q}_1^{\,2})^2}{4k^2\tilde{t}_1\vec{k}_1^{\,2}}
$$

$$
+\frac{x\vec{q}_2^{\,2}(\vec{q}_1^{\,2}(\vec{q}_1^{\,\prime}\vec{k}_1)-\vec{q}_1^{\,\prime\,2}(\vec{q}_1\vec{k}_1))}{2(1-x)k^2\tilde{t}_1\vec{k}_1^{\,2}}-\frac{\vec{q}_2^{\,2}(\vec{q}_1^{\,\prime}\vec{q})}{2(1-x)k^2\tilde{t}_1}\Bigg\}+\Bigg\{\vec{q}_i\leftrightarrow\vec{q}_i^{\,\prime}\Bigg\}\,, \tag{59}
$$

where $\tilde{t}_1^{\prime}$ is obtained from $\tilde{t}_1$ (51) by the substitution $\vec{q}_1\rightarrow\vec{q}_1^{\,\prime}$,

$$
\Lambda=((1-x)k_1-xk_2)_{\perp}\ \ ,\ \ \Sigma=\vec{\Lambda}^{\,2}+x(1-x)\vec{k}^{\,2}. \tag{60}
$$

Unfortunately, the integral (57) can not be expressed in terms of elementary functions (and dilogarithms) at arbitrary ϵ. Therefore we present the result (see details of the integration in Appendix B) in a "combined" form, leaving untouched the terms in $f_2(k_1,k_2)$ which can not be integrated in elementary functions.

$$
\mathcal{K}^{(8)}_{RRGG}(\vec{q}_1,\vec{q}_2;\vec{q})=\frac{4g^4N^2\Gamma(1-\epsilon)}{(4\pi)^D\pi^{1+\epsilon}}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\Bigg\{\frac{\left(\vec{k}^{\,2}\right)^{\epsilon-1}}{4\epsilon}\Big[\ (\vec{q}_1^{\,\prime 2}\vec{q}_2^{\,2}+\vec{q}_2^{\,\prime 2}\vec{q}_1^{\,2})
$$

$$
\times\left(\frac{1}{\epsilon}+\psi(1)+\psi(1+\epsilon)-2\psi(1+2\epsilon)-\frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)}\right)-\frac{\vec{q}^{\,2}\vec{k}^{\,2}}{\epsilon}\Big]
$$

$$
+\frac{\left(\vec{q}^{\,2}\right)^{\epsilon+1}}{4\epsilon}\left(-\frac{1}{\epsilon}+\psi(1)-\psi(1-\epsilon)+2\psi(1+\epsilon)-2\psi(1+2\epsilon)\right.
$$

$$
\left.-\frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)}\right)-\frac{11+7\epsilon}{4\epsilon(1+2\epsilon)(3+2\epsilon)}\vec{q}_1^{\,2}\vec{q}_2^{\,2}\frac{\left(\vec{q}_1^{\,2}\right)^{\epsilon}-\left(\vec{q}_2^{\,2}\right)^{\epsilon}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}
$$

$$
+\frac{\vec{q}^{\,2}}{\epsilon(1+2\epsilon)}\frac{\left(\vec{q}_1^{\,2}\right)^{\epsilon+1}-\left(\vec{q}_2^{\,2}\right)^{\epsilon+1}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}+\frac{\vec{q}^{\,2}}{4\epsilon}\left(\frac{1}{2\epsilon}-\psi(1+\epsilon)+\psi(1+2\epsilon)\right)
$$

$$\times\left(\left(\vec q_1^{\,2}\right)^{\epsilon}+\left(\vec q_2^{\,2}\right)^{\epsilon}\right)+\frac{1}{8\epsilon(1+2\epsilon)(3+2\epsilon)}$$

$$\times\left[\left(2(1+\epsilon)\vec q_1^{\,2}\vec q_2^{\,2}\left(\left(\vec q_1^{\,2}\right)^{\epsilon}-\left(\vec q_2^{\,2}\right)^{\epsilon}\right)-\epsilon\left(\vec q_1^{\,2}+\vec q_2^{\,2}\right)\left(\left(\vec q_1^{\,2}\right)^{\epsilon+1}-\left(\vec q_2^{\,2}\right)^{\epsilon+1}\right)\right)\right.$$

$$\times\left(\frac{\vec k^{\,2}}{\left(\vec q_1^{\,2}-\vec q_2^{\,2}\right)^3}\left(\vec q_1^{\,2}+\vec q_2^{\,2}+2\vec q_1^{\,\prime 2}+2\vec q_2^{\,\prime 2}-2\vec k^{\,2}-2\vec q^{\,2}\right)-\frac{\vec q_1^{\,\prime 2}-\vec q_2^{\,\prime 2}}{(\vec q_1^{\,2}-\vec q_2^{\,2})^2}\right)$$

$$\left.+\frac{\left(\left(\vec q_1^{\,2}\right)^{\epsilon+1}-\left(\vec q_2^{\,2}\right)^{\epsilon+1}\right)}{\left(\vec q_1^{\,2}-\vec q_2^{\,2}\right)}\left(\epsilon\left(\vec q_1^{\,\prime 2}+\vec q_2^{\,\prime 2}-\vec k^{\,2}\right)-2(1+\epsilon)\vec q^{\,2}\right)\right]$$

$$\left.+\frac{\Gamma(1+2\epsilon)}{4\Gamma^2(1+\epsilon)}I(\vec q_1,\vec q_2;\,\vec q)\right\}+\frac{4g^4N^2}{(4\pi)^D\pi^{1+\epsilon}}\Gamma(1-\epsilon)\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\{q_1\leftrightarrow q_1',\ q_2\leftrightarrow q_2'\},\tag{61}$$

where

$$I(\vec q_1,\vec q_2;\,\vec q)=4\hat{\mathcal S}\int_0^1\frac{dx}{(1-x)_+x}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}$$

$$\left(\frac{x\vec q_2^{\,2}}{2\vec\Lambda^2\tilde t_1}\left[\frac{x\left(\vec q_1^{\,2}(\vec k_1\vec q_1^{\,\prime})-\vec q_1^{\,\prime 2}(\vec k_1\vec q_1)\right)}{\vec k_1^{\,2}}-(\vec q\vec q_1^{\,\prime})\right]-\frac{x^2(1-x)}{4\vec\Lambda^2\tilde t_1}\right.$$

$$\left.\times\left[\frac{\vec q_1^{\,\prime 2}\vec q_2^{\,4}}{\Sigma}+\frac{\vec q_1^{\,4}\vec q_2^{\,\prime 2}-\vec q_1^{\,2}\vec q^{\,2}\vec k^{\,2}}{\vec k_1^{\,2}}\right]\right)-\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\frac{(\vec k^{\,2})^{\epsilon-1}}{2\epsilon}(\vec q_1^{\,2}\vec q_2^{\,\prime 2}+\vec q_2^{\,2}\vec q_1^{\,\prime 2}-\vec q^{\,2}\vec k^{\,2})$$

$$-\frac{(\vec q^{\,2})^2}{4}\frac{1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\int\frac{d^{2+2\epsilon}l}{(\vec q_1-\vec l)^2(\vec q_1^{\,\prime}-\vec l)^2}\ln\left(\frac{\vec l^{\,2}(\vec l-\vec k)^2}{\vec q^{\,4}}\right).\tag{62}$$

The first of the symmetries (12) of the two-gluon contribution is explicit in (61); the second is also easily seen. The properties (11) are not so evident. It takes some job to demonstatate their existence. In particular, one has to calculate the function $I(\vec q_1,\vec q_2;\,\vec q)$ at $\vec q_2=0$ and $\vec q_2^{\,\prime}=0$. It is not very easy, but possible (see for details Appendix C), so that we have checked the fulfilment of Eqs.(11) for the two-gluon contribution at arbitrary ϵ.

In the limit $\epsilon\to 0$ we obtain

$$\mathcal K_{RRGG}^{(8)}(\vec q_1,\vec q_2;\vec q)=\frac{g^4N^2\Gamma(1-\epsilon)}{(4\pi)^D\pi^{1+\epsilon}}\left\{\left(\vec k^{\,2}\right)^{\epsilon-1}\left(\vec q_1^{\,\prime 2}\vec q_2^{\,2}+\vec q_2^{\,\prime 2}\vec q_1^{\,2}-\vec q^{\,2}\vec k^{\,2}\right)\right.$$

$$\times\left(\frac{1}{\epsilon^2}-\frac{11}{6\epsilon}+\frac{67}{18}-4\zeta(2)+\epsilon\left(-\frac{202}{27}+9\zeta(3)+\frac{11}{6}\zeta(2)\right)\right)$$

$$
+\vec{q}^{\,2}\left[\frac{1}{2}\left(\frac{1}{\epsilon}+\ln\vec{q}^{\,2}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}^{\,4}}\right)-2\zeta(2)\right.
$$

$$
\left.+\frac{11}{6}\left(\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}\vec{q}^{\,2}}\right)+\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)+\frac{1}{4}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)+\frac{1}{4}\ln^2\left(\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\right)\right]+\frac{\vec{q}_1\vec{q}_2}{3}
$$

$$
-\frac{11}{6}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})-\frac{1}{6}\left(11-\frac{\vec{k}^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\left(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+4(\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime})-2\vec{q}^{\,2}\right)+\frac{\vec{q}_1^{\,\prime\,2}-\vec{q}_2^{\,\prime\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\right)
$$

$$
\left.\times\left(\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\vec{q}_1^{\,2}-\vec{q}_2^{\,2}\right)+I(\vec{q}_1,\vec{q}_2;\,\vec{q})\right\}
$$

$$
+\frac{g^4N^2\Gamma(1-\epsilon)}{(4\pi)^D\pi^{1+\epsilon}}\left\{\vec{q}_i\longleftrightarrow\vec{q}_i^{\,\prime}\right\}. \tag{63}
$$

In this limit the function $I(\vec{q}_1,\vec{q}_2;\,\vec{q})$ takes the form

$$
I(\vec{q}_1,\vec{q}_2;\,\vec{q})=\frac{1}{2}\int_0^1\frac{dx}{(\vec{q}_1(1-x)+\vec{q}_2x)^2}\ln\left(\frac{\vec{q}_1^{\,2}(1-x)+\vec{q}_2^{\,2}x}{\vec{k}^{\,2}x(1-x)}\right)
$$

$$
\times[\vec{q}^{\,2}(\vec{k}^{\,2}-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})+2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}+\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}}(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})]
$$

$$
+\frac{\vec{q}^{\,2}}{2}\left(4\zeta(2)-\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)-\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}-\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{4\vec{k}^{\,2}}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,4}}\right)
$$

$$
-\frac{\vec{q}^{\,2}}{4}\left[\left(\frac{1}{\epsilon}+\ln\vec{q}^{\,2}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,\prime\,2}}{\vec{q}^{\,8}}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1^{\,\prime\,2}}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_2^{\,\prime\,2}}\right)\right]. \tag{64}
$$

The integral in (64) can be presented in another form:

$$
\int_0^1\frac{dx}{(\vec{q}_1(1-x)+\vec{q}_2x)^2}\ln\left(\frac{\vec{q}_1^{\,2}(1-x)+\vec{q}_2^{\,2}x}{\vec{k}^{\,2}x(1-x)}\right)
$$

$$
=\int_0^\infty\frac{dz}{z+\vec{k}^{\,2}}\frac{1}{\sqrt{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+z)^2-4\vec{q}_1^{\,2}\vec{q}_2^{\,2}}}
$$

$$
\times\ln\left(\frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+z+\sqrt{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+z)^2-4\vec{q}_1^{\,2}\vec{q}_2^{\,2}}}{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+z-\sqrt{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+z)^2-4\vec{q}_1^{\,2}\vec{q}_2^{\,2}}}\right). \tag{65}
$$

It is possible also to express the integral in (64) in terms of dilogarithms, but this expression is not very convenient:

$$
\int_0^1 \frac{dx}{(\vec{q}_1(1-x)+\vec{q}_2 x)^2} \ln\left(\frac{\vec{q}_1^{\,2}(1-x)+\vec{q}_2^{\,2}x}{\vec{k}^{\,2}x(1-x)}\right) = -\frac{2}{|\vec{q}_1||\vec{q}_2|\sin\phi}
$$

$$
\times\left[\ln\rho \arctan\frac{\rho\sin\phi}{(1-\rho\cos\phi)} + Im\left(L\left(\rho\exp i\phi\right)\right)\right], \tag{66}
$$

where ϕ is the angle between $\vec{q}_1$ and $\vec{q}_2$,

$$
\rho = min\left(\frac{|\vec{q}_1|}{|\vec{q}_2|}, \frac{|\vec{q}_2|}{|\vec{q}_1|}\right), \quad L(z) = \int_0^z \frac{dt}{t}\ln(1-t). \tag{67}
$$

6 The non-forward octet BFKL kernel

The general form of the kernel (for arbitrary representation $\mathcal{R}$ of the colour group in the t-channel) is given by Eq.(2). The "virtual" part is universal (does not depend on $\mathcal{R}$) and is determined by the gluon Regge trajectory, which is given by Eqs.(3)-(5). (Remind, that in this paper we consider pure gluodynamics. The quark part of the kernel was considered in [18]). The "real" part, related to real particle production in the Reggeon-Reggeon collisions, in the NLA is given by the one-gluon and the two-gluon contributions considered in Section 3 and Section 5 respectively. Since the radiative corrections to the effective vertex of the one-gluon production are known only in the limit $\epsilon \to 0$, the total "real" part of the kernel can be obtained only in this limit. It is given by the sum of (23) and (63). After powerful cancellations (in particular, between the terms with singularities $1/\epsilon^2$ and all terms with $(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})$ in denominators) we obtain

$$
\mathcal{K}_r^{G\,(8)}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{g^2N}{2(2\pi)^{D-1}}\left\{\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right)\right.
$$

$$
\times\left(\frac{1}{2} + \frac{g^2N\Gamma(1-\epsilon)(\vec{k}^{\,2})^{\epsilon}}{(4\pi)^{2+\epsilon}}\left(-\frac{11}{6\epsilon}+\frac{67}{18}-\zeta(2)\right.\right.
$$

$$
\left.\left.\left.+\epsilon\left(-\frac{202}{27}+7\zeta(3)+\frac{11}{6}\zeta(2)\right)\right)\right) + \frac{g^2N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\left[\vec{q}^{\,2}\left(\frac{11}{6}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}^{\,2}\vec{k}^{\,2}}\right)\right.\right.
$$

$$
\left.+\frac{1}{4}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,\prime 2}}{\vec{q}^{\,2}}\right)+\frac{1}{4}\ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_2^{\,\prime 2}}{\vec{q}^{\,2}}\right)+\frac{1}{4}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)
$$

$$
-\frac{\vec q_1\,\vec q_2\quad+\vec q_2\,\vec q_1}{2\vec k^{\,2}}\ln^2\left(\frac{\vec q_1}{\vec q_2^{\,2}}\right)+\frac{\vec q_1\,\vec q_2\quad-\vec q_2\,\vec q_1}{\vec k^{\,2}}\ln\left(\frac{\vec q_1}{\vec q_2^{\,2}}\right)\left(\frac{11}{6}-\frac{1}{4}\ln\left(\frac{\vec q_1\,\vec q_2}{\vec k^{\,4}}\right)\right)
$$

$$
+\frac{1}{2}[\vec q^{\,2}(\vec k^{\,2}-\vec q_1^{\,2}-\vec q_2^{\,2})+2\vec q_1^{\,2}\vec q_2^{\,2}-\vec q_1^{\,2}\vec q_2^{\,\prime\,2}-\vec q_2^{\,2}\vec q_1^{\,\prime\,2}+\frac{\vec q_1^{\,2}\vec q_2^{\,\prime\,2}-\vec q_2^{\,2}\vec q_1^{\,\prime\,2}}{\vec k^{\,2}}(\vec q_1^{\,2}-\vec q_2^{\,2})]
$$

$$
\times\int_0^1\frac{dx}{(\vec q_1(1-x)+\vec q_2 x)^2}\ln\left(\frac{\vec q_1^{\,2}(1-x)+\vec q_2^{\,2}x}{\vec k^{\,2}x(1-x)}\right)\Bigg]\Bigg\}+\frac{g^2N}{2(2\pi)^{D-1}}\left\{\vec q_i\longleftrightarrow\vec q_i^{\,\prime}\right\}. \tag{68}
$$

After the cancellation of the terms $\sim 1/\epsilon^2$ the leading singularity of the kernel is $1/\epsilon$. It turns again into $\sim 1/\epsilon^2$ after subsequent integrations of the kernel because of the singular behaviour of the kernel at $\vec k^{\,2}=0$. The additional singularity arises from the region of small $\vec k^{\,2}$, where $\epsilon|\ln\vec k^{\,2}|\sim 1$. Therefore we have not expanded in ϵ the term $\left(\vec k^{\,2}\right)^{\epsilon}$. The terms $\sim\epsilon$ are taken into account in the coefficient of the divergent at $\vec k^{\,2}=0$ expression in order to save all nonvanishing in the limit $\epsilon\to 0$ contributions after the integrations.

The symmetries (12) of the kenel are easily seen. The first of them is explicit in (68). To notice the second it is sufficient to change $x\leftrightarrow(1-x)$ in the integral in (68).

In order to check that the kernel (68) turns into zero at zero transverse momenta of the Reggeons (11) one has to know the behaviour of the integral in (68). A suitable for this purpose representation is given in (65). From this representation one can see that singularities of the integral at zero transverse momenta of the Reggeons are not more than logarithmic. After this no problems remain to verify (11).

In conclusion let us note that in [17] the octet kernel was obtained using as a basis the bootstrap relation and a specific ansatz to solve it. Our results disagree with the results obtained in [17]. To see the disagreement it is sufficient to observe that the kernel obtained in [17] is expressed in terms of elementary functions. We conclude that the ansatz used in [17] is not correct.

Appendix A

In this Appendix we present the details of the calculation of the convolution $f_a(k_1,k_2)$ (46).

It is suitable to represent the tensor $c^{\mu\nu}(k_1,k_2)$ in the form

$$c^{\mu\nu}(k_1,k_2)=l_1^{\mu\nu}+l_2^{\mu\nu}+l_3^{\mu\nu}, \tag{A.1}$$

where

$$l_1^{\mu\nu}=-\frac{x\vec{q}_1^{\,2}}{k^2}\left[\frac{k^\mu_{2\perp}k^\nu_{2\perp}}{\Sigma}+\frac{k^\mu_{1\perp}((1-x)k_1-k_2)^\nu_\perp}{(1-x)\vec{k}_1^{\,2}}+\frac{g^{\mu\nu}_\perp}{2}\frac{(1-x)}{\Sigma}\left(\frac{\Sigma}{(1-x)}-\frac{\vec{k}_1^{\,2}}{x}\right)\right], \tag{A.2}$$

$$l_2^{\mu\nu}=\frac{1}{k^2}\left[-\frac{q^\mu_{1\perp}\Lambda^\nu_\perp}{(1-x)}+\left(k-\frac{x\vec{k}^2}{\vec{k}_1^{\,2}}k_1\right)^\mu_\perp q^\nu_{1\perp}\right.$$
$$\left.-\frac{2x}{\vec{k}_1^{\,2}}k^\mu_{1\perp}\left(\frac{(\vec{q}_1\vec{k}_1)}{(1-x)}k_2-(\vec{q}_1\vec{k})k_1\right)^\nu_\perp-(\vec{q}_1\vec{\Lambda})g^{\mu\nu}_\perp\right]. \tag{A.3}$$

$$l_3^{\mu\nu}=-\frac{xk^\mu_{1\perp}(q_1-k_1)^\nu_\perp}{\vec{k}_1^{\,2}}+\frac{(q_1-k_1)^\mu_\perp(q_1-k_1)^\nu_\perp}{\tilde{t}_1}+\frac{g^{\mu\nu}_\perp}{2}\frac{(1-x)}{\tilde{t}_1}(\vec{q}_1^{\,2}-2\vec{k}_1\vec{q}_1). \tag{A.4}$$

Analogous decomposition is made for the tensor $c'_{\mu\nu}(k_1,k_2)$ obtained from $c_{\mu\nu}(k_1,k_2)$ by the substitution $\vec{q}_i\leftrightarrow\vec{q}_i^{\,\prime}$, $i=1,2$ and we denote $l'_{n\mu\nu}$ ($n=1,2,3$) the tensors $l_{n\mu\nu}$ after this substitution.

The calculation of the products $l_n^{\mu\nu}l'_{n'\mu\nu}$ is significantly simpler than the calculation of the whole convolution $f_a(k_1,k_2)$ (46), though still rather tedious. The results are:

$$l_1^{\mu\nu}l'_{1\mu\nu}=\frac{x^2\vec{q}_1^{\,\prime\,2}\vec{q}_1^{\,2}}{k^4}\left\{-k^2\left(\frac{(\vec{k}_1\vec{k}_2)}{\vec{k}_1^{\,2}\Sigma}+\frac{(1-x)}{x\Sigma}-\frac{1}{x(1-x)\vec{k}_1^{\,2}}\right)\right.$$
$$\left.+\left(\frac{1+\epsilon}{2}\right)\left[\frac{(1-x)}{\Sigma}k^2\left(1-2x+\frac{(2\vec{\Lambda}\vec{k})}{k^2}\right)\right]^2\right\}, \tag{A.5}$$

$$l_2^{\mu\nu}l'_{1\mu\nu}=\frac{x\vec{q}_1^{\,\prime\,2}}{2k^4}\left\{-2(1+\epsilon)(1-x)\left(1-2x+\frac{(2\vec{\Lambda}\vec{k})}{k^2}\right)\frac{(\vec{q}_1\vec{\Lambda})k^2}{\Sigma}-k^2(\vec{q}_1\vec{k}_1)\right.$$
$$\left.\times\left(\frac{xk^2}{\vec{k}_1^{\,2}\Sigma}-\frac{(1-x)}{\Sigma}-\frac{(1-3x+x^2)}{(1-x)\vec{k}_1^{\,2}}\right)+k^2(\vec{q}_1\vec{k})\left(\frac{(1-2x)}{\Sigma}-\frac{2}{\vec{k}_1^{\,2}}\right)\right\}, \tag{A.6}$$

$$l_3^{\mu\nu}l'_{1\mu\nu}=\frac{x\vec{q}_1^{\,\prime\,2}}{2k^2}\left\{(1+\epsilon)(1-x)^2\frac{k^2}{\Sigma\tilde{t}_1}(\vec{q}_1^{\,2}-2\vec{k}_1\vec{q}_1)\left(1-2x+\frac{(2\vec{\Lambda}\vec{k})}{k^2}\right)-\frac{1}{2(1-x)\vec{k}_1^{\,2}\tilde{t}_1}\right.$$

$$\times\Big(\vec{q}_1^{\,2}(\vec{q}_2^{\,2}+(1-x)(\vec{q}_1^{\,2}-\vec{k}^{\,2}))-2(1-x)k^2(\vec{q}_1^{\,2}-\vec{k}_1\vec{q}_1)\Big)$$

$$-\frac{\vec{q}_2^{\,2}}{2\Sigma\tilde{t}_1}\left(\vec{q}_2^{\,2}-4(1-x)(\vec{q}_1^{\,2}-2\vec{k}_1\vec{q}_1)\right)$$

$$-\frac{4(1-x)^2\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}{2x(1-x)\tilde{t}_1}+\frac{x\vec{k}^{\,2}(2\vec{q}_1\vec{k}_2-\vec{k}^{\,2})}{2\vec{k}_1^{\,2}\Sigma}$$

$$+\frac{1}{\Sigma}\left((1-x)(\vec{q}_1^{\,2}-\vec{k}_1\vec{q}_1)+\frac{(1-2x)}{2}(\vec{q}_2^{\,2}-\vec{k}^{\,2})\right)$$

$$+\frac{1}{2(1-x)\vec{k}_1^{\,2}}\left(-2(1-3x+x^2)(\vec{q}_1\vec{k}_1)+(1-x)\vec{k}^{\,2}+x\vec{q}_2^{\,2}-2\vec{q}_1^{\,2}\right)\Bigg\}, \qquad (A.7)$$

$$l_2^{\mu\nu}(q_1)l'_{2\mu\nu}=\frac{x(1-x)}{k^2}\left[\frac{(\vec{q}_1\vec{q}_1^{\,\prime})}{2}\left(\frac{1}{(1-x)^2}+\frac{\vec{k}^{\,2}}{\vec{k}_1^{\,2}}\right)-\frac{(\vec{q}_1\vec{k}_1)(\vec{q}_1^{\,\prime}\vec{\Lambda})}{(1-x)\vec{k}_1^{\,2}}\right]$$

$$+(1+\epsilon)\frac{(\vec{q}_1\vec{\Lambda})(\vec{q}_1^{\,\prime}\vec{\Lambda})}{k^4}+\{\vec{q}_i\leftrightarrow\vec{q}_i^{\,\prime}\}, \qquad (A.8)$$

$$l_3^{\mu\nu}l'_{3\mu\nu}=x^2\frac{(\vec{q}_1\vec{q}_1^{\,\prime})}{2\vec{k}_1^{\,2}}-\frac{\vec{q}^{\,2}(\vec{q}_1\vec{q}_1^{\,\prime})}{4\tilde{t}_1\tilde{t}_1^{\prime}}+\frac{(1+\epsilon)(1-x)^2}{4}$$

$$\times\frac{(\vec{q}_1^{\,2}-2(\vec{k}_1\vec{q}_1))}{\tilde{t}_1}\frac{(\vec{q}_1^{\,\prime\,2}-2(\vec{k}_1\vec{q}_1^{\,\prime}))}{\tilde{t}_1^{\prime}}-\frac{x(1-x)}{2\vec{k}_1^{\,2}}\frac{(\vec{q}_1^{\,\prime\,2}-2(\vec{k}_1\vec{q}_1^{\,\prime}))(\vec{k}_1\vec{q}_1)}{\tilde{t}_1^{\prime}}$$

$$-\frac{\vec{q}_1^{\,\prime\,2}-2(\vec{k}_1\vec{q}_1^{\,\prime})}{2\tilde{t}_1\tilde{t}_1^{\prime}}\left[(1-2x)(\vec{q}_1\vec{q})+\frac{2x(\vec{k}_1\vec{q}_1)}{\vec{k}_1^{\,2}}(\vec{q}(\vec{q}_1-\vec{k}_1))\right]+\{\vec{q}_1\leftrightarrow\vec{q}_1^{\,\prime},\vec{q}\to-\vec{q}\}, \qquad (A.9)$$

$$l_3^{\mu\nu}l'_{2\mu\nu}=-(1+\epsilon)(1-x)(\vec{q}_1^{\,2}-2(\vec{k}_1\vec{q}_1))\frac{(\vec{q}_1^{\,\prime}\vec{\Lambda})}{k^2\tilde{t}_1}+x(1-x)\frac{(\vec{q}_1^{\,\prime}\vec{k}_1)}{2\vec{k}_1^{\,2}\tilde{t}_1}(\vec{q}_1^{\,2}-2(\vec{k}_1\vec{q}_1))$$

$$+\frac{x}{k^2\vec{k}_1^{\,2}}\left[(\vec{k}_1\vec{q}_1)(\vec{q}_1^{\,\prime}\vec{\Lambda})-\Big((\vec{q}_1^{\,\prime}\vec{k}_1)(\vec{q}_1\vec{k})-(\vec{q}_1\vec{k}_1)(\vec{q}_1^{\,\prime}\vec{k})\Big)\right]-\frac{x^2(\vec{q}_1\vec{q}_1^{\,\prime})}{2\vec{k}_1^{\,2}}-\frac{x(\vec{q}_1\vec{q}_1^{\,\prime})}{2(1-x)k^2}$$

$$-\frac{x(\vec{q}_1\vec{q}_1^{\,\prime})}{2\tilde{t}_1\vec{k}_1^{\,2}}(\vec{q}_1^{\,2}-(\vec{q}_1\vec{k}_1))+\frac{\vec{q}_1^{\,2}(\vec{q}_1^{\,\prime}\vec{k}_1)-(\vec{k}_1\vec{q}_1)(\vec{q}_1^{\,\prime}\vec{q}_1)}{k^2\tilde{t}_1}\left[2+\frac{x}{2\vec{k}_1^{\,2}}\left(\frac{\vec{q}_1^{\,2}+\vec{k}^{\,2}}{1-x}-k^2\right)\right]$$

$$+\frac{x\vec{q}_1^{\,2}}{4(1-x)\tilde{t}_1\vec{k}_1^{\,2}k^2}\left[(\vec{q}_1\vec{q}_1^{\,\prime})(\vec{q}_1^{\,2}+(2x-1)\vec{k}^{\,2})-2x(\vec{q}_1^{\,\prime}\vec{k})\vec{q}_1^{\,2}\right]$$

$$+\frac{1}{4(1-x)\tilde t_1 k^2}\Big\{(\vec q_1\vec q_1{}')\Big[\vec q_1^{\,2}(3-4x)-(1-2x)^2\vec k^{\,2}\Big]-\vec q_1^{\,2}(\vec q_1{}'\vec k)$$

$$\times 2(1+4x(1-x))+4(\vec q_1\vec q_1{}')(\vec q_1\vec k)\Big\}+\frac{x}{4(1-x)\vec k_1^{\,2}k^2}\Big[2\vec q_1^{\,2}(\vec q_1{}'\vec\Lambda)+(\vec q_1\vec q_1{}')(\vec q_1^{\,2}+\vec k^{\,2})\Big]$$

$$-x(1-x)\frac{(\vec q_1\vec q_1{}')\vec k^{\,2}}{2k^2\vec k_1^{\,2}}+\Big[(\vec k_1\vec q_1)(\vec q_1{}'\vec k)-(\vec q_1{}'\vec k_1)(\vec q_1\vec k)\Big]\left(\frac{2}{k^2\tilde t_1}+\frac{x\vec q_1^{\,2}}{(1-x)\vec k_1^{\,2}\tilde t_1 k^2}\right)\,. \tag{A.10}$$

With the help of (A.1)-(A.10) and the equations obtained from them by the substitution $\vec q_i\leftrightarrow\vec q_i{}'$ we arrive at (59).

Appendix B

In this Appendix we present the details of the calculation of the integrals in (57) with $f_a(k_1,k_2)$ given by (59). Firstly let us remind the denotation used:

$$\vec k=\vec k_1+\vec k_2=\vec q_1-\vec q_2,\;\;\vec\Lambda=(1-x)\vec k_1-x\vec k_2,\;\;\vec q_i{}'=\vec q_i-\vec q,\;i=1,2;$$

$$k^2=\frac{\Big((1-x)\vec k_1-x\vec k_2\Big)^2}{x(1-x)},\;\;\Sigma=\vec\Lambda^{\,2}+x(1-x)\vec k^{\,2},$$

$$\tilde t_1=-\frac{1}{x}\Big((1-x)\vec k_1^{\,2}+x(\vec k_1-\vec q_1)^2\Big) \tag{B.1}$$

and $\tilde t_1{}'$ is obtained from $\tilde t_1$ by the substitution $\vec q_1\to\vec q_1{}'$.

It is easy to see, that

$$J_0=\int_0^1\frac{dx}{x(1-x)}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left\{\frac{1+\epsilon}{k^2}\left[\frac{x^2(1-x)}{\Sigma}\vec q_1{}'^{\,2}\right.\right.$$

$$\left.\times\left((\vec q_1\vec\Lambda)+(1-2x)(1-x)\frac{\vec q_1^{\,2}(\vec\Lambda\vec k)}{\Sigma}\right)+\frac{(\vec q_1{}'\vec\Lambda)(\vec q_1\vec\Lambda)}{k^2}\right]$$

$$\left.+\frac{x(1-x)\vec q_1{}'^{\,2}}{k^{\,2}\Sigma}\left(\frac{x(1-x)\vec q_1^{\,2}}{2k^{\,2}\Sigma}k_\perp^\mu k_\perp^\nu\;-\frac{k_\perp^\mu q_{1\perp}^\nu}{k^2}\right)\Big(2(1+\epsilon)\Lambda_\mu\Lambda_\nu+\vec\Lambda^{\,2}g_{\mu\nu}^\perp\Big)\right\}=0\,. \tag{B.2}$$

The zero appears as a result of the integration over $\vec k_1$ (or, equivalently, over $\vec\Lambda$). The first two terms here gives zero due to parity, the third - due to the dimensional regularization, and the remaining terms - due to the isotropy of the transverse space leading to the replacement $2(1+\epsilon)\Lambda_\mu\Lambda_\nu \to -\vec{\Lambda}^{\,2} g^{\perp}_{\mu\nu}$ after the angular integration.

The calculation of the contributions of the terms with the denominators Σ, Σ^2, $k^2\Sigma$ and $k^2\vec{k}^{\,2}$ is straightforward. We obtain, using usual Feynman parametrization if necessary

$$
\begin{aligned}
I_\Sigma &= \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\Sigma} = \frac{-1}{\epsilon}[x(1-x)\vec{k}^2]^{\epsilon}, \\
I_{\Sigma^2} &= \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\Sigma^2} = [x(1-x)\vec{k}^2]^{\epsilon-1}\ , \\
I_{\Lambda^2\Sigma} &= \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{\Lambda}^2\Sigma} = \frac{1}{\epsilon}[x(1-x)\vec{k}^2]^{\epsilon-1}\ , \\
I_{\Lambda^2 k_1^2} &= \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{\Lambda}^{\,2}\vec{k}_1^{\,2}} = \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}[x^2\vec{k}^2]^{\epsilon-1}\ .
\end{aligned}
\tag{B.3}
$$

The subsequent integration of these terms over x can be done without difficulties.

The integrals over $\vec{k}_1$ from the terms with the denominators $\vec{k}_1^{\,2}\Sigma$ and $\vec{k}_1^{\,2}\tilde{t}_1$ can not be expressed through elementary functions at arbitrary ϵ. Nevertheles, these terms do not create problems. For them it is convenient to make integration over $\vec{k}_1$,

$$
\begin{aligned}
I_{k_1^2\Sigma} &= \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^{\,2}\Sigma} = \left(\vec{k}^{\,2}\right)^{\epsilon-1}\int_0^1 \frac{dz}{(xz(1-xz))^{(1-\epsilon)}}\ , \\
I_{k_1^2 t_1} &= \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^{\,2}\tilde{t}_1} = -x\left(\vec{q}_1^{\,2}\right)^{\epsilon-1}\int_0^1 \frac{dz}{(xz(1-xz))^{1-\epsilon}}\ ,
\end{aligned}
\tag{B.4}
$$

then to introduce the variable $y = xz$ instead of z and to change the order of the integrations over x and y, after that the integrals can be easily calculated.

In this way we obtain

$$
J_1 = \int_0^1 \frac{dx}{(1-x)_+ x}\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left\{\frac{x^2\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{4\vec{k}_1^{\,2}\Sigma} + \frac{x\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{4(1-x)\vec{k}_1^{\,2}k^2}\right.
$$
$$
+\left(\frac{x(1-x)}{\Sigma}\right)^2\frac{\vec{q}_1^{\,\prime\,2}\vec{q}_1^{\,2}}{2}\left(\frac{1+\epsilon}{2} - (3+2\epsilon)x(1-x)\right)
$$

$$
-\frac{x(1-x)\vec{q}_1}{2k^2\Sigma}\Bigg[(1-x)\left((1+\epsilon)(2(\vec{k}_1\vec{q}_1)-x\vec{q}_1^{\,2})-\epsilon x(\vec{k}^{\,2}-\vec{q}_2^{\,2})\right)
$$

$$
\left.\left.-(1+\epsilon)\vec{k}_2^{\,2}+2\vec{q}_2^{\,2}\right]\right\}
$$

$$
=\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\frac{\vec{q}_1^{\,\prime 2}\vec{q}_2^{\,2}}{2\epsilon\vec{k}^{\,2}}\left(\vec{k}^{\,2}\right)^{\epsilon}\left\{\psi(1)+\psi(\epsilon)-2\psi(2\epsilon)-\frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)}\right\}, \quad \text{(B.5)}
$$

$$
J_2=\int_0^1\frac{dx}{(1-x)_+x}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{x\vec{q}_1^{\,2}\vec{q}^{\,2}}{4\tilde{t}_1\vec{k}_1^{\,2}}=-\frac{\Gamma^2(\epsilon)}{4\Gamma(2\epsilon)}\left(\vec{q}_1^{\,2}\right)^{\epsilon}\vec{q}^{\,2}[\psi(\epsilon)-\psi(2\epsilon)]. \quad \text{(B.6)}
$$

The integrals with $\tilde{t}_1\tilde{t}_1'$ in denominators can be calculated with the help of the trick used in [18]. Let us consider in (59) the first such term. It can be presented as

$$
-(\vec{q}_1\vec{q})(1-x)\frac{(\vec{q}_1^{\,\prime\,2}-2(\vec{k}_1\vec{q}_1^{\,\prime}))}{\tilde{t}_1\tilde{t}_1'}=(\vec{q}_1\vec{q})(1-x)\left[\frac{1}{\tilde{t}_1}+\frac{\vec{k}_1^{\,2}}{x\tilde{t}_1\tilde{t}_1'}\right]. \quad \text{(B.7)}
$$

The first term in the R.H.S. can be integrated over k_1 and then over x . For the second it seems more convenient to begin with the integration over x in (57) getting

$$
\int_0^1\frac{dx}{x(1-x)}(\vec{q}_1\vec{q})(1-x)\frac{\vec{k}_1^{\,2}}{x\tilde{t}_1\tilde{t}_1'}=\frac{(\vec{q}_1\vec{q})}{(\vec{q}_1^{\,\prime}-\vec{k}_1)^2-(\vec{q}_1-\vec{k}_1)^2}\ln\frac{(\vec{q}_1^{\,\prime}-\vec{k}_1)^2}{(\vec{q}_1-\vec{k}_1)^2}\ . \quad \text{(B.8)}
$$

With the help of the representation

$$
\frac{1}{(\vec{q}_1^{\,\prime}-\vec{k}_1)^2-(\vec{q}_1-\vec{k}_1)^2}\ln\frac{(\vec{q}_1^{\,\prime}-\vec{k}_1)^2}{(\vec{q}_1-\vec{k}_1)^2}
$$

$$
=\int_0^1 dz\,\frac{1}{z(\vec{q}_1^{\,\prime}-\vec{k}_1)^2+(1-z)(\vec{q}_1-\vec{k}_1)^2}\ , \quad \text{(B.9)}
$$

the integration over k_1 and the subsequent integration over z become trivial and give

$$
J_3=-\int_0^1\frac{dx}{x(1-x)}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}(\vec{q}_1\vec{q})(1-x)\frac{(\vec{q}_1^{\,\prime\,2}-2(\vec{k}_1\vec{q}_1^{\,\prime}))}{\tilde{t}_1\tilde{t}_1'}
$$

$$
=-\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\frac{(\vec{q}_1\vec{q})}{\epsilon(1+2\epsilon)}\left[\left(\vec{q}^{\,2}\right)^{\epsilon}-\left(\vec{q}_1^{\,2}\right)^{\epsilon}\right]. \quad \text{(B.10)}
$$

Analogously we obtain

$$J_4 = \int_0^1 \frac{dx}{x(1-x)} \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{(1+\epsilon)(1-x)^2}{4} \frac{(\vec{q}_1^{\,\prime\,2} - 2(\vec{k}_1\vec{q}_1^{\,\prime}))(\vec{q}_1^{\,2} - 2(\vec{k}_1\vec{q}_1))}{\tilde{t}_1\tilde{t}_1^{\prime}}$$

$$= -\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)} \frac{(1+\epsilon)}{8\epsilon} \frac{\left((\vec{q}_1^{\,\prime 2})^{1+\epsilon} + (\vec{q}_1^{\,2})^{1+\epsilon} - (\vec{q}^{\,2})^{1+\epsilon}\right)}{(1+2\epsilon)(3+2\epsilon)}. \qquad (B.11)$$

The terms in $f_a(k_1, k_2)$ with the denominator $k^2\tilde{t}_1$ and with x^n in numerators at natural n can be calculated performing the integration over $\vec{k}_1$ at fixed Feynman parameter z, then making the change of variable $y = xz$:

$$\int_0^1 \frac{dx}{x(1-x)} \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{x^n}{k^2\tilde{t}_1}$$

$$= -\int_0^1 dx \int_0^1 dz \frac{x^{n+1}}{\{xz[x(1-z)\vec{q}_2^{\,2} + (1-x)\vec{q}_1^{\,2}]\}^{1-\epsilon}}$$

$$= -\int_0^1 dy\, y^{\epsilon-1} \int_y^1 dx \frac{x^n}{[x(\vec{q}_2^{\,2} - \vec{q}_1^{\,2}) + \vec{q}_1^{\,2} - y\vec{q}_2^{\,2}]^{1-\epsilon}} \;. \qquad (B.12)$$

The change of variable $y = xz$ has been performed in the last equality. This integral can be now calculated integrating first over x and then over y. The complete calculation for all such terms in (59) is long, but straightforward. The integration of the terms

$$\frac{1}{k^2\tilde{t}_1}\left[-\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}\frac{(1-x)\vec{k}_1^{\,2}}{\Sigma} + \frac{1+\epsilon}{2}(1-x)\frac{\vec{k}_1^{\,2}\vec{k}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\Sigma}\right]$$

can be done quite analogously, since under the transformation (49) they acquire the form of the terms discussed above. In this way we obtain:

$$\int_0^1 \frac{dx}{x(1-x)} \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{1}{k^2\tilde{t}_1}\Big[2((\vec{q}_1\vec{q}_2)(\vec{k}_1\vec{q}_1^{\,\prime}) - (\vec{q}_1^{\,\prime}\vec{q}_2)(\vec{k}_1\vec{q}_1))$$

$$+x\vec{q}_2^{\,\prime\,2}\vec{q}_1^{\,2} - x\vec{k}^{\,2}(\vec{q}_1\vec{q}) + \vec{q}_1^{\,2}(\vec{q}_1^{\,\prime}\vec{q}) - \vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}\frac{(1-x)\vec{k}_1^{\,2}}{\Sigma}\Big]$$

$$= \frac{\Gamma(\epsilon)\Gamma(1+\epsilon)}{2\Gamma(2+2\epsilon)}\left[\vec{q}_1^{\,\prime\,2}\left(\vec{q}_1^{\,2}\right)^{\epsilon} + \frac{(\vec{q}^{\,2} - \vec{q}_1^{\,2})}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})}\left(\left(\vec{q}_1^{\,2}\right)^{\epsilon}\left(\vec{q}_1^{\,2} + \vec{q}_2^{\,2}\right) - 2\left(\vec{q}_2^{\,2}\right)^{1+\epsilon}\right)\right], \qquad (B.13)$$

$$\int_0 \frac{dx}{x(1-x)} \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{(1+\epsilon)}{2} \frac{(1-x)}{k^2\tilde{t}_1}$$

$$\left\{ (\vec{q}_1^{\,2} - 2(\vec{k}_1\vec{q}_1))(\vec{q}_1^{\,\prime\,2}(1-x) - 2(\vec{q}_1^{\,\prime}\vec{\Lambda})) + \frac{\vec{k}_1^{\,2}\vec{k}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\Sigma} \right\}$$

$$= \frac{\Gamma(\epsilon)\Gamma(2+\epsilon)}{4\Gamma(4+2\epsilon)} \left[\left(2(1+\epsilon)\vec{q}_1^{\,2}\vec{q}_2^{\,2} \left(\left(\vec{q}_1^{\,2}\right)^{\epsilon} - \left(\vec{q}_2^{\,2}\right)^{\epsilon} \right) \right.\right.$$

$$\left. - \epsilon \left(\vec{q}_1^{\,2} + \vec{q}_2^{\,2}\right) \left(\left(\vec{q}_1^{\,2}\right)^{\epsilon+1} - \left(\vec{q}_2^{\,2}\right)^{\epsilon+1} \right) \right)$$

$$\times \left(\frac{\vec{k}^{\,2}}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})^3} \left(\vec{q}_1^{\,2} + \vec{q}_2^{\,2} + 2\vec{q}_1^{\,\prime 2} + 2\vec{q}_2^{\,\prime 2} - 2\vec{k}^{\,2} - 2\vec{q}^{\,2} \right) - \frac{\vec{q}_1^{\,\prime 2} - \vec{q}_2^{\,\prime 2}}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})^2} \right)$$

$$\left. + \frac{\left(\left(\vec{q}_1^{\,2}\right)^{\epsilon+1} - \left(\vec{q}_2^{\,2}\right)^{\epsilon+1} \right)}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})} \left(\epsilon \left(\vec{q}_1^{\,\prime 2} + \vec{q}_2^{\,\prime 2} - \vec{k}^{\,2} \right) - 2(1+\epsilon)\left(\vec{q}^{\,2} - \vec{q}_1^{\,2} \right) \right) \right] . \tag{B.14}$$

All the calculations discussed before were done exactly at arbitrary ϵ. It can not be done for remaining terms. They contribute into the function $I(\vec{q}_1, \vec{q}_2; \vec{q})$ (62). We have used the following equality:

$$\int_0^1 \frac{dx}{(1-x)_+ x} \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{(\vec{q}^{\,2})^2}{8\tilde{t}_1\tilde{t}_1'}$$

$$= -\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \frac{(\vec{q}^{\,2})^{1+\epsilon}}{16} \Big[\frac{2}{\epsilon} - 2\psi(1) + 2\psi(1-\epsilon) - 4\psi(1+\epsilon)$$

$$+ 4\psi(1+2\epsilon) \Big] - \frac{(\vec{q}^{\,2})^2}{8} \int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{\ln\left(\vec{l}^{\,2}/\vec{q}^{\,2}\right)}{(\vec{q}_1 - \vec{l})^2(\vec{q}_1^{\,\prime} - \vec{l})^2} . \tag{B.15}$$

This equality can be obtained performing the integration over x first, using the representation (B.9) and the integral

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{\ln\left(\vec{l}^{\,2}/\vec{q}^{\,2}\right)}{\vec{l}^{\,2}(\vec{q} - \vec{l})^2}$$

$$= -\frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)} \frac{(\vec{q}^{\,2})^{\epsilon}}{\vec{q}^{\,2}} \Big[\frac{1}{\epsilon} - 2\left(\psi(1) - \psi(1-\epsilon) + \psi(1+\epsilon) - \psi(1+2\epsilon)\right) \Big]. \tag{B.16}$$

The last integral can be easily obtained with the help of the generalised Feynman parametrisation.

Using the integrals calculated above we come to the representation (61) for the two-gluon contribution for the kernel.

In the limit $\epsilon \to 0$ the expression (64) for $I(\vec{q}_1, \vec{q}_2; \vec{q})$ was obtained performing the integration over $\vec{k}$ in (62) first. We have

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{1}{k^2\tilde{t}_1\vec{k}_1^2} \simeq -\frac{(1-x)}{\vec{q}_1^{\,2}\vec{k}^{\,2}} \left\{ \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} x^{2\epsilon-1} \left(\vec{k}^{\,2}\right)^{\epsilon} \right.$$

$$+ \frac{\left(\vec{q}_1^{\,2} - \vec{q}_2^{\,2}\right)}{\left(\vec{q}_1^{\,2}(1-x) + \vec{q}_2^{\,2}x\right)} \left[\ln\left(x \left(\vec{q}_1^{\,2}(1-x) + \vec{q}_2^{\,2}x \right)^2 \right) + \frac{2 - \left(\vec{q}_1^{\,2}(1-x)\right)^{\epsilon}}{\epsilon} \right]$$

$$\left. - \frac{\left(2\vec{k}(\vec{q}_1(1-x) + \vec{q}_2 x)\right)}{(\vec{q}_1(1-x) + \vec{q}_2 x)^2} \left[\ln\left(\frac{\vec{q}_1^{\,2}\left(\vec{q}_1^{\,2}(1-x) + \vec{q}_2^{\,2}x\right)}{x\vec{k}^{\,2}} \right) + \frac{1 - \left(\vec{q}_1^{\,2}(1-x)\right)^{\epsilon}}{\epsilon} \right] \right\}, \tag{B.17}$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{\vec{k}_1}{k^2\tilde{t}_1\vec{k}_1^2} \simeq -x(1-x) \left\{ \frac{\vec{q}_1}{\vec{q}_1^{\,2}(\vec{q}_1(1-x) + \vec{q}_2 x)^2} \right.$$

$$\left[\ln \frac{\left(\vec{q}_1^{\,2}\left(\vec{q}_1^{\,2}(1-x) + \vec{q}_2^{\,2}x\right)\right)}{x\vec{k}^{\,2}} + \frac{1 - \left(\vec{q}_1^{\,2}(1-x)\right)^{\epsilon}}{\epsilon} \right]$$

$$+ \frac{\vec{k}}{x\vec{k}^{\,2}} \left[\frac{1}{\left(\vec{q}_1^{\,2}(1-x) + \vec{q}_2^{\,2}x\right)} \left(\ln \frac{\left(x\left(\vec{q}_1^{\,2}(1-x) + \vec{q}_2^{\,2}x\right)^2 \right)}{\vec{q}_1^{\,2}(1-x)} + \frac{1}{\epsilon} \right) \right.$$

$$\left.\left. - \frac{1}{(\vec{q}_1(1-x) + \vec{q}_2 x)^2} \ln \frac{\left(\vec{q}_1^{\,2}(1-x) + \vec{q}_2^{\,2}x\right)}{x(1-x)\vec{k}^{\,2}} \right] \right\}, \tag{B.18}$$

$$\int \frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \frac{\ln\left(\vec{l}^{\,2}/\vec{q}^{\,2}\right)}{(\vec{q}_1 - \vec{l})^2(\vec{q}_1^{\,\prime} - \vec{l})^2}$$

$$\simeq \frac{1}{\vec{q}^{\,2}} \left[\left(\frac{1}{\epsilon} + \ln \vec{q}^{\,2} \right) \ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}}{(\vec{q}^{\,2})^2} \right) + \frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1^{\,\prime\,2}} \right) \right]. \tag{B.19}$$

The integrals (B.17),(B.18) are calculated for arbitrary small $\vec{k}^{\,2}$ and for x arbitrary close to zero or unity.

The approximate form (64) for $I(q_1, q_2;\, q)$ can be obtained using Eqs.(B.17)-(B.19) and the relation

$$\int_0^1 \frac{x dx}{(\vec{q}_1(1-x)-\vec{q}_2 x)^2} \ln\left(\frac{\vec{q}_1^{\,2}(1-x)+\vec{q}_2^{\,2}x}{\vec{k}^{\,2}x(1-x)}\right)$$

$$= \frac{\vec{k}^{\,2}+\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}{2\vec{k}^{\,2}} \int_0^1 \frac{dx}{(\vec{q}_1(1-x)-\vec{q}_2 x)^2} \ln\left(\frac{\vec{q}_1^{\,2}(1-x)+\vec{q}_2^{\,2}x}{\vec{k}^{\,2}x(1-x)}\right)$$

$$-\frac{1}{2\vec{k}^{\,2}}\left(L(1-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}})-L(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}})\right)-\frac{1}{4\vec{k}^{\,2}}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,4}}\right). \qquad \text{(B.20)}$$

Appendix C

Due to the symmetries (12) of the kernel (61) it is sufficient to show that the kernel turns into zero at zero transverse momentum of one of the Reggeons, let say, at $\vec{q}_2 = 0$. But even in this special case the integration in $I(\vec{q}_1, \vec{q}_2;\, \vec{q})$ (62) can not be done in terms of elementary functions. Fortunately, the expression (62) can be simplified at $\vec{q}_2 = 0$ and presented as

$$I(\vec{q}_1, \vec{q}_2;\, \vec{q})|_{\vec{q}_2=0} = -\frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)}\vec{q}^{\,2}\left(\vec{k}^{\,2}\right)^{\epsilon}[\psi(1)+\psi(\epsilon)-2\psi(2\epsilon)]$$

$$-\frac{(\vec{q}^{\,2})^2}{4}\frac{1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\int\frac{d^{2+2\epsilon}l}{\vec{l}^{\,2}(\vec{l}-\vec{q})^2}\ln\left(\frac{\vec{l}^{\,2}(\vec{l}-\vec{k})^2}{\vec{q}^{\,4}}\right). \qquad \text{(C.1)}$$

Note, that at $\vec{q}_2 = 0$ we have $\vec{q}_1 = \vec{k}$ and $\vec{q}_2^{\,\prime} = -\vec{q}$, $\vec{q}_1^{\,\prime} = \vec{k} - \vec{q}$.

Evidently, the integral term in (C.1) excludes for the piece of the kernel (61) without the substitution $\vec{q}_i \leftrightarrow \vec{q}_i^{\,\prime}$ the possibility to turn alone into zero at $\vec{q}_2 = 0$. Therefore we need to calculate $I^{\,\prime}(\vec{q}_1, \vec{q}_2;\, \vec{q}) \equiv I(\vec{q}_1^{\,\prime}, \vec{q}_2^{\,\prime};\, -\vec{q})$ at $\vec{q}_2 = 0$. This function can be also simplified. Not, that as well as in the preceeding case simplifications can not be done for separate terms in (62) and are possible only due to the definite combination of them. We obtain:

$$I(\vec{q}_1^{\,\prime}, \vec{q}_2^{\,\prime};\, -\vec{q})|_{\vec{q}_2=0} = \frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)}\vec{q}^{\,2}\left[\left(\vec{q}_1^{\,\prime\,2}\right)^{\epsilon}(\psi(\epsilon)-\psi(2\epsilon))\right.$$

$$\left.-\left(\vec{k}^{\,2}\right)^{\epsilon}(\psi(1)-\psi(2\epsilon))-\left(\vec{q}^{\,2}\right)^{\epsilon}(\psi(1)-\psi(1-\epsilon)+2\psi(\epsilon)-2\psi(2\epsilon))\right]$$

$$-\frac{(\vec{q}^{\,2})^2}{4}\frac{1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\int\frac{d^{2+2\epsilon}l}{\vec{l}^{\,2}(\vec{l}-\vec{q})^2}\ln\left(\frac{\vec{l}^{\,2}}{(\vec{l}-\vec{k})^2}\right) . \qquad \text{(C.2)}$$

In the sum of (C.1) and (C.2) the terms with $\ln(\vec{l}-\vec{k})^2$ cancel each other, after that the integrals can be calculated and we obtain:

$$(I(\vec{q}_1,\vec{q}_2;\,\vec{q})+I(\vec{q}_1^{\,\prime},\vec{q}_2^{\,\prime};\,-\vec{q})|_{\vec{q}_2=0}$$
$$=\frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)}\vec{q}^{\,2}\left[\left(\vec{q}_1^{\,\prime\,2}\right)^\epsilon(\psi(\epsilon)-\psi(2\epsilon))-\left(\vec{k}^{\,2}\right)^\epsilon(2\psi(1)+\psi(\epsilon)-3\psi(2\epsilon))\right.$$
$$\left.-\left(\vec{q}^{\,2}\right)^\epsilon(2\psi(1)-2\psi(1-\epsilon)+3\psi(\epsilon)-3\psi(2\epsilon))\right] . \qquad \text{(C.3)}$$

With this result to show that the kernel (61) turns into zero at $\vec{q}_2 = 0$ (and, due to the symmetries (12) at zero transverse momentum of any of the Reggeons) becomes a simple task.

References

[1] V.S. Fadin, E.A. Kuraev and L.N. Lipatov, Phys. Lett. **B60** (1975) 50; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Zh. Eksp. Teor. Fiz. **71** (1976) 840 [Sov. Phys. JETP **44** (1976) 443]; **72** (1977) 377 [**45** (1977) 199]; Ya.Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.

[2] L.N. Lipatov and V.S. Fadin, Zh. Eksp. Teor. Fiz. Pis'ma **49** (1989) 311 [Sov. Phys. JETP Lett. **49** (1989) 352]; Yad. Fiz. **50** (1989) 1141 [Sov. J. Nucl. Phys. **50** (1989) 712].

[3] V.S. Fadin and L.N. Lipatov, Nucl. Phys. **B406** (1993) 259; V.S. Fadin, R. Fiore and M.I. Kotsky, Phys. Lett. **B389** (1996) 737.

[4] V.S. Fadin, Zh. Eksp. Teor. Fiz. Pis'ma **61** (1995) 342; V.S. Fadin, R. Fiore and A. Quartarolo, Phys. Rev. **D 53** (1996) 2729; M.I. Kotsky and V.S. Fadin, Yad. Fiz. **59**(6) (1996) 1; V.S. Fadin, R. Fiore and M.I. Kotsky, Phys. Lett. **B359** (1995) 181; Phys. Lett. **B387** (1996) 593.

[5] V.S. Fadin and R. Fiore, Phys. Lett. **B294** (1992) 286; V.S. Fadin, R. Fiore and A. Quartarolo, Phys. Rev. **D 50** (1994) 5893; Phys. Rev. **D 50** (1994) 2265.

[6] V.S. Fadin and L.N. Lipatov, Nucl. Phys. **B477** (1996) 767; V.S. Fadin, M.A. Kotsky and L.N. Lipatov, Phys. Lett. **B415** (1997) 97; Yad. Fiz. **61**(6) (1998) 716.

[7] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. **B242** (1990) 97; Nucl. Phys. **B366** (1991) 135; G. Camici and M. Ciafaloni, Phys. Lett. **B386** (1996) 341, Nucl. Phys. **B496** (1997) 305; V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky, Phys. Lett. **B422** (1998) 287; Yad. Fiz. **62**(6) (1999) 1.

[8] V.S. Fadin, L.N. Lipatov, Phys. Lett. **B429** (1998) 127; M. Ciafaloni and G. Camici, Phys. Lett. **B430** (1998) 349.

[9] D.A. Ross, Phys. Lett. **B431** (1998) 161; Yu.V. Kovchegov and A.H. Mueller, Phys. Lett. **B439** (1998) 423; J. Blümlein, V. Ravindran, W.L. van Neerven and A. Vogt, in *Brussels 1998, Deep inelastic scattering and QCD* 211-216; E.M. Levin, Nucl.Phys. **B545** (1999) 481; N. Armesto, J. Bartels, M.A. Braun, Phys. Lett. **B442** (1998) 459; G.P. Salam, JHEP **8907** (1998) 19; M. Ciafaloni and D. Colferai, Phys. Lett. **B452** (1999) 372; M. Ciafaloni, D. Colferai and G.P. Salam, Phys.Rev. **D60** (1999) 114036; R.S. Thorne, Phys.Rev. **D60** (1999) 054031; S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov, JETP Lett. **70** (1999) 155.

[10] M.T. Grisaru, H.J. Schnitzer, Phys. Lett. **B30** (1973) 811; Phys. Rev. **D8** (1973) 4498; L.N. Lipatov, Yad. Fiz. **23** (1976) 642.

[11] Ya.Ya. Balitskii, L.N. Lipatov, V.S. Fadin, in: Proceeding of Leningrad Winter School, Physics of Elementary Particles, Leningrad 1979, pp. 109-149.

[12] J. Blumlein, V. Ravindran, W.L. van Neerven, Phys. Rev. **D58** (1998) 091502; V. Del Duca, C.R. Schmidt, Phys. Rev. **D59** (1999) 074004.

[13] V.S. Fadin, R. Fiore, Phys. Lett. **B440** (1998) 359.

[14] M. Braun, hep-ph/9901447.

[15] V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa, BUDKERINP/99-61, UNICAL-TH 99/3, hep-ph/9908264.

[16] V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa, BUDKERINP/99-62, UNICAL-TH 99/4, hep-ph/9908265.

[17] M. Braun, G.P. Vacca, Phys. Lett. **B454** (1999) 319.

[18] V.S. Fadin, R. Fiore, A. Papa, Phys. Rev. **D60** (1999) 074025.

[19] M. Braun, G.P. Vacca, hep-ph/9901447.

[20] J. Bartels, Nucl. Phys. **B175** (1980) 365.

[21] V.S. Fadin, preprint Budker INP 98–55, `hep-ph/9807528`; talk given at the International conference "LISHEP98", February 14-20, 1998, Rio de Janeiro, Brazil; to be published in the Proceedings.