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ENERGY RECUPERATOR FOR A SHEET BEAM OF MAGNETIZED ELECTRONS

(a numerical simulation)

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Abstract

Problem of energy recuperation for an electron sheet beam used to drive a mm-waves generator is considered. The main features of the solved problem are presence of magnetic field guiding beam electrons and significant energy and angular spreads of the electrons in the waste beam. We analyse two schemes of recuperator which differ by configurations. In the first one separation of electrons is realised by using of transverse magnetic field due to difference of Larmour radius of electrons with different energy of their longitudinal motion. In the second variant a spatial periodic transverse magnetic field is added to the longitudinal one on the deceleration section of electron trajectory. Resonance between bounce electron motion and cyclotron one gives strong increasing of Larmour radius for a small group of electrons with chosen particle energy. As a result, the electrons with resonance energy fall out to small local part of channel wall where collector plates are placed. Comparison of two schemes of the recuperator is also given in the paper.

Аннотация

Рассматривается задача рекуперации энергии электронного ленточного пучка, использованного для накачки генератора миллиметровых волн. Основными особенностями решаемой проблемы являются наличие ведущего магнитного поля и значительный энергетический и угловой разбросы электронов пучка, выходящего из генератора. Анализируются два варианта рекуператора с различной конфигурацией. В первом используется разделение электронов поперечным магнитным полем за счет различия ларморовских радиусов электронов с разной продольной энергией. Во втором варианте на участке торможения электронов к продольному магнитному полю добавляется периодическое в пространстве поперечное магнитное поле. Резонанс между ондуляторным движением электронов и циклотронным приводит к увеличению ларморовского радиуса для небольшой группы электронов с данной энергией частиц. В результате электроны с резонансной энергией попадают на небольшие участки боковой стенки канала, где размещены коллекторные пластины. В работе также проведено сравнение двух схем рекуператора.

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1. Introduction

1.1 Formulation of problem

In order to increase efficiency of energy transfer at electron beam applications one needs to return energy from a waste beam to the energy storage of an accelerator. Before today, a lot of recuperators were proposed for realising this process. For example, problem of beam energy recuperation was considered for the case of small energy spread [1,2] of the beam electrons and of absence of guiding magnetic field [3].

In case of using of E-beams to drive mm-wave generators [4,5] a waste beam has large energy and angle spreads after passing through the generator. Therefore, returning the energy from beam electrons to a power supply system becomes very complicated problem. In this paper we consider two schemes of recuperator for a magnetized sheet beam used in experiment on microwave generation. Below we discuss principles of recuperator operation

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1.2 Recuperator with a single collector

At first, let us consider the simple scheme of beam recuperation on a collector with fixed retarding potential. An energy spectrum of the beam is put as homogeneous in energy interval ε_1, ε_2 (fig. 1.1), where A_0 is a normalisation factor, $A_0 = N/(\varepsilon_2 - \varepsilon_1)$, N is a number of beam electrons passed per a time unit. Retarding potential of the collector corresponds to electron energy ε_0. The power of a spectrum part is determined as $W = \int \frac{dN}{d\varepsilon} \varepsilon d\varepsilon$. In this case, the total power of a beam is equal to $W_{full} = N(\varepsilon_2 + \varepsilon_1)/2$. Electrons with energy $\varepsilon < \varepsilon_0$ are reflected by the collector, and the power of this part of the beam is $W_{refl} = N/2 \cdot (\varepsilon_0^2 - \varepsilon_1^2)/(\varepsilon_2 - \varepsilon_1)$.

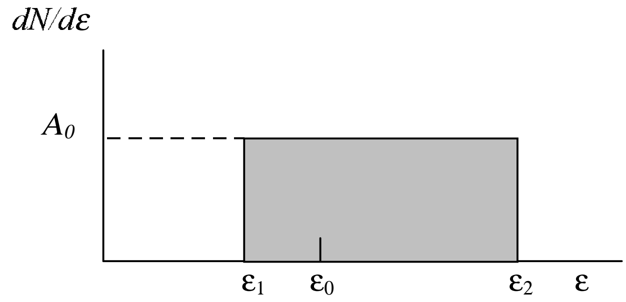

Fig. 1.1.

Power of other part of the beam that is absorbed by the collector, is equal

$$W_{ab} = \begin{cases} N/2 \cdot \left(\varepsilon_2^2 - \varepsilon_0^2\right)/\left(\varepsilon_2 - \varepsilon_1\right), & \varepsilon_0 \geq \varepsilon_1 \\ W_{full}, & \varepsilon_0 < \varepsilon_1 \end{cases}, \tag{1.1}$$

Some part of this power (W_{heat}) is spent to heat the collector and other one (W_{rec}) is returned to power supply system: $W_{ab} = W_{heart} + W_{rec}$ .The returned power depends on the potential of the collector as follows

$$W_{rec} = \begin{cases} N\varepsilon_0\left(\varepsilon_2 - \varepsilon_0\right)/\left(\varepsilon_2 - \varepsilon_1\right), & \varepsilon_0 \geq \varepsilon_1 \\ N\varepsilon_0, & \varepsilon_0 < \varepsilon_1 \end{cases} \tag{1.2}$$

We shall define efficiency of recuperation as $\eta(\varepsilon_0) = W_{rec} / W_{full}$. The efficiency has maximum at the energy $\varepsilon_0 = \max(\varepsilon_1, \varepsilon_2 / 2)$. This maximum is equal to

$$\eta_{max} = \begin{cases} 2\varepsilon_1/\left(\varepsilon_2 + \varepsilon_1\right), & \varepsilon_1 \geq \varepsilon_2 / 2 \\ \varepsilon_2^2 / 2\left(\varepsilon_2^2 - \varepsilon_1^2\right), & \varepsilon_1 < \varepsilon_2 / 2 \end{cases} \tag{1.3}$$

If the minimum energy of electrons of the beam is $\varepsilon_1 = 0$ then the efficiency is $\eta_{\max} = 0.5$. In case $\varepsilon_1 = \varepsilon_2 / 2$ it will be $\eta_{\max} = 2/3 \sim 0.67$.

The main advantages of the single collector scheme of the recuperator are its simplicity and tolerance to a magnetic field. But for the beams with broad energy spectrum the following features should be taken into account: 1) a rather low efficiency; 2) large part of the beam is reflected back into microwave generator and influences on its operation. To depress these disadvantages it is necessary to use multi-collector recuperation.

1.3 Multi-collector recuperator

For absorption of a beam with a broad energy spectrum as on fig. 4.1.1, n collectors with regular potentials corresponding to electron energies $\varepsilon_i = \varepsilon_0 + \Delta\varepsilon\,(i-1)$, where $i = 1,\ldots,n$; $\Delta\varepsilon = (\varepsilon_2 - \varepsilon_1)/n$ may be used (fig. 1.2). These collectors must absorb the beam electrons in the small energy interval $\Delta\varepsilon$. For this case the efficiency of recuperation is characterised with value of energy, returned to the power supply system, similar to (1.2)

$$W_{rec} = \frac{N}{n}\sum_{i=1}^{n} \varepsilon_i = \varepsilon_1 \frac{N}{n} + \frac{\varepsilon_2 + \varepsilon_1}{2} N \frac{n-1}{n}$$

Such a way we receive

$$\eta = \frac{2}{n}\frac{\varepsilon_1}{\varepsilon_2 + \varepsilon_1} + \frac{n-1}{n}\,. \qquad (1.4)$$

One can see that increasing of number of collectors increases the efficiency of such "ideal" recuperator to 100%. In this paper we shall consider two multi-collector schemes, suitable for practical realization.

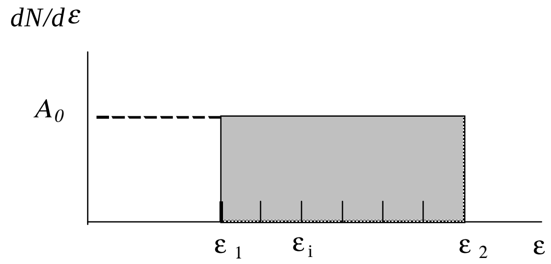

Fig. 1.2.

2. A recuperator with separation of electrons by a non-uniform magnetic field

2.1 General scheme

It is obvious, that the process of recuperation should consist of two stages, at least. The first one provides deceleration of a part of the fast electron beam up to low energies, which will ensure the electron absorption on a recuperator collectors without damage of covering materials. The second stage consists in a transportation of decelerated electrons from a beam to collectors.

The first stage is traditionally realised by sequential passing of a beam through gaps where a decelerating electrical field (fig. 2.1) is supplied. For the second stage of process it is possible to use static cross magnetic fields.

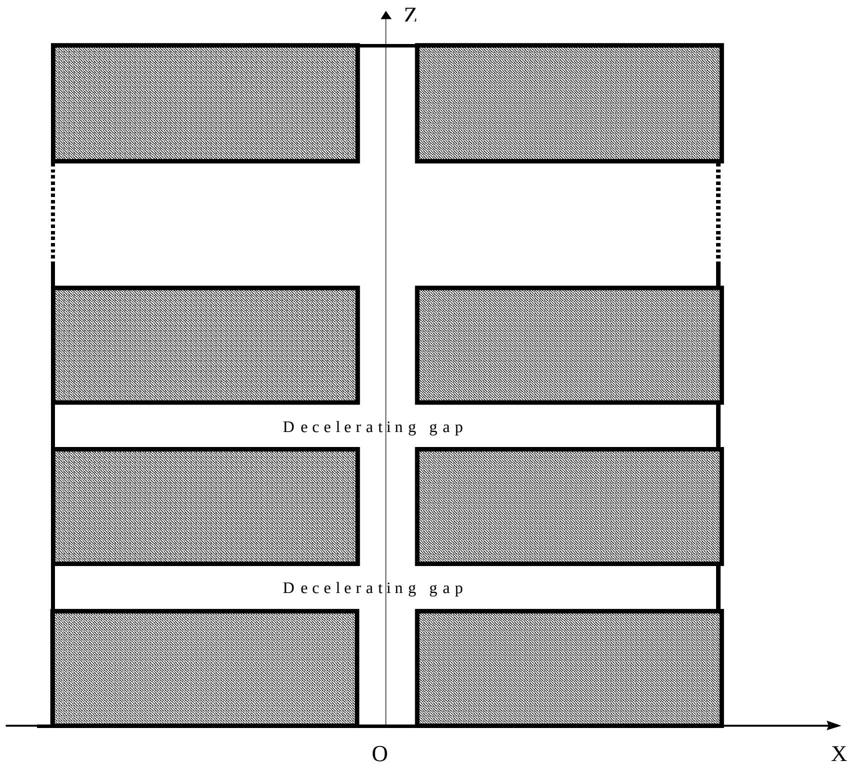

Fig. 2.1. The basic scheme of sectioned recuperator.

2.2 Configuration of electrical and magnetic fields

Magnetic fields

For spatial separation of electrons with various energies it is possible to use a transversal constant homogeneous magnetic field.

We shall consider a motion of electron in this field. Assume, that it has only two components: along axes OX and OZ:

$$\vec{B} = B_x\vec{e}_x + B_z\vec{e}_z, \quad \frac{B_z}{B_x} = \mathrm{tg}(\alpha) < 1. \tag{2.1}$$

We suppose that in an initial moment of time the velocity of a particle is directed along an axis Z: $v_z(0) = v_0$ (fig. 2.2).

The analytical solution of the motion equations (see [6]) results in the dependence of velocity and coordinates of a particle on time accordingly to the following formulas:

$$v_x = v_0 \sin(\alpha)\cos(\alpha)(1-\cos(\omega t)), \quad x = v_0 \cos(\alpha)\sin(\alpha)\left(t - \frac{\sin(\omega t)}{\omega}\right),$$

$$v_y = v_0 \cos(\alpha)\sin(\omega t), \quad y = \frac{v_0 \cos(\alpha)}{\omega}(1-\cos(\omega t)),$$

$$v_z = v_0(\sin^2(\alpha) + \cos^2(\alpha)\cos(\omega t)), \quad z = v_0(\sin^2(\alpha)\, t + \frac{\cos^2(\alpha)\sin(\omega t)}{\omega},$$

$$\omega = \frac{ec|\vec{B}|}{W}. \tag{2.2}$$

Here $W = mc^2$ is a total energy of electron; e, $m = \gamma m_0$ are its charge and a mass of rest, g is a relativistic factor, c is the light velocity.

It follows from the considered formulas that electron, originally moving in OZ direction, begins to move in a magnetic field along a screw line with an axis along a magnetic field (which direction has an angle α with an axis of coordinates OX) and with Larmour radius

$$r = \frac{v_0}{\omega}\cos(\alpha).$$

In particular, if coordinate of electron is $z = z_{\max}$,

$$z_{max} = \frac{v_0 W}{ec|B|}(cos^2 \alpha \, sin \, wt_m + sin^2 \alpha \cdot wt_m), wt_m = \frac{\pi}{2} + arcsin(tg^2 \alpha) \qquad (2.3)$$

then direction of its motion along an axis Z changes on the opposite one.

The linear dependence of Larmour radius of the electron on its velocity can be used for spatial separation of slow electrons from faster ones.

If the size of area along axis OZ, where constant magnetic field is given, is equal to *l*, then it is possible to remove electrons with $z_{\max} < l$ in such a field.

The calculations show that slow electrons are removed from a beam, but more high-energetic ones gaining some perturbations from magnetic field continue moving in the initial direction. Since the large amplitude of trajectories perturbations for high-energetic particles can lead to undesirable loss of electrons on walls it is possible to compensate perturbations by applying a magnetic field of an opposite direction (see fig. 2.2) in area $l < z < 2 \cdot l$.

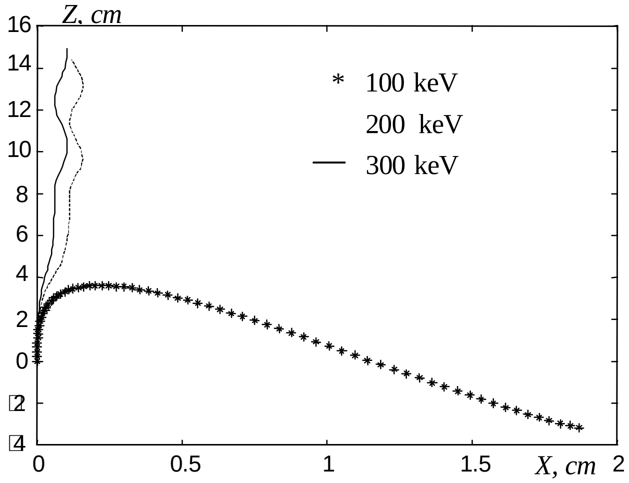

Рис.2.2. Trajectory of particles with different energy in a constant magnetic field, dependent on coordinate *z*:

Bx=0.35 kGs,	*Bz*=0.04 *kGs*	*for z* < 4 cm
Bx=–0.35 kGs	*Bz*=–0.04 *kGs*	*for 4* < *z* < 8 cm
Bx=*0, Bz*=3 kGs		*for z*> 8 cm

For the consideration of recuperator with absorption of particles on walls, it is necessary for a condition of electrons removing from a beam

$$z_{max}(v_0) < l,$$

to add a condition of absorption of a particle within the area of external homogeneous field. Otherwise particles can begin a motion in an opposite direction, or will move parallel to a motion of a beam along recuperator's walls, forming an additional space charge and disturbing a structure of an initial beam. This condition for the motion equations (2.2) in the homogeneous magnetic field within an interval $0 < z < z_{\max}$ may be written as:

$$0 < z(t) < l, \text{ for } x < a, \qquad (2.4)$$

where a is a width of the recuperator channel.

These two conditions allow us to determine a required configuration of a magnetic field for given values of threshold electron energy and magnetic field $\vec{B}$ (or the size l of the area occupied by a field).

In particular, for small angles $tg\,\alpha << 1$ the value of a magnetic field within a region of length l necessary for extracting of electrons with energy smaller than W_{th} from a beam will be determined by the expression $B \approx B_x \approx \sqrt{2W_{th}m_e}\,\dfrac{c}{e\,l}$ with the field incline angle to the axis OX

$$Sin\alpha \approx \frac{a}{\pi r} \cong \frac{a}{\pi l},$$

Thus, the following scheme of one section of recuperator (fig. 2.3) might be considered to provide a fall out process of low-energy electrons from a beam to collector.

If one keeps in mind a recuperation of electrons with energy less than 100 keV within one section, the parameters of magnetic fields may be the following:

Table 2.1

Parameters	A separating gap	A compensating gap	A stabilizing gap
length of gap, cm	4	4	2
B_x, kGs	0.35	–0.35	0
B_z, kGs	0.04	–0.04	3.0

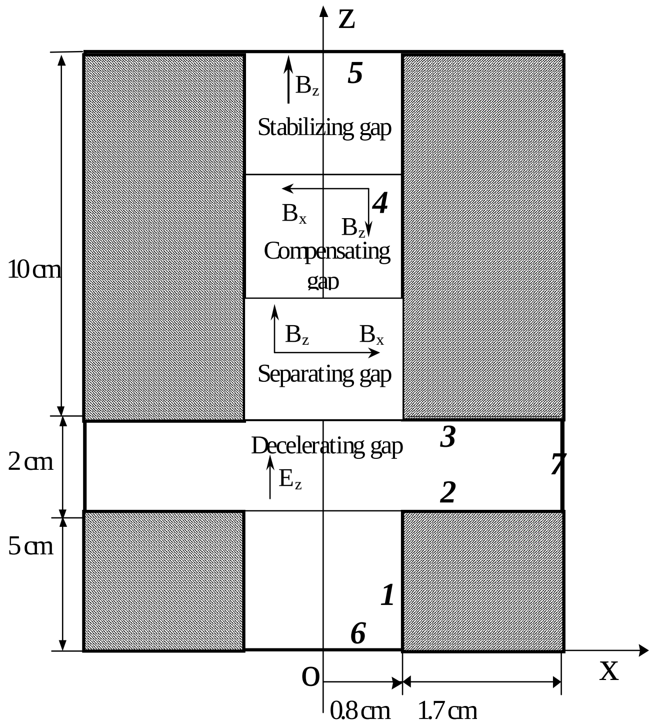

Fig. 2.3. The basic scheme of a recuperator section.

Electrostatic fields in a decelerating gap

Geometry of one section of recuperator is represented on fig. 2.3, the internal walls of recuperator are assumed to be metal.

The electrical fields in a decelerating gap are defined by potentials that are considered to be given on metal surfaces and external borders of the area. For calculation of potentials inside recuperator, it is necessary to solve the Laplace equation $\Delta\varphi = 0$ with the following boundary conditions:

- on surfaces ***1, 2, 6*** constant potential $\varphi(x) = 0$ kV is given;
- on surfaces ***3, 4, 5*** constant potential $\varphi(x) = 100$ kV is given;
- on a surface *7* linearly growing potential $\varphi(z) = 50^{*}(z - 5\ \mathrm{cm})$ kV is given;
- the axis OZ is an axis of symmetry.

The Laplace equation is solved numerically. A difference representation of it looks as follows $\Delta\varphi = \dfrac{\varphi_{i+1,j} + \varphi_{i-1,j} + \varphi_{i,j+1} + \varphi_{i,j-1} - 4\,\varphi_{i,j}}{h^2}$.

The problem is solved in *Mathlab* environment by establishment solution method with the subsequent data transferring to the program *EMC2D*. An initial approximation is zero. The steps of a difference grid on axis X and Z are identical and equal to 0.05 cm.

The calculated electrostatic fields E_x and E_z are shown. on fig. 2.4 *a)* and *b)*

The fields' values are given in units of [V/m]

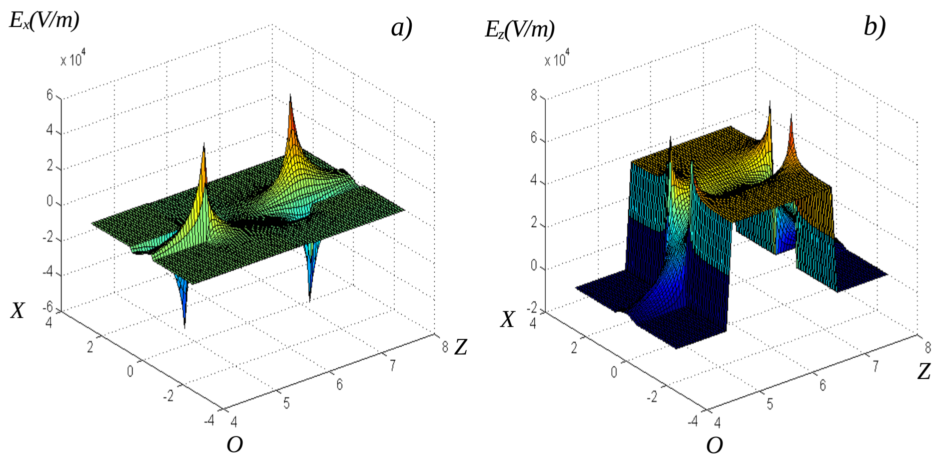

Fig. 2.4a) and b). An electrostatic fields Ex and Ez within a decelerating gap of recuperator.

2.3 Numerical simulation of an electron beam passing through the recuperator unit

Formulation of a problem.

Into the system, represented on fig. 2.5, consisting of three separating gaps, the beam symmetric in respect to axis Z with uniform distribution of electrons on energies in range from 200 up to 500 keV is injected. Beam thickness is 0.6 cm. An electrical field decreasing energy of particles on 100 keV is applied within a gap. A combination of magnetic fields intended for removal of low-energy particles with energy 100–150 keV from a beam (table 2.1, fig. 2.3) is created between decelerating gaps. The consecutive system of electrical and magnetic fields will allow, gradually reducing energy of a beam, to remove from it the low-energy particles which energy can be used for the second time.

Electron beam motion in external electromagnetic fields (the interaction of particles is not taken into account).

All electrons in a beam are divided in 6 groups on energy, designated by different colours (or by different intensities of black colour on figure):

Number of group	1	2	3	4	5	6
Energy (keV)	E<150	150≤E<250	250≤E<300	300≤E<400	400≤E<500	E≥500
Colour	black	green	red	blue	violet	yellow

Electron beam behaviour in recuperator is represented on fig. 2.5 *a)*.

One can see on the figure that low-energetic electrons are removed from a beam and fall on recuperator walls. At the same time electrons with higher energy continue a motion, though the bea quamlity becomes worse.

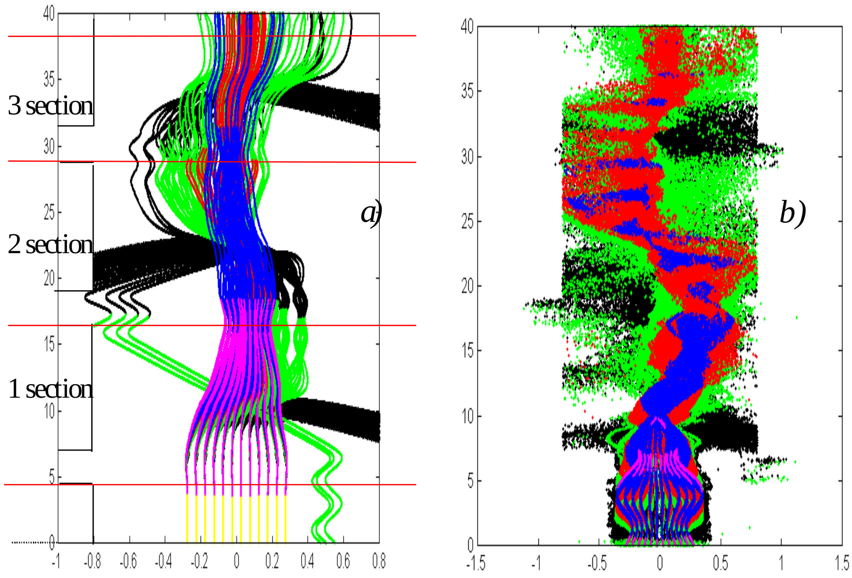

Fig. 2.5 a) and b). Nonself-consistent and self-consistent motion of an electron beam through recuperator for different separation fields,
Bx=0.35 kGs, *Bz*=0.04 kGs.

Self-consistent motion of an electron beam in recuperator

An approximation, in which interaction between particles is ignored, is valid in the case when electron density in a beam is small. In practice intensive beams with linear density of a current ~1 kA/cm are of interest. In this case, a nonself-consistent model describes rather well behaviour of a beam only at initial stage. Since some moment, it is necessary to take into account interaction of particles in a beam.

The fig. 2.5 б) demonstrates the space distribution of particles in recuperator, obtained in self-consistent calculation for a beam with linear density of a current equal 1 kA/cm. One can see from a figure that though a beam does not scatter it is bent. It can result in falling high-energy particles to walls of the device when a beam passes the subsequent sections of recuperator. Nevertheless, low-energy particles are removed from a beam in each separating gap.

The displacement of a beam along an axis X is possible to decrease by reducing of a separating gap, but consequently this reducing lead to increasing longitudinal and transversal magnetic fields in the gap.

We can obtain from the estimation that for the channel with a transversal width of 1.6 cm and a separating gap of 1 cm the following values of magnetic fields are:

Bx=1.15 kGs, Bz=0.8 kGs.

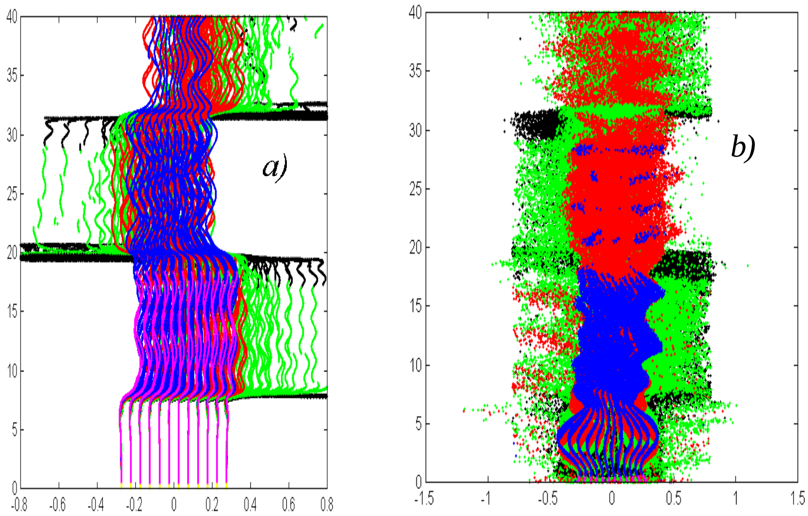

Fig. 2.6 a) and b). Nonself-consistent and self-consistent motion of an electron beam through recuperator for separating fields values Bx=1.15 kGs, Bz=0.8 kGs.

Fig. 2.6 *a)* represents for nonself-consistent case the electron beam behaviour in recuperator with a separating gap of 1 cm and the values of magnetic fields in a gap Bx=1.15 kGs, Bz=0.8 kGs. The fields in a compensating gap of width 1 cm are equal to Bx=–1.15 kGs, Bz=–0.8 kGs respectively. The size of a stabilising gap has been increased up to 8 cm and the value of magnetic fields in it has remained as Bx=0, Bz=3 kG*s*. Low-energy particles leave a beam, and the beam itself is not bent practically.

Spatial distribution of particles in recuperator, obtained for self-consistent calculation for a beam with stationary current density of emission is presented on fig. 4.2.6 *b)*. In self-consistent formulation the particles flying into a separating gap, have *x*-component of velocity. Nevertheless, a main part of low-energy particles leaves area.

3. Recuperator with a resonant increasing of transversal velocity

3.1 Principle of operation

The presence of a driving magnetic field where an electron beam with broad energy spectrum spread makes a problem of recuperation much more complicated. Here a resonant scheme of recuperation is offered, taking into account the peculiarities of geometry of a sheet beam. It distinguishes from the standard systems by using a resonance on Larmour frequency.

We consider a sheet beam of width *2d* with electron energy in a range $(\varepsilon_1, \varepsilon_2)$, passed system of generation of microwaves and being transported in a longitudinal magnetic field B_z. The side walls of transport channel of width *2a* and length *L* are made of conducting strips (collectors), extended across a direction of a motion of a beam and separated by insulator strips. Potential of collector strips is equal to zero on an entrance and decreases along a recuperator length down to value $-\varepsilon_2 / e$, where $e > 0$ is the elementary charge. Following to traditions of ideology of free electrons laser, we add to a longitudinal magnetic field a transversal field B_x that is periodic in space with a periodicity length λ and satisfies to a condition of resonance $\omega = eB_z / \gamma m_0 c = \Omega = 2\pi p_z^* / \lambda \gamma m_0$. Here γ is relativistic factor, m_0 is an electron mass of rest and p_z^* is its longitudinal momentum, provided resonance between cyclotron ω and undulating Ω frequency. An electrical field formed by conducting strips decelerates beam electrons, being entered to system, decreases their longitudinal velocity, that reaches the resonant value. Due to increase of Larmour radius electrons can reach collectors, and can be absorbed with respectively small kinetic energy. If this increased transversal energy is insufficient for absorption, electrons stop in longitudinal motion and then come back in the same electrical field, increasing longitudinal velocity. When they reach the resonant value again, the transversal energy increases again until electron will touch collector plate. Its longitudinal energy corresponds to momentum p_z^* and transversal energy – to Larmour radius, at which the electron orbit touches a wall. These components determine residual electron energy in the considered scheme of recuperator. Probability of electron absorption at return way is more, because here the increase of transversal energy at a resonance is compensated by work of a longitudinal electrical field on returned beam electrons.

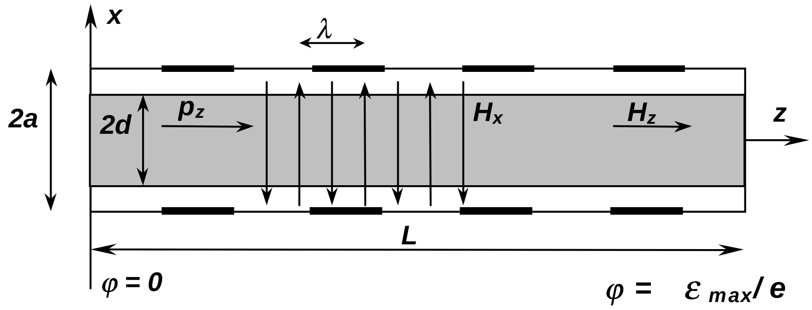

Fig. 3.1. A scheme of resonant recuperator.

Basic elements of the scheme and the system of coordinates are shown on fig. 3.1. A blacked strip picks out the area of passage of a beam.

An amount and the spatial distribution of collector plates, specifying also potential of space, are determined both by desirable accuracy of a longitudinal electrical field and by required spatial resolution of energy groups of decelerated electrons.

3.2 The initial equations

The motion of electrons in an electromagnetic field is determined by a usual way

$$\frac{d\vec{p}}{dt} = -e\vec{E} - \frac{e}{c}\left[\vec{V} \times \vec{B}\right], \qquad (3.1)$$

in fields

$\vec{E} = \{0,0,E_z\}$, $\vec{B} = \{B_x, B_y, B_z\} = \{B_t \mathrm{ch}\,(kx)\cos((kz),0, B_0 - B_t \mathrm{sh}\,(\mathrm{kx})\sin(\mathrm{kz})$, where $k = 2\pi/\lambda$. After designating $\alpha = B_x / B_z$, using complex representation for cross components of a momentum $p = p_x + i\,p_y$ and after adding the equation for longitudinal coordinate z, we rewrite (3.1) as

$$\begin{cases} \dfrac{dp}{dt} = i\omega p - i\omega\alpha p_z\,, \\ \dfrac{dp_z}{dt} = -eE_z + \omega\alpha p_y\,, \\ z = \int\limits_0^t p_z / \gamma m_0 \cdot dt + z_0 \end{cases} \tag{3.2}$$

Assuming a cross magnetic field as small in comparison with longitudinal one: $B_t << B_0$, we shall accept approximately $\omega \sim \omega_0 = eB_0 / \gamma\, m_0 c$ and for near-axis area of a beam we get $\alpha \sim B_t / B_0 \cos(kz)$, $B_z = B_0$.

We shall take into account in (3.2) a Larmour rotation by substituting $p = p_0(t)\exp(i\omega_0 t)$

$$\begin{cases} \dfrac{dp_0}{dt} = -i\omega_0 \dfrac{B_t}{B_0} cos(kz) exp(-i\omega_0 t)\cdot p_z \\ \dfrac{dp_z}{dt} = -eE_z + \omega_0 \dfrac{B_t}{B_0} cos(kz) Im(p_0\, exp(i\omega_0 t)) \\ z = \int\limits_0^t p_z / \gamma m_0 \cdot dt + z_0 \end{cases} \tag{3.3}$$

$$\sqrt{(p_0^2 + p_z^2)c^2 + m_0^2 c^4} = W_0 + e\varphi(z)$$

The last equation notes the conservation law for energy and is a consequence of two first ones with relativistic definition of total energy W_0.

3.3 Characteristics and scales of the decision

From a view of the equations (3.3) it is possible to make the following conclusions about properties of the solution and to estimate meanings of separate parameters and variables.

- The system is resonant under condition of $kz = \omega_0 t$, or $kp_z = eB_z / c$.
- At a resonance the rate of change cross components of a momentum can be appreciated from an expression $\Delta p_0 \sim \omega_0 t / 2 \cdot B_t / B_0 \cdot p_z$.
- At a deviation of a momentum p_x from resonant value $kp_z / \gamma m - \omega_0 = \Delta\omega$ the maximum meaning of cross components of a momentum p_0 is determined by a time of beats period $\Delta p_0 \sim \omega_0 \pi / \Delta\omega \cdot B_t / B_0 \cdot p_z$.

For maintenance of a resonance at $B_z = \mathrm{const}$ it is necessary to support a longitudinal momentum close to resonant value $p_z^* = eB_0 / c \cdot \lambda / 2\pi$. Thereby, a following expression would be fulfilled

$$eE_z \sim \omega_0 \frac{B_t}{B_0} \cos^2(\omega_0 t)\, p_0 \sim \omega_0 \left(\frac{B_t}{B_0}\right)^2 \frac{\omega_0 t}{4} p_z^* \qquad (3.4)$$

that results from the second equation of system (3.3) after averaging on a period of oscillations. The expression (3.4) determines a longitudinal electrical field and rate of its increasing along a trajectory of electron. It is necessary for supporting of its longitudinal momentum during grow of transversal one. We put $p_0 \sim p_{\max} = eB_0 a' / c$, $a' \sim (a - d/2)$ for estimate a field E_z in (3.4). In this case, the most part of electrons falls on side walls of the channel. A transversal momentum of electron may arise up to p_0 at a resonance during $t \sim eB_0 a' / c \cdot B_0 / B_t \cdot 2 / (\omega_0 p_z^*)$. This estimate gives maximum value of a longitudinal electrical field inside recuperator

$$eE_z^{\max} \sim \frac{\omega_0}{2} \frac{B_t}{B_0} \frac{eB_0 a'}{c} = \frac{a'}{2} \frac{B_t}{B_0} \frac{(eB_0)^2}{\gamma m_0 c^2} \qquad (3.5)$$

As one can see from an expression (3.4), the condition of supporting of a resonance cannot be fulfilled strictly for electrons with different energy simultaneously. It demands the increase of an electrical field value proportionally to distance passed by electron from a coordinate of a beginning of resonant interaction. This coordinate is essentially different for electrons with different total energy. Therefore in a mode of beats we shall choose an electrical field with some value of a field $E_z \le E_z^{\max}$, optimal for support of a resonance and that is constant on length of system.

As well as in simple version of recuperator, space charge influences on a value of E_z, together with potentials of side electrodes – strips, especially for a large current of beam, comparable to a limiting current. It amplifies near to a point of stop of electrons at their longitudinal movement, where their contribution to charge concentration is maximal, they pass this part of a way twice (or more, at occurrence longitudinal oscillations of their motion). In principle, it is possible to reduce a space charge, by moving a beam apart with a magnetic field along its greater cross size, but evidently, in some narrow limits. This factor can be correctly taken into account only by the joint solving of the equations of a motion and Poisson equation as a self-consistent problem.

3.4 Choice of physical parameters of a problem

We shall set the initial data and define numerical values of parameters for computer simulation of recuperator. Based of conditions of experiment on microwaves generation we shall define the following data for recuperator and beam. A longitudinal magnetic field is B_0 = 2.5 kG*s,* the thickness of a beam $2d$ = 0.5 cm and size of gap between sides of the channel $2a$ = 1.4 cm, are determined by a magnetic field and geometry of a beam leaving from the generator, in addition divorced along the greater cross size in 2 times. Linear density of a current is $J = 100$ A/cm, the energy spectrum of electrons is presumably in a range $W = 500 - 1000$ keV, angular spread of velocities is about 5°. Cross width of a beam is 20 cm or more; it allows us to consider the geometry of problem as plane one and such a way a cross width does not include in a problem itself. The undulate magnetic field $B_t / B_0 = 0.2$ is formed by coils and so its step of periodicity λ is supposed to be not less than cross size of the recuperator channel. We shall set a step λ using condition of a resonance for electron with a residual longitudinal momentum p_z^* (that appropriates to energy in non relativistic approximation, $\varepsilon_z^0 = (p_z^*)^2 / 2m$: $\lambda = 2\pi / \sqrt{2m_0 c^2 \varepsilon_z} / eB_0 \approx 0.67\,[\mathrm{cm}]\,\sqrt{\varepsilon_z\,[\mathrm{keV}]} / B_0\,[\mathrm{kGs}]$. Accepting $\varepsilon_z^0 = 100$ keV and $B_0 = 2.5$ kGs, we get $\lambda = 2.68$ cm that is larger than width of the channel. The complete residual energy of electrons e_{kin} is sum of longitudinal ε_z^0 and transversal energy $\varepsilon_0 = p_{\max}^2 / 2m \sim 180$ keV and is thus, about 300 keV. In principle, it may be reduced by additional moving a beam apart with a magnetic field due to reduction both a magnetic field and a value of a resonant longitudinal momentum. For above mentioned conditions the maximum electrical field will be $E_z^{\max} \sim 6.3 \cdot 10^4$ V/cm.

3.5 Numerical simulation

The solution of a complete self-consistent problem was carried out with a package of the applied codes POISSON-2 [7]. In particular it allows us to calculate electrical fields formed by electrodes and space charge of beam electrons, the initial energy and angular characteristics of which are set on entrance of system. Electrons move in an external magnetic and self-consistent electrical field from entrance border up to a contact with side walls of a channel. It was set 5 angular (with angular velocities corresponded to 0 and $\pm 5^{\circ}$ in two cross directions) and 6 energetic (with initial energies 0.5 - 1 MeV through 0.1 MeV) groups of trajectories with identical currents. They began on entrance border, that is of 5 mm width, in five points on coordinate x (see fig. 3.1): one central, two border and two intermediate trajectories for each energy and for each angle. Thus, the beam was represented by 150 trajectories. Potential of side collectors was set variable along beam motion, linearly changing in gaps between collectors, potential of which also linearly increased from value $U = 0$ on an entrance up to $U = -1$ MV at the end of the channel through 100 kV. Length of gaps and collectors was set identical, equal $\Delta z = 10$ cm, so that total length of the channel was 200 cm. It corresponds to a longitudinal electrical field 10 kV/cm in gaps.

The results of modelling are shown on fig. 3.2. On an axis of ordinates is energy of electrons for each energy subgroup, on an axis of abscissa is shown coordinate, on which the appropriate trajectories are absorbed on some collector of the channel. "Bars" characterize an error of simulation in conservation low for total energy of electron. The dashed line corresponds to distribution of potentials of a side wall and plates on length of system.

The influence of a space charge to distribution of absorption of electrons on length of system for given parameters of a beam and of recuperator is insignificant.

The energy of a beam, entered into recuperator, is distributed as follows. About 60% of energy comes back to power supply system, 5–7% is lost with reflected back particles, and ~35% of energy of a beam is transformed to heat on plates of a collector or on insulators. As one can see, for those parameters of a beam the efficiency of recuperation in such system appear even less, than in the simple one-collector scheme considered in Introduction. The losses in recuperator are determined, mainly, by residual energy of particles. Really, for a considered beam the average energy of electrons is $\varepsilon = 750\,\text{keV}$ and for residual energy $\varepsilon_{\text{kin}} = 280\,\text{keV}$ the efficiency of recuperation will be $\eta \sim 63\,\%$.

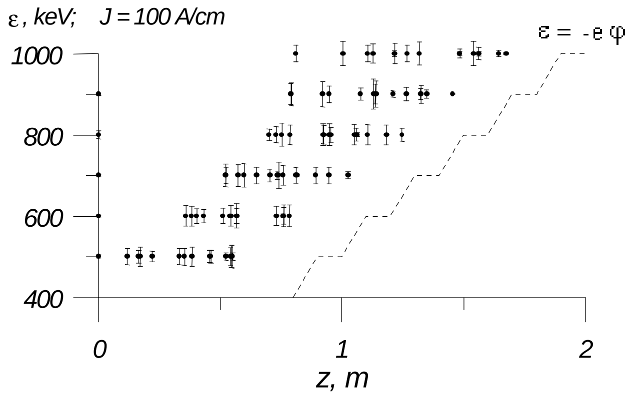

Fig. 3.2. Distribution of absorbed electrons on recuperator length;
$B_0 = 2.5$ *kGs*, $\varepsilon_0 \sim 180$ *keV*, $\varepsilon_z^0 = 100$ *keV*, η=62%.

Residual kinetic energy of electrons depends on value of a longitudinal magnetic field and space step of undulating field. Moreover, the step is limited by width of the channel (more precisely, distance between coils, creating undulate magnetic field) and allowable non-homogeneity of a longitudinal field. For reduction of residual energy we choose the following parameters of a beam. By increasing its cross width (after an exit from the microwave generator) twice (i.e., up to 40 cm), we shall reduce thus a driving magnetic field up to B_0 = 1.25 kGs and set a step of undulate magnetic field equal to 2 cm. Thus, linear density of a beam current will be ~50 A/cm, residual cross energy will be about $\varepsilon_0 \sim 45$ keV, and longitudinal one $\varepsilon_z^0 \sim 15$ keV. The pitch-angles of electron velocities will be ~2.5 degrees. We choose a longitudinal electrical field equal to 5 kV/cm, that is less than its limiting value, equal to $E_z^{\max} \sim 16\,\text{kV/cm}$. For this purpose we set a longitudinal gaps and length of plates equal to 20 cm, and total length of system equal to 4 meters.

For such parameters of system and beam the efficiency of recuperation has increased ($\eta \sim 79\,\%$), but there was appeared the appreciable quantity of reflected electrons, carrying away ~14% of energy of a beam. The last 7% of energy are transformed to heat on collectors. Calculation without a space charge of a beam has shown that its influence at these parameters also is insignificant. In absence of a space charge the efficiency of recuperation of system is $\eta \sim 82\%$. Thus about

11% of energy of a beam are reflected and 7% is lost on collectors. Note, that the maximal efficiency for residual energy of electrons $\varepsilon_{kin} \sim 60$ keV is ~92%. The actual efficiency is lower of maximal on 10–15% because of reflected electrons.

The second version looks as more preferable from considered versions if to bear in mind a thermal loading on a collector. The main disadvantage of it is noticeable number of reflected back electrons.

Actually, it is expected that the space charge of a beam will be compensated by ions, being generated by the beam itself in the channel. On the other hand, on channel walls a formation of plasma, extending on a magnetic field, is possible which can break electrical isolation in gaps. Therefore it is necessary to reduce a value of an electrical field in gaps and density of a current, collected on plates and insulators. The second version also looks as more preferable from these positions.

The efficiency of recuperation of electron beam energy with a wide energy spectrum was checked for a beam with uniform distribution of electrons on an energy spectrum in a range 100–1000 keV. The result of simulation is shown on fig. 3.3. In this case, as well as in case of absence of a space charge of a beam, the efficiency of recuperation was ~80% at thermal effect on collectors ~10% and reflection of 10% energy. The maximal meaning of efficiency expects ~88% with residual energy of electrons ~60 keV. We would remind, that using of simple recuperator for such beam its efficiency does not exceed 50% (see (1.3)), at that 25% of energy is reflected and 25% transforms to heat.

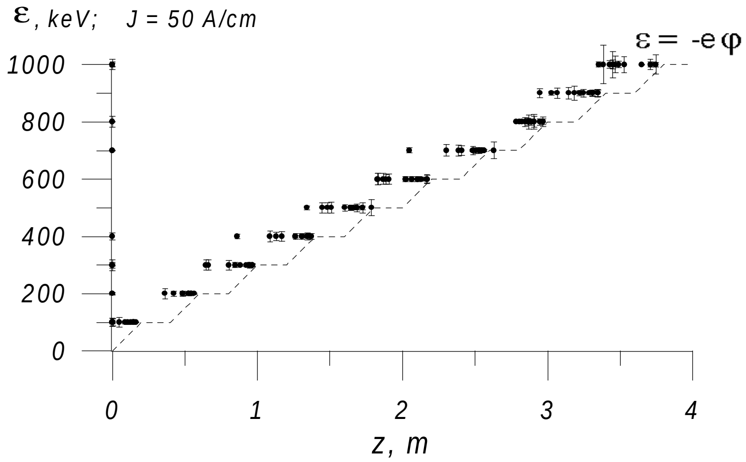

Fig. 3.3. Distribution of absorbed electrons of beam with wide energy spectrum along recuperator length in magnetic field B_0=*1.25 kGs*. Residual energies are $\varepsilon_0 \sim$ *45 keV*, $\varepsilon_z^0 \sim$ *15 keV*, an efficiency is $\eta \sim 80\%$.

The influence of angular spread of velocities was checked by calculations with parameters of fig. 3.3, but with twice large pitch-angles ($\pm 5^\circ$). As well as it was possible to expect, the efficiency of recuperation has fallen up to ~75% with increase of losses on heating up to ~15%.

At last, reduction of a decelerating electrical field from 10 kV/cm down to 2.5 kV/cm results to decreasing energy reflected electrons from ~20–25% down to ~5%. It means that this value of field $E_z = 2.5$ kV/cm is more optimal than previous value 10 kV/cm. The decreasing of electric field between plates of a collector is favourable for electrical strength of system too.

Thus, the considered method allows us to receive good results for recuperation of beams with a wide energy spectrum and small angular spread of velocities. As to beams with narrow spectrum $(\varepsilon_2 - \varepsilon_1)/\varepsilon_2 \ll 1$, close to monochromatic ones, using of one-collector recuperator may be more effective for them.

4. Conclusions

Two types of recuperation systems for a sheet electron beam with a typical energy 1 MeV and a linear current density in the interval 0.05–1 kA/cm are considered.

It is shown that the variant of the recuperator described in sections 4.2 and 4.3 allows one to make recuperation of electron energy with a good efficiency for a wide energy spectrum. The main obstacle to realise the first variant is necessity to generate vacuum magnetic fields with large gradients and required configuration. As to the second type, its disadvantages are to reach good electrical insulation at presence of beam electrons that is not simple task and to depress influence of a returned electron flow on the operation of the mm-wave generator is also complicated problem.

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