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НОВОСИБИРСК

SUPERLUMINAL VELOCITY OF PHOTONS IN A GRAVITATIONAL BACKGROUND

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Abstract

The influence of radiative corrections on the photon propagation in a gravitational background is investigated without the low-frequency assumption $\omega \ll m$. The conclusion is made in this way that the velocity of light can exceed unity.

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1. The question addressed in the present article was raised many years ago by Drummond and Hathrell [1]. They noted that tidal gravitational forces on the photons, induced by radiative corrections, in general alter the characteristics of propagation, and pointed out that due to it photons may travel in some cases at speeds greater than unity. To be more precise, in a local inertial frame the induced curvature terms in the Maxwell equation survive and modify the light cone in different ways for different polarizations. The analogous conclusion for the neutrino in a Friedman metric was made somewhat later by Ohkuwa [2]. Recently the results of Ref. [1] were generalized for charged black holes by Daniels and Shore [3].

The approach of Ref. [1] consisted in expanding the contribution to the photon effective action from one-loop vacuum polarization to the lowest order in the inverse electron mass squared $1/m^2$. Therefore, their result by itself refers strictly speaking to low-frequency photons with $\omega \ll m$ only. Meanwhile, the velocity of the wave-front propagation in a dispersive medium is determined by the asymptotics of the refraction index $n(\omega)$ at $\omega \to \infty$ (see, e.g., [4]). It is argued however in Ref. [1] that due to the dispersion relation for the refraction index $n(\omega)$, its high-frequency asymptotics $n(\infty)$ is related to the low-frequency one $n(0)$ as follows:

$$n(\infty) = n(0) - \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega}\,\mathrm{Im}\,n(\omega). \tag{1}$$

Then, since $\mathrm{Im}\,n(\omega)$ is nonnegative,

$$n(\infty) \leq n(0)$$

which would guarantee the superluminal propagation of the wave front. The shortcoming of this argument, as pointed out in Ref. [5], is that the sign of Im $n(\omega)$ in the problem of interest is not fixed, generally speaking. Indeed, the physical meaning of the condition

$$\text{Im } n(\omega) \geq 0$$

is that in a homogenious medium without instabilities (particle creation) the wave amplitude can decrease only, due to the loss of particles from the beam. However, in an inhomogenious medium (and this is the case of a gravitational background) the processes of the beam focusing and bunching are possible, leading to the increase of the wave amplitude which corresponds to

$$\text{Im } n(\omega) \leq 0.$$

The analysis performed in Ref. [1] has demonstrated that even if the superluminal propagation takes place indeed in this way, it does not violate causality. Still the effect discussed is quite unexpected and interesting, and it is certainly worth efforts to find out whether the predicted phenomenon is a true one or just a result of an inadequate approximation. In the present paper the problem is addressed without the low-frequency assumption $\omega \ll m$. In this way we come to the conclusion that in a gravitational background photons can propagate indeed with superluminal velocities.

2. We will start with the discussion of the general structure of the photon-graviton vertex. To the lowest order in the momenta k' and k of the outgoing and incoming photons respectively, there is the well-known minimal interaction:

$$\frac{1}{2}\kappa h_{\mu\nu} T_{\mu\nu}. \qquad (2)$$

Here $\kappa h_{\mu\nu}$ is the deviation of the metric from the flat one:

$$\kappa h_{\mu\nu} = g_{\mu\nu} - \delta_{\mu\nu}, \quad \kappa^2 = 32\pi G$$

where G is the Newton constant. The matrix element of the energy-momentum tensor of electromagnetic field is

$$T_{\mu\nu} = -F'_{\mu\lambda}F_{\nu\lambda} - F'_{\nu\lambda}F_{\mu\lambda} + \frac{1}{2}\delta_{\mu\nu}F'_{\kappa\lambda}F_{\kappa\lambda} \qquad (3)$$

where

$$F'_{\mu\lambda} = i(k'_\mu e'_\lambda - k'_\lambda e'_\mu),$$

$$F_{\nu\lambda} = -i(k_\nu e_\lambda - k_\lambda e_\nu),$$

and e'_μ, e_ν are the polarization vectors of the outgoing and incoming photons, respectively.

To investigate other possible structures for this vertex, let us introduce vectors

$$p_\mu = k'_\mu + k_\mu, \quad q_\mu = k'_\mu - k_\mu$$

orthogonal on mass shell, $p_\mu q_\mu = 0$. The next two independent vectors can be conveniently chosen as

$$g_{1\mu} = k'_\nu F_{\nu\mu}, \quad g_{2\mu} = k_\nu F'_{\nu\mu}.$$

Then the general vertex contains beside (2) three other symmetric second-rank tensors, bilinear in F', F and orthogonal to q:

$$\tau_{1\mu\nu} = p_\mu p_\nu F'_{\kappa\lambda}F_{\kappa\lambda}, \qquad (4)$$

$$\tau_{2\mu\nu} = (q^2\delta_{\mu\nu} - q_\mu q_\nu)F'_{\kappa\lambda}F_{\kappa\lambda}, \qquad (5)$$

$$\tau_{3\mu\nu} = g_{1\mu}g_{2\nu} + g_{1\nu}g_{2\mu} = k'_\alpha F_{\alpha\mu}k_\beta F'_{\beta\nu} + k'_\alpha F_{\alpha\nu}k_\beta F'_{\beta\mu}; \qquad (6)$$

$$q_\mu \tau_{i\mu\nu} = 0.$$

The interaction of $\tau_{2,3}$ with an external gravitational field can be immediately rewritten in a covariant form:

$$\kappa h_{\mu\nu}\tau_{2\mu\nu} = R\, F'_{\kappa\lambda}F_{\kappa\lambda}, \qquad (7)$$

$$\kappa h_{\mu\nu}\tau_{3\mu\nu} = -R_{\mu\nu\kappa\lambda}F'_{\mu\nu}F_{\kappa\lambda} \qquad (8)$$

where R and $R_{\mu\nu\kappa\lambda}$ are the scalar curvature and the Riemann tensor, respectively.

As to $\tau_{1\mu\nu}$, its interaction reduces to

$$\kappa h_{\mu\nu}\tau_{1\mu\nu} = 4R_{\mu\nu\kappa\lambda}F'_{\mu\nu}F_{\kappa\lambda} - RF'_{\kappa\lambda}F_{\kappa\lambda} + 2q^2\kappa h_{\mu\nu}T_{\mu\nu}. \qquad (9)$$

Let us note also that as reducible in this sense is the widely used covariant structure with the Ricci tensor $R_{\mu\nu}$:

$$R_{\mu\nu}F'_{\mu\lambda}F_{\nu\lambda} = \frac{1}{4}RF'_{\kappa\lambda}F_{\kappa\lambda} - \frac{1}{4}q^2\kappa h_{\mu\nu}T_{\mu\nu}. \qquad (10)$$

Thus, the most general tensor structure for the photon-graviton vertex, valid at any frequencies and momentum transfers, can be presented as

$$\frac{1}{2}\kappa h_{\mu\nu}T_{\mu\nu}f_1(q^2) + R_{\mu\nu\kappa\lambda}F'_{\mu\nu}F_{\kappa\lambda}f_2(q^2) + RF'_{\kappa\lambda}F_{\kappa\lambda}f_3(q^2). \qquad (11)$$

The lowest order QED contribution to the form-factors f_i was calculated in Refs. [6, 7]. The first nontrivial terms of their expansion in q^2 are (see [1]):

$$f_1 = 1 + \frac{11\alpha}{720\pi}\frac{q^2}{m^2}, \quad f_2 = -\frac{\alpha}{360\pi m^2}, \quad f_3 = -\frac{\alpha}{144\pi m^2}. \tag{12}$$

Similar analysis can be performed for the neutrino-graviton interaction. As to the structure with $R_{\mu\nu\kappa\lambda}$, it is kinematically impossible at all for a spin 1/2 particle. The interaction with the scalar curvature is forbidden for a two-component neutrino by helicity arguments. Therefore, the neutrino-graviton vertex is reduced effectively to interaction (2) with a form-factor. An identity analogous to (10) allows one to use an alternative form: the pointlike minimal interaction of the energy-momentum tensor with $h_{\mu\nu}$ plus the interaction of the same energy-momentum tensor with $R_{\mu\nu}$. Just this last form was used in Ref. [2].

3. Passing over at last to the photon propagation problem, let us emphasize that the form-factors in amplitude (11) depend on the momentum transfer only, but not on the photon energy itself. Of course, this property is in no way confined to the lowest order loop calculated in Refs. [6, 7], but refers to a general vertex with two on-mass-shell particles. Moreover, when light propagates in a gravitational field of a macroscopic length scale L, the typical impact parameters $\sim L$ are large as compared to the Compton wave-length m^{-1} (or any other dimensional parameter possibly involved in the radiative corrections), and therefore one can confine to the values of the form-factors f_i at $q^2 = 0$.

The lowest order correction discussed modifies the Maxwell equation in the region where $R_{\mu\nu} = R = 0$, and at $\omega L \gg 1$, as follows [1]:

$$D_\mu F^{\mu\nu} + \xi R^{\mu\nu}_{\rho\tau} D_\mu F^{\rho\tau} = 0; \; \xi = \frac{\alpha}{90\pi m^2}. \tag{13}$$

The structure $\xi R^{\mu\nu}_{\rho\tau}$ in this expression can be considered obviously as an anisotropic contribution to a refraction index which in general leads to a superluminal photon velocity.

However, the photon interaction with a gravitational background, induced by radiative corrections, certainly contains terms of higher order in curvature. How will they influence the photon propagation?

As distinct from the three-particle vertex discussed, the diagrams generating the terms nonlinear in R (from now on R is a generic notation for $R_{\mu\nu\kappa\lambda}$, $R_{\mu\nu}$, R) have more external lines and therefore certainly depend on

the photon energy. But by dimensional reasons it is quite natural to expect that it is R/ω^2 which serves as a parameter for the high-frequency expansion of the photon-background interaction [2]. Such a behaviour in the high-frequency limit is much more natural than the expansion in R/m^2 with mass singularities in the asymptotic region. In this sense the photon-graviton vertex is an exception: being ω-independent (kind of a subtraction constant in the dispersion relation), it has no choice at $q^2 \ll m^2$ but generate linear terms of the type R/m^2. Thus the terms nonlinear in R die out at $\omega \to \infty$ and do not influence the wave-front propagation. On the other hand, there is a case when those nonlinear terms are certainly inessential at any frequency: that of a weak gravitational background.

Since now the low-frequency limit can be abandoned, the situation changes as well with the problem whether the phenomenon of superluminal propagation is observable, at least in principle. The difficulty pointed out in Refs. [1, 3] is as follows. If the curvature length scale is L ($R \sim L^{-2}$), then according to Eq.(13) the velocity shift caused by the radiative correction is

$$\delta v \sim \frac{\alpha}{m^2 L^2}. \tag{14}$$

The time available for examining signals is also L. So, the corresponding position discrepancy of a signal constitutes

$$\delta s \sim L\delta v \sim \frac{\alpha}{m^2 L} \ll \frac{1}{m}. \tag{15}$$

It is not exactly clear how such a distance can be resolved with frequencies $\omega \ll m$ discussed in Refs. [1, 3]. Of course, going beyond the low-frequency approximation removes this difficulty in principle.

The arguments presented in this work give strong reasons to believe that the effect of superluminal propagation of photons in a gravitational background does exist.

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[2] By the way, this is exactly what happens when one considers the effect of the curvature tidal forces on the propagation of a finite wave packet in classical gravity [5]. Even to linear approximation the effective refraction index for a photon of the helicity λ propagating along the z axis in a space with vanishing $R_{\mu\nu}$, reduces in a local inertial frame to

$$n = 1 + \frac{1}{2\omega^2}(R_{1212} + i\lambda R_{1230}).$$

References

[1] I.T. Drummond and S.J. Hathrell, Phys.Rev. **D22** (1980) 343

[2] Y. Ohkuwa, Prog.Theor.Phys. **65** (1981) 1058

[3] R.D. Daniels and G.M. Shore, Nucl.Phys. **B425** (1994) 634

[4] M.A. Leontovich, in *L.I. Mandelstam, Lectures on Optics, Relativity and Quantum Mechanics* (Nauka, Moscow, 1972, p.308)

[5] A.D. Dolgov and I.B. Khriplovich, Zh.Eksp.Teor.Fiz. **85** (1983) 1153 [Sov.Phys. JETP **58** (1983) 671]

[6] F.A. Berends and R. Gastmans, Ann.Phys. (N.Y.) **98** (1976) 225

[7] K.A. Milton, Phys.Rev. **D15** (1977) 2149

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