СИБИРСКОЕ ОТДЕЛЕНИЕ АН СССР
ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ

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T-ODD ASYMMETRY IN HEAVY PARTICLE DECAYS

ПРЕПРИНТ 80-65

Новосибирск

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A b s t r a c t

The possibility of studying T-odd correlations of the type $A = \varepsilon^{ijk} n^i n_+^j n_-^k (\vec{n}\vec{Q})$ in e^+e^--annihilation is discussed. Here $\vec{n}$ is the direction of a beam and $\vec{n}_+, \vec{n}_-$ are the unit vectors of two opposite charge particles to be detected, and $\vec{Q}$ is some superposition of the moments of final particles. Such correlations arise due to the electromagnetic production of a τ-lepton or charmed particles with a subsequent weak decay violating CP-parity.

Calculations were made in the model of CP-invariance of the Weinberg type /1/. In this case, the effect for the semilepton decay of a D-meson ($D \to K^* \mu\nu \to K\pi\mu\nu$) is $\sim \frac{1}{\sqrt{2}} \left(\frac{m_\mu}{2\,\text{GeV}}\right)^2 \sim 2 \cdot 10^{-3}$ and, correspondingly, for the Cabibbo-suppressed decay of a τ-lepton ($\tau \to K^* \bar{\pi} \nu \to K\pi\bar{\pi}\nu$) this effect is $\sim \frac{1}{\sqrt{2}} \left(\frac{m_K}{2\,\text{GeV}}\right)^2 \sim 5 \cdot 10^{-2}$.

1. So far, the violation of CP-invariance has been revealed in the decays of neutral K-mesons only. Experimental study of T-odd asymmetry in other processes is a matter of some difficulty not only for the reason that the expected CP-parity violation is small but also due to the fact that T-odd correlations arising in the weak decays are caused by both the CP-invariance violation and final-state particle interaction. However, in the decay of the type $K^{\pm}\to\pi^0\mu^{\pm}\nu$ ($D^{\pm}\to K^0\mu^{\pm}\nu$) the masking background is much less than the effect of CP-invariance violation /2/ and onservation of the transverse muon polarization at the level $5\cdot10^{-3}$ (for K-decay) and at the level $2\cdot10^{-2}$ (for D-decay) would be indicative of T-parity nonconservation.

Unfortunately, the experimental measurement of the polarization of a fast muon is a rather complicated problem. For this reason, of interest is the study of T-odd correlations of the type $A=\varepsilon^{ijk}n^i n^j_+ n^k_-(\vec n\vec Q)$ composed from the particle moments only.

Correlations which are similar to A arise in the processes of the following type:

$$\begin{aligned} e^+(-\vec n)+e^-(\vec n)\to\gamma^*\to\big[&D^+D^-(F^+F^-,\tau^+\tau^-)+D^+D^-(F^+F^-,\tau^+\tau^-)\big] \\ &\hookrightarrow a^+(\vec n_+)+a^-(\vec n_-)+X \qquad \searrow a^-(\vec n_-)+a^+(\vec n_+)+X \end{aligned} \qquad (1)$$

Here $a^+(\vec n_+)$, $a^-(\vec n_-)$ is the particle-antiparticle pair (or two particles with opposite charges and unknown identities; for instance π^+K^-, π^-K^+), $\vec n_+,\vec n_-$ are the directions of escape of these particles, $\vec n$ is the direction os the beam, X are all other (except for a^+,a^-) particles produced in the decay and the moments of which are included in the expression for $\vec Q$.

First of all, the following important details of the processes under consideration should be noted: a) the T-odd correlation composed from the moments only occurs in the decays whose number of final particles is not less than 4, i.e. $X \geq 2$. This remark is obvious, since the T-odd correlation $\sim \varepsilon^{\alpha\beta\gamma\delta} P^{\alpha}_{D(F,T)} P^{\beta}_{+} P^{\gamma}_{-} P^{\delta}_{X}$ includes 4 independent moments. b) After integration over all directions of escape of the $D(F,T)$-particles the correlation $\sim \vec{n}_{D(F,T)} (\vec{n}_+ \times \vec{n}_-)$ is converted to the correlation $\vec{n}(\vec{n}_+ \times \vec{n}_-)(\vec{n}\vec{Q})$, just which is measured in the experiment. c) The correlations under discussion arise not only due to the violation of CP-invariance in heavy particle decays but also due to the strong and electromagnetic interactions in a final state. However, as it has been mentioned in Ref./3/, the interactions in the final state do not affect an average value of the correlation $\langle A \rangle$ over all events. Thus, the study of $\langle A \rangle$ provides information about violation of CP-invariance.

To calculate the T-odd part of an amplitude, the CP-invariance violation model of the Weinberg type /1/ is used. CP-parity nonconservation in such a model is due to the exchange of charged Higgs bosons.

In the present paper the semilepton processes will be considered; the corresponding effective quark-lepton interaction violating CP-invariance is described in Ref./2/.

Prior to calculation of the concrete processes, let us present the estimates of the expected effects for the semilepton decays of D-mesons (a) and hadron decays of τ-leptons (b).

a) In the semilepton decays of a D-meson

$$D^+ \to \pi^+(\vec{n}_+) + \pi^-(\vec{n}_-) + \mu^+ + \nu, \quad D^+ \to \rho^0 + \mu^+ + \nu, \quad \rho^0 \to \pi^+(\vec{n}_+) + \pi^-(\vec{n}_-)$$

produced in e^+e^--annihilation the correlation $\vec{n}(\vec{n}_+ \times \vec{n}_-)(\vec{n}\vec{Q})$ is measured. The term violating CP-invariance for this process is proportional to

$$G_F \frac{m_\mu m_c}{m_c^2} \langle \rho(\pi\pi) | S(P) | D \rangle m_\mu \sim G \frac{m_\mu m_c}{m_c^2} \frac{m_D^2}{m_c} m_\mu \sim$$
$$\sim (G m_D^2) \left(\frac{m_\mu^2}{m_c^2} \right) \sim 5 \cdot 10^{-3} (G m_D^2)$$

Here $S(P)$ is the scalar (pseudoscalar) current of the Higgs boson, $m_c \sim 2$ GeV, m_μ, m_c are the masses of the muon and c-quark, respectively. As it is seen from the estimate, the smallness of CP-violating contribution, as compared to the main term, arises from the fact that the $S(P)$-current changes the helicity of the particle and its matrix elements are proportional to the mass of the fermion.

The process considered above is a Cabibbo-suppressed one. However, one can consider the decays

$$D^- \to K^{0*} \mu^- \bar{\nu}, \quad K^{0*} \to K^+(\vec{n}_+) + \pi^-(\vec{n}_-) \qquad D^+ \to \overline{K^{0*}} \mu^+ \nu, \quad \overline{K^{0*}} \to K^-(\vec{n}_-) + \pi^+(\vec{n}_+)$$

proceeding with a quite large branching ratio 3÷4% /4/.

In this case, in order that the cancel of T-odd correlations due to the strong interaction may take place, the K- and π-mesons shouldn't be distinguished in the experiment. The effect is the same and is of the order of 10^{-3}.

b) In the hadron decays of a τ-lepton

$$\tau \to \rho \nu \bar{\nu}, \quad \rho \to \pi\pi$$

produced in e^+e^--annihilation the correlation $(\vec{n}\vec{Q})\vec{n}(\vec{n}_+ \times \vec{n}_-)$ is measured.

The term violating CP-parity in the processes of this type is proportional to

$$G \frac{m_\tau m_u}{m_c^2} \langle \rho(\pi\pi), \pi | S(P) | 0 \rangle m_\tau \sim G \frac{m_\tau m_u}{m_c^2} \frac{m_\pi^2}{m_u} m_\tau \sim G m_\tau^2 \left(\frac{m_\pi^2}{m_c^2} \right)$$

Here m_u is the mass of a u -quark.

Some increase (in comparison with the quantity 10^{-3}) of the CP-violating part of the amplitude occurs in the decays

$$\tau \to K^* \pi \nu \quad (\hookrightarrow K\pi) \qquad \tau \to \rho K \nu \quad (\hookrightarrow \pi\pi) \qquad \tau \to \pi\pi K \nu$$

It is due to the fact that the matrix elements $S(P)$ of the hadron state current are numerically large

$$\langle \pi K^* (\pi\pi K, \rho K) | P | 0 \rangle \sim \frac{m_K^2}{m_s}$$

(m_s - is the mass of a strange quark), and hence the effect is proportional to

$$G \frac{m_\tau m_s}{m_c^2} \langle \pi\pi K | P | 0 \rangle m_\tau \sim G m_\tau^2 \left(\frac{m_K^2}{m_c^2} \right) \sim 5 \cdot 10^{-2} (G m_\tau^2)$$

However, the branching ratios of such decays are strongly suppressed ($\sim \sin^2\theta_c$) in comparison with the corresponding decays not containing K-mesons, the gain in time for statistics is small and is of the order of

$$\sim \frac{\sin^2\theta_c}{\cos^2\theta_c} \left(\frac{m_K^2}{m_\pi^2} \right)^2 \sim 5$$

The next point is devoted to the decay $D^+ \to \overline{K^{0*}} \mu^+ \nu$. An explicit expression for the differential cross section of the process

$$e^+ e^- \to \gamma^* \to D^+ D^- \qquad\qquad (2)$$
$$D^+ \to \overline{K^{0*}} \mu^+ \nu, \quad \overline{K^{0*}} \to K^- \pi^+ \qquad + \qquad D^- \to K^{0*} \mu^- \nu, \quad K^{0*} \to K^+ \pi^-$$

is derived, which is integrated over neutrino and all intermediate states and contains T-odd correlations caused by CP-invariance violation. Analogous expressions cal also be easily derived for the F -meson or τ -lepton decays. The integrals for F, τ -decays in our calculations are identical to those appearing in the calculation of the process (2) and they are calculated in Appendix. We do not write out here the corresponding formulae for the F, τ -decays because they are cunbersome.

2. Let us divide the calculation of the differential cross section of the process (2) into three stages: a) the amplitudes of the processes $D^+ \to \overline{K^{0*}} \mu^+ \nu$ and $\overline{K^{0*}} \to K^- \pi^+$ with the given polarization of a $\overline{K^{0*}}$-meson are written; b) the differential probability of the decay $D^+ \to \overline{K^{0*}} \mu^+ \nu \to K^- \pi^+ \mu^+ \nu$ is found, which is integrated over the directions of escape of the $\overline{K^{0*}}$ and is summed over its polarizations; c) if we know the differential cross section of the production of a pair of D -mesons $d\sigma(e^+e^- \to D^+ D^-)$ and use the expression for $dw(D^+ \to K^- \pi^+ \mu^+ \nu)$, obtained at the stage (b), one can integrate over all directions of escape of the $D^+ D^-$-mesons and neutrino. As a result, we obtain an expression for $\frac{d\sigma}{d^3p_K d^3p_\pi d^3p_\mu}$, which contains the T-odd correlation $(\vec{n}\vec{q})\,\vec{n}(\vec{n}_\mu \times \vec{n}_\pi)$ due to CP-parity violation.

Let us write the transition amplitude $D^+ \to \overline{K^{0*}} \mu^+ \nu$ in the form

$$M = \frac{G\cos\theta_c}{\sqrt{2}} \bar{\nu}\gamma_\alpha(1+\gamma_5)\mu \langle \overline{K^{0*}}(e^\lambda, q) | \bar{s}\gamma_\alpha(1+\gamma_5)c | D^+(k_1) \rangle -$$
$$- i \frac{G\cos\theta_c}{m_c^2} \frac{V_1^2}{V_3^2} m_\mu m_c \bar{\nu}_L \mu_R \langle \overline{K^{0*}}(e^\lambda, q) | \bar{s}_L c_R | D^+(k_1) \rangle \qquad (3)$$

We have used here the expression for an effective CP-violating quark-lepton interaction taken from Ref./2/.

The most general expression for hadron matrix elements is of the form

$$\langle \overline{K^{0*}}(e^\lambda, q) | \bar{s}\gamma_\alpha(1+\gamma_5)c | D^+ \rangle = \frac{ih}{m_D} \varepsilon^{\alpha\beta\gamma\delta} k_{1\beta} q_\gamma e_\delta + f m_D e^\alpha + \frac{g}{m_D}(ek_1) q_\alpha + \frac{\tilde{g}}{m_D}(ek_1) k_1^\alpha \qquad (4)$$

$$\langle \overline{K^{0*}}(e^\lambda, q) | \bar{s}_L c_R | D^+(k_1) \rangle = -(ek_1) t \qquad (5)$$

Here $h, f, g, \tilde{g}, t$ are dimensionless formfactors of the corresponding transitions which depend on the momentum transfer $(k_1 - q)^2$;

and k_1, q are the moments of the D - and $\overline{K^{0*}}$-mesons; m_μ, m_c - are the masses of the muon and c-quark, respectively; e^λ is the vector of the polarization $\overline{K^{0*}}$, $m_c \sim 2$ GeV.

The second term in Eq.(3) is due to the exchange of a Higgs boson and leads to the violation of CP-invariance. The equation (3) includes the quantity V_1^2/V_3^2, the ratio of vacuum mean Higgs fields. Although V_1^2/V_3^2 is an unknown parameter of the model, we do not see any reasons for which this parameter could be very large or too small. Therefore, in numerical estimates we shall assume V_1^2/V_3^2 to be equal to unity.

The T-odd correlation in the expression for the probability of the decay $D^+ \to \overline{K^{0*}} \mu^+ \nu$ is a result of the interference of a vector current (which is described by the formfactor h (4) and a pseudoscalar current described by the formfactor t (5). The remaining terms do not give the contribution to the effect under discussion and describe the ordinary T-invariant terms.

The energy spectrum of the $D^+ \to \overline{K^{0*}} \mu^+ \nu$ decay, its branching ratio, and other problems (related to the formfactors $f, g, \tilde{g}$), not dealing with the CP-invariance violation, was a matter of interest of a number of papers /5/. For this reason, we shall not dwell upon the discussion of the corresponding T-invariant terms and set $g = \tilde{g} = 0$, and the formfactor f is conserved with the aim of estimating the scale of a relative value of the CP-violating term in comparison to the CP-invariant term.

Taking into account the fact that the transition amplitude $\overline{K^{0*}}(e^\lambda, q) \to K^-(P_K) + \pi^+(P_\pi)$ is representable in the form

$$M(\overline{K^{0*}} \to K^- \pi^+) = R\, e_\lambda (P_K - P_\pi) \qquad (6)$$

(R is some constant), let us write out the differential probability of the $D^+ \to K^- \pi^+ \mu^+ \nu$ decay proceeding via the $\overline{K^{0*}}$-resonance:

$$dW = B(\overline{K^{0*}} \to K^- \pi^+) \frac{3 G^2 f^2 \delta[(P_K + P_\pi)^2 - m_{K^*}^2]\, m_{K^*}^4 m_D}{(2\pi)^6 (m_{K^*}^2 - m_K^2)^3} \times$$

$$\times \Big\{ \frac{1}{2 m_{K^*}^2} (m_{K^*}^2 - m_K^2)^2 (P_\mu k_1 - P_\mu \ell) + (k_1 m)(P_\mu m) - (P_\mu m)^2 - \qquad (7)$$

$$- \frac{1}{\sqrt{2}} \tau \frac{m_\mu^2}{m_c^2} \frac{V_1^2}{V_3^2} \frac{(k_1 m)}{m_D^2} \varepsilon^{\alpha\beta\gamma\delta} k_{1\alpha} P_{\mu\beta} P_{K\gamma} P_{\pi\delta} \Big\} \delta^4(k_1 - \ell - P_\nu) \frac{d^3 P_K}{E_K} \frac{d^3 P_\pi}{E_\pi} \frac{d^3 P_\mu}{E_\mu} \frac{d^3 P_\nu}{E_\nu}$$

Here P_K, P_π, P_μ, P_ν are the four-moments K^-, π^+, μ^+, ν, respectively; $\ell^\alpha = P_K^\alpha + P_\pi^\alpha + P_\mu^\alpha$, $m^\alpha = (P_K - P_\pi)^\alpha - \frac{m_K^2}{m_{K^*}^2}(P_K + P_\pi)^\alpha$, $B(\overline{K^{0*}} \to K^- \pi^+)$ is the corresponding branching ratio of the $\overline{K^{0*}}$-decay, $\tau = \frac{2ht}{f^2} \frac{m_c}{m_D}$.

In derivation of Eq.(7) we have took into account the fact that the probability of the $\overline{K^{0*}} \to K^- \pi^+$ decay averaged over initial polarizations of the $\overline{K^{0*}}$-meson is equal to (the pion mass is neglected throughout):

$$W(\overline{K^{0*}} \to K^- \pi^+) = \frac{R^2 m_{K^*}}{48\pi} \Big(1 - \frac{m_K^2}{m_{K^*}^2}\Big)^3 \qquad (8)$$

Here R is the same constant as that in Eq.(6). From (7) it is seen that a relative value of the CP-violating term is

$$\sim \tau \frac{m_\mu^2}{m_c^2} \frac{V_1^2}{V_3^2}$$

The formfactors h, f have been defined in various models /5/ (numerically $h/f \sim 1$) and the formfactor t can be estimated by taking the divergence from the expression (4). With $g, \tilde{g}$ neglected, we have $t \approx \frac{m_D}{m_c} h f$. Hence, we expect that $\tau \approx \frac{2ht}{f^2} \frac{m_c}{m_D} \sim 1$ with an accuracy of up to the formfactor 1.5÷2.

Uncertainty in estimation of the quantity τ is connected with both the noticeable dependence of the functions f, h, t on the square of the transmitted moments ($0 \le (k_1-q)^2 \le m_{\mu}^2$) and the non-unique, model-dependent predictions for the formfactors.

Note, that only the element $\sim \vec{k}_1(\vec{p}_K \times \vec{p}_\pi)$ is a matter of our interest in the expression (7), since it is that which leads to the structure under study $(\vec{n}\vec{Q})\vec{n}(\vec{n}_+ \times \vec{n}_-)$ after integration over d^3k_1 . (Here the directions of escape of $K^-(\pi^+)$-mesons are denoted by $\vec{n}_-(\vec{n}_+)$.) The interest to this correlation, as it has been mentioned in Introduction, is due to the fact that strong interactions do not influence the mean value of the structure $(\vec{n}\vec{Q})\vec{n}(\vec{n}_+ \times \vec{n}_-)$ /3/.

Other correlations from the expression (7) (for instance, $\sim \vec{k}_1(\vec{p}_\mu \times \vec{p}_\pi)$) are of less interest, because the masking background of strong interactions exceeds considerably the effect of CP-invariance violation.

It should be mentioned that in order to measure the correlation $\vec{k}_1(\vec{p}_K \times \vec{p}_\pi)$, it is desirable to have quite fast D - mesons, since the smallness of the effect arises due to both the exchange of a Higgs boson ($\sim m_\mu^2/m_c^2$) and the smallness proportional to the moment $\sim |\vec{k}_1|$ of a D -meson (for instance, $|\vec{k}_1|/m_D \sim 1/4$ in the $\psi''(3770) \to D\bar{D}$ decay).

Let us turn now to the calculation of the differential cross section of the process (2). To this end, $d\sigma(e^+e^- \to \gamma^* \to D^+D^-)$ is taken in the form

$$d\sigma = \frac{16\alpha^2 F^2(P^2)}{P^6}\left[(k_1P_1)(k_1P_1) - \frac{1}{4}m_D^2 P^2\right]\frac{d^3k_1}{2\omega_1}\frac{d^3k_2}{2\omega_2}\,\delta^4(P-k_1-k_2) \qquad (10)$$

Here $P = P_1 + P_2$ and the electromagnetic formfactor is determined as

$$\langle D^+(k_1)D^-(k_2)|J_\mu^{el}|0\rangle = (k_1-k_2)_\mu F(P^2) \qquad (11)$$

With the expression (7) for dW and expression (10) for $d\sigma$ both taken into account, let us write out the differential cross section of the process (2), which is integrated over neutrino and intermediate states as follows:

$$\begin{aligned} &d\sigma(e^+e^- \to \gamma^* \to D^+D^- \\ &\qquad\qquad \hookrightarrow \overline{K^{0*}}\mu^+\nu \to K^-\pi^+\mu^+\nu) = \\ &= B(D^+ \to \overline{K^{0*}}\mu^+\nu)B(\overline{K^{0*}} \to K^-\pi^+)\frac{16\alpha^2F^2(P^2)}{P^6}\frac{36}{\pi^3}\frac{\delta[(p_K+p_\pi)^2 - m_{K^*}^2]}{m_D^6\left(1-\frac{m_K^2}{m_{K^*}^2}\right)J(z)} \times \\ &\Big\{\frac{1}{2}\frac{(m_{K^*}^2-m_K^2)^2}{m_{K^*}^2}\Big[(p_\mu l)^\alpha P_1^\beta P_2^\gamma I^{\alpha\beta\gamma} - \frac{1}{4}m_D^2P^2P_1^\alpha I^\alpha - (p_\mu l)P_1^\alpha P_2^\beta I^{\alpha\beta} + \frac{1}{4}m_D^2P^2(p_\mu l)I\Big] + \\ &+ (p_\mu m)\Big[P_1^\alpha P_2^\beta m^\gamma I^{\alpha\beta\gamma} - \frac{1}{4}m_D^2P^2m^\alpha I^\alpha\Big] - (p_\mu m)^2\Big[P_1^\alpha P_2^\beta I^{\alpha\beta} - \frac{1}{4}m_D^2P^2 I\Big] + \\ &+ \frac{\tau}{\sqrt{2}}\frac{m_\mu^2}{m_c^2}\frac{V_1^2}{V_3^2}\frac{1}{m_D^2}\varepsilon^{\alpha\beta\gamma\delta}p_\mu^\beta p_K^\gamma p_\pi^\delta\Big[m^\rho P_1^\sigma P_2^\chi I^{\alpha\rho\sigma\chi} - \frac{1}{4}m_D^2P^2m^\rho I^{\rho\alpha}\Big]\Big\} \times \\ &\times \frac{d^3\vec{p}_K\, d^3\vec{p}_\pi\, d^3\vec{p}_\mu}{E_K\,E_\pi\,E_\mu} \end{aligned} \qquad (12)$$

Here $J(z) = 1/8 + 9z^2 - 8z^3 - 9/8z^4 - 9/2z^2\ln\frac{1}{z} - 6z^3\ln\frac{1}{z} \approx 0.05$, $z = \frac{m_{K^*}^2}{m_D^2} \sim 0.23$

$B(D^+ \to \overline{K^{0*}}\mu^+\nu)$, $B(\overline{K^{0*}} \to K^-\pi^+)$ are the corresponding branching ratios of the corresponding decays, I^α – the integrals taken in the following way:

$$(I, I^\alpha, I^{\alpha\beta}, I^{\alpha\beta\gamma}, I^{\alpha\beta\gamma\delta}) = \int\frac{d^3k_1}{2\omega_1}\frac{d^3k_2}{2\omega_2}\frac{d^3k_\nu}{2E_\nu}\,\delta^4(P-k_1-k_2)\,\delta^4(k_1-l-p_\mu)(1, k_\alpha, k_{\alpha\beta}, \ldots) \qquad (13)$$

The integrals (13) are rather cumbersome and their calculation is made in Appendix.

In deviation of Eq.(12), the following expression for the probability of the $D^+ \to \overline{K^{0*}}\mu^+\nu$ decay has been used:

$$W(D^+ \to \bar{K}^{0*}\mu^+\nu) = \frac{f^2 G^2 \cos^2\theta_c\, m_D^5}{3\cdot 2^7 \pi^3}\frac{1}{z}\,\mathcal{J}(z) \qquad (14)$$

In the formula (14) we neglect the mass of a muon and also the term proportional to $\sim h^2$ (the corresponding contribution is of the order of 15% with respect to the total probability of the process).

It shoudl be emphasized once again that from the whole set of terms in Eq.(12) violating CP-invariance of interest are only those terms which are of the form $(\vec{n}\vec{Q})\vec{n}(\vec{P}_{\pi}\times\vec{P}_{\mu})$ in the three-dimensional description. It is obvious that the term with $I^{\rho\alpha}$ cannot lead to such a correlation and in the term proportional to $I^{\alpha\rho\sigma\chi}$ the contribution is given only by those components which contain the following structures:

$$(g^{\alpha\sigma}t_\rho t_\chi + g^{\alpha\chi}t_\rho t_\sigma),\ (g^{\alpha\sigma}g^{\rho\chi} + g^{\alpha\chi}g^{\rho\sigma}),\ (g^{\alpha\sigma}t^\chi P^\rho + g^{\alpha\chi}P^\rho t^\sigma)$$

Here $t_\beta = l_\beta - \frac{lP}{P^2}P_\beta$. It is accounted for the fact that the index α in the expression $m^\rho P_1^\sigma P_2^\chi I^{\alpha\rho\sigma\chi}$ from Eq.(12) must belong to the combination of moments $(P_1-P_2)_\alpha = (0, \vec{h}2E)$ rather than $P_\alpha, t_\alpha, m_\alpha$; otherwise we do not obtain the required correlation. Taking into account this circumstance and using Eq.(12), let us write the final form of the T-odd correlation under study (the common factor in front of the braces in (12) is omitted):

$$\frac{z}{\sqrt{2}}\frac{m_\mu^2}{m_c^2}\frac{V_1^2}{V_3^2}\frac{1}{m_D^2}\frac{I}{16}\frac{1}{2}\,\varepsilon^{\alpha\beta\gamma\delta}(P_1-P_2)^\alpha (P_\mu)^\beta P_K^\gamma P_\pi^\delta \times$$
$$\times\left\{R\left(X^2-\frac{R}{2t^2}\right)(mt)\left((P_2t)-(P_1t)\right)+\frac{1}{2}R^2\left((P_2m)-(P_1m)\right)+XR(mP)\left((P_2t)-(P_1t)\right)\right\} \qquad (15)$$

Here I, X, R are determined and calculated in Appendix and are of the following form:

$$I = \frac{\pi}{4}\frac{1}{\sqrt{-t^2P^2}}, \quad X = \frac{l^2 - lP + m_D^2}{t^2}, \quad R = \frac{1}{2}\left(4m_D^2 - P^2 - X^2t^2\right) \qquad (16)$$

3. In conclusion, let us dwell upon some peculiarities of calculating the asymmetry in the $\tau \to K\pi^+\pi^-\nu$ decay.

a) The process $\tau^- \to K^-\pi^+\pi^-\nu$ can go through the following channel:

$$\tau^- \to \nu Q^-,\ Q^- \to K^-\pi^+\pi^- \qquad \tau^- \to \nu\pi^-\bar{K}_0^*,\ \bar{K}_0^* \to K^-\pi^+ \qquad \tau^- \to \nu K^-\rho^0,\ \rho^0 \to \pi^+\pi^- \qquad \tau^- \to \nu K^-\pi^+\pi^-$$

The $\tau^- \to \nu Q_1^-$ decay is likely to take place (the corresponding branching ratio is $\sim 0.4\%$ /6/). However, this channel does not give the contribution to the T-odd asymmetry, because the matrix element leading to violation of CP-invariance is equal to zero, $\langle Q|\bar{s}(P)|0\rangle = 0$ (the Higgs boson cannot be converted to a vector particle). Due to this fact, the time needed for collection of the statistics required increases highly ($K^-\pi^+\pi^-$ outside the region Q^-). The remaining channels give the contribution $\sim \frac{m_\pi^2}{m_c^2}$ to the effects under discussion and the final result (if the experimentally possible channels are not distinguished) contains the sum of all contributions with allowance for the branching ratios of each of the channels. Remind that there is no similar problem for the $D \to K^*\mu\nu$ decay, since the nonresonant contribution to the $D \to K\pi\mu\nu$ decay is much less than the resonant contribution /5/.

b) For a process of the type $\tau \to K^{0*}\pi\nu$, a quantity analogous to z (recall that z is expressed via the ratio of the interference, CP-violating, and the main, T-even, terms) may be estimated by the method used in Ref./7/, by assuming the momentum transfer $q^2 \sim m_\tau^2$ to be a quite large quantity. In this case, it is easy to estimate the quantity $z \sim \frac{m_\tau^2}{m_c^2}$, which determines the degree of CP-invariance violation.

c) As it has been earlier mentioned, the T-odd correlation in the $D \to K^{*}\mu\nu$ decay has an additional smallness proportional to $|\vec{k}_1|$ (k_1 - is the moment of the produced heavy particle) in comparison with the main term. For a τ -lepton, the analogous kinematic suppression is more significant and proportional to $|\vec{k}_1|^3$. It is accounted for by the fact that the matrix element squared for the process of producing scalar particles is proportional to $|\vec{k}_1|^2$, and for the spinor particles ~ 1. At the same time, in both cases the term which depends on the angles and leads, after integration, to a necessary correlation has the smallness proportional to $(\vec{k}_1\vec{P}_3)^2 \sim |\vec{k}_1|^2$. Thus, the measurement of the T-odd correlation in the τ -lepton decay is more reasonable at quite high energies when $|\vec{k}_1| \sim m_\tau$.

The author is grateful to A.I.Vainshtein, I.B.Khriplovich, and V.L.Chernyak for helpful discussions and critical remarks.

APPENDIX

The results of integration of the expression (13) are here presented:

$$I = \int \frac{d^3k_1}{2\omega_1}\frac{d^3k_2}{2\omega_2}\frac{d^3P_3}{2E_3}\,\delta^4(P-k_1-k_2)\,\delta^4(k_1-\ell-P_3) = \frac{\pi}{4\sqrt{-t^2P^2}} \qquad (A.1)$$

The vector t_α is defined as follows:

$$t_\alpha = \ell_\alpha - \frac{\ell P}{P^2}P_\alpha$$

Convenience in its use is accounted by the fact that $(tP) = 0$. In addition, in the centre-of-mass of the colliding particles $t_\alpha = (0\,\vec{\ell})$.

$$I_\alpha = \frac{1}{2}I(P_\alpha + Xt_\alpha) \qquad (A.2)$$

Here $\quad X = \frac{1}{t^2}(\ell^2 - \ell P + m_D^2)$

$$I_{\alpha\beta} = \frac{I}{4}\Big[g_{\alpha\beta}R + P_\alpha P_\beta\Big(1-\frac{R}{P^2}\Big) + t_\alpha t_\beta\Big(X^2 - \frac{R}{t^2}\Big) + X(P_\alpha t_\beta + P_\beta t_\alpha)\Big] \qquad (A.3)$$

Here $\quad R = \frac{1}{2}(4m_D^2 - P^2 - X^2t^2)$

$$I_{\alpha\beta\gamma} = \frac{I}{8}\Big\{\Big(1-\frac{3R}{P^2}\Big)P_\alpha P_\beta P_\gamma + X\Big(X^2-\frac{3R}{t^2}\Big)t_\alpha t_\beta t_\gamma + R(P_\alpha g_{\beta\gamma} + \alpha\to\beta\to\gamma) +$$
$$+ XR(t_\alpha g_{\beta\gamma} + \alpha\to\beta\to\gamma) + \Big(X^2 - \frac{R}{t^2}\Big)(P_\alpha t_\beta t_\gamma + \alpha\to\beta\to\gamma) + \qquad (A.4)$$
$$+ X\Big(1-\frac{R}{P^2}\Big)(t_\alpha P_\beta P_\gamma + \alpha\to\beta\to\gamma)\Big\}$$

The symbol $\alpha\to\beta\to\gamma$ denotes the summetrization over all indices:

$$I_{\alpha\beta\gamma\delta} = \frac{I}{16}\Big\{P_\alpha P_\beta P_\gamma P_\delta\Big[6\Big(1-\frac{R}{2P^2}\Big)^2 - 5\Big] + t_\alpha t_\beta t_\gamma t_\delta\Big[6\Big(X^2-\frac{R}{2t^2}\Big)^2 - 5X^4\Big] +$$
$$+ R\Big(1-\frac{R}{2P^2}\Big)\Big[P_\alpha P_\beta g_{\gamma\delta} + \alpha\to\beta\to\gamma\to\delta\Big] + R\Big(X^2 - \frac{R}{2t^2}\Big)\Big[t_\alpha t_\beta g_{\gamma\delta} + \alpha\to\beta\to\gamma\to\delta\Big] + \qquad (A.5)$$

$$+\frac{1}{2}R^2\left[g_{\alpha\beta}g_{\gamma\delta}+\alpha\to\beta\to\gamma\to\delta\right]+\left[\frac{5}{2}\frac{R^2}{\rho^2t^2}+\chi^2-\frac{4m_\delta^2R}{\rho^2t^2}\right]\left(\rho_\alpha\rho_\beta t_\gamma t_\delta+\alpha\to\beta\to\gamma\to\delta\right)+$$

$$+\chi\left(\chi^2-\frac{3R}{t^2}\right)\left(\rho_\alpha t_\beta t_\gamma t_\delta+\alpha\to\beta\to\gamma\to\delta\right)+\chi\left(1-\frac{3R}{\rho^2}\right)\left(t_\alpha\rho_\beta\rho_\gamma t_\delta+\alpha\to\beta\to\gamma\to\delta\right)+$$

$$+\chi R\left(\rho_\alpha t_\beta g_{\gamma\delta}+\alpha\to\beta\to\gamma\to\delta\right)\Big)$$

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